Commensurability Oscillations in Magnetoplasmons of a Density-Modulated Two-Dimensional Electron Gas

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The intra-Laudau-band magnetoplasma spectrum of a periodically density-modulated two-dimensional electron gas is shown to exhibit unusual oscillatory behavior. Such oscillations (periodic in $1/B$) are seen to arise from the commensurability of the spatial period of the modulation with the cyclotron-orbit size at the Fermi level.

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Chaplik[1] anticipated that a two-dimensional electron system under a periodic density modulation should exhibit interesting magnetotransport and response properties. Not until very recently, however, has such a structure of high quality been realized with the advent of microfabrication technology.[2,3] Magnetotransport measurements performed on such systems (with the magnetic field perpendicular to the 2D electron gas) by Weiss *et al.* and others[4] revealed novel magnetoresistance oscillations, similar to the Schubnikov–de Haas (SdH) oscillations, but with very different periods and a much weaker temperature dependence. It was shown[4] that this oscillatory behavior is a consequence of the commensurability of the length scales of the system, the cyclotron radius R_c and the modulation period a, governed by the resonance condition

$$2R_c=(j+\phi)a,\quad j=1,2,\ldots, \tag{1}$$

where ϕ is a constant phase shift. The spatial density modulation broadens the Landau levels into minibands whose widths oscillate as functions of the magnetic field. The electronic states are thus substantially altered,[5,6] resulting in a modulated density of states, as shown by magnetocapacitance measurements.[7] It is to be expected that the observed oscillatory behavior of the magnetoresistance is but one manifestation of the density-of-states modulation, as the electronic density of states governs a wide variety of dc and frequency-dependent response and transport phenomena, as well as thermodynamic properties (e.g., electronic specific heat, magnetic susceptibility) of the system. One of the most important of these properties is the collective excitation spectrum (plasmons) of the 2D electrons. We present here our investigation of the magnetoplasmons in a density-modulated two-dimensional electron gas (2DEG), in conjunction with an analysis of the dynamic, nonlocal dielectric response function, paying particular attention to the modulation-induced effects. Our analysis shows that the magnetoplasma spectrum clearly reflects the electronic density-of-states modulation, in the form of oscillating magnetoplasma frequencies, satisfying the commensurability condition of Eq. (1).

We consider a 2DEG of unmodulated areal density n_0, electron effective mass m, and charge $-e$, contained in the x-y plane, subject to a uniform magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$. In the Landau gauge [with the vector potential given by $\mathbf{A}=(0,Bx,0)$], the unperturbed wave functions have plane-wave structure in the y direction, and are subject to the well-known oscillator Hamiltonian in the x direction ($\hbar=c=1$ here)

$$H_0=-(1/2m)d^2/dx^2+\tfrac{1}{2}m\omega_c^2(x-x_0)^2, \tag{2}$$

where $\omega_c=eB/m$ is the cyclotron frequency, and $x_0=-k_y/m\omega_c$ is the coordinate of the cyclotron orbit center. These eigenstates may be written as

$$\Psi_{nk_y}(x,y)=L^{-1/2}\exp(ik_yy)u_n(x,x_0),$$

where L is a normalization length in the y direction along which the motion is free-electron-like, and $u_n(x,x_0)$ is the normalized wave function of a harmonic oscillator centered at x_0. The spatial modulation is modeled by a sinusoidal potential in the x direction, $H'=V_0\cos(2\pi x/a)$. In accordance with experimental conditions[4] the modulation potential may be treated as a perturbation, with the constant V_0 about an order of magnitude smaller than the Fermi energy E_F. Standard perturbation theory yields the energy eigenvalues to first order in H' as[4]

$$E_n(x_0)=(n+\tfrac{1}{2})\omega_c+V_n\cos(2\pi x_0/a), \tag{3}$$

where $V_n=V_0\exp(-X/2)L_n(X)$, with $X=(2\pi/a)^2/2m\omega_c$, and $L_n(X)$ is a Laguerre polynomial. With this, the formerly sharp Landau levels [defined by the eigenvalues of the unperturbed Hamiltonian H_0, $E_n=(n+\tfrac{1}{2})\omega_c$, degenerate with respect to k_y] are now broadened into minibands (called Landau bands henceforth) by the potential modulation, and the k_y degeneracy is lifted. Furthermore, the Landau bandwidth ($\sim|V_n|$) oscillates as a function of n, since $L_n(X)$ is an oscillatory function of its index n.[8] These observations have been set forth by the authors of Ref. 4 in the interpretation of their magnetoresistance data. As we have indicated above, the modulation-induced change in the electronic density of states should also be manifested in the dielectric response and collective excitations of the 2DEG.

The dynamic and static response properties of an electron system are all embodied in the structure of the density-density correlation function. For the present case of a density-modulated 2DEG in a perpendicular magnetic field, we have determined it in the random-phase approximation (RPA) treating the spatial modulation potential perturbatively, with the result[9]

$$\pi(q_x,q_y,\omega)=\pi_0(q_x,q_y,\omega)/[1-U(q)\pi_0(q_x,q_y,\omega)]\,. \tag{4}$$

Here, $U(q)=2\pi e^2/\kappa q$, $q=(q_x^2+q_y^2)^{1/2}$, κ is the background dielectric constant, and $\pi_0(q_x,q_y,\omega)$ is the noninteracting electron density-density correlation function given by

$$\pi_0(q_x,q_y,\omega)=(m\omega_c/\pi a)\sum_{n,n'}C_{nn'}(q^2/2m\omega_c)\int_0^a dx_0[f(E_n(x_0+x_0'))-f(E_{n'}(x_0))] \times[E_n(x_0+x_0')-E_{n'}(x_0)+\omega+i\delta]^{-1}\,, \tag{5}$$

where $f(E)$ is the Fermi-Dirac distribution function, $x_0'=-q_y/m\omega_c$, and

$$C_{nn'}(x)=(n_2!/n_1!)x^{n_1-n_2}e^{-x}[L_{n_2}^{n_1-n_2}(x)]^2\,,$$

with $n_1=\max(n,n')$, $n_2=\min(n,n')$, and $L_n^l(x)$ an associated Laguerre polynomial. Equation (5) includes a factor of 2 arising from spin degeneracy which is not resolved in the magnetic field range (0.4–0.8 T) investigated in the experiments.[4] The Fermi energy of the 2DEG is fixed, for a given temperature and magnetic field, by the unperturbed 2D electron density through the relation $n_0=(m\omega_c/\pi a)\sum_n\int_0^a dx_0 f(E_n(x_0))$, where the sum over n runs through all the occupied Landau bands.

The real and imaginary parts of the density-density correlation function as given by Eqs. (4) and (5) are the essential ingredients for theoretical considerations of such diverse problems as high-frequency and steady-state transport, static and dynamic screening, and correlation phenomena. Whereas the RPA treatment presented here is by its nature a high-density approximation which has been eminently successful in analyzing 3D bulk systems, it has also enjoyed much success in the study of collective excitations in lower-dimensional systems such as planar semiconductor superlattices[10-12] and quantum-wire structures,[13] both with and without an applied magnetic field. Such success has been convincingly attested by the excellent agreement of RPA predictions of plasmon spectra with experiments.[14-20] In this connection it should be pointed out that the exchange effect at finite wave vectors ought to be considered, as indicated by Kallin and Halperin.[21] However, in our study of plasma excitations here we shall be mainly concerned with the long-wavelength regime, where exchange contributions are negligible. Furthermore, in the absence of any theory of magnetoplasmons for the case of a density-modulated 2DEG, our RPA prediction will serve as a benchmark for more sophisticated calculations.

The plasma modes are readily furnished by the singularities of the function $\pi(q_x,q_y,\omega)$, namely the roots of the longitudinal dispersion relation $1-U(q)\mathrm{Re}\pi_0(q_x,q_y,\omega)=0$. Employing Eq. (5) this becomes (P denotes principal value integral)

$$1=(2\pi e^2/\kappa q)(m\omega_c/\pi a)\sum_{n,n'}C_{nn'}(q^2/2m\omega_c)\mathrm{P}\int_0^a dx_0[f(E_n(x_0+x_0'))-f(E_{n'}(x_0))][E_n(x_0+x_0')-E_{n'}(x_0)+\omega]^{-1}\,, \tag{6}$$

along with the condition $\mathrm{Im}\pi_0(q_x,q_y,\omega)=0$ to ensure long-lived excitations. These normal modes originate from two kinds of virtual electronic transitions: those involving different Landau bands (inter-Landau-band plasmons), and those within a single Landau band (intra-Landau-band plasmons). Inter-Landau-band plasmons encompass the local principal 2D magnetoplasma mode and the Bernstein-like plasma resonances,[22] all of which involve excitation frequencies greater than the Landau band separation ($\sim\omega_c$). On the other hand, intra-Landau-band plasmons resonate at frequencies comparable with the bandwidths, and the existence of this new class of modes is conditioned upon the introduction of finite widths to the Landau levels. The occurrence of such intra-Landau-band plasmons is accompanied by regular oscillatory behavior (in $1/B$) of the SdH type, as shown by Que and Kirczenow[23] in the case of a tunneling planar superlattice, where the wave-function overlap of electrons on adjacent quantum wells provides the broadening mechanism. Such SdH-type oscillations result from the emptying out of electrons from successive Landau bands when they pass through the Fermi level as the magnetic field is increased. The amplitude of the SdH oscillations is a monotonic function of the magnetic field, when the Landau bandwidth is independent of the band index n. On the contrary, in the present case of a density-modulated 2DEG, the Landau bandwidths oscillate as functions of the band index n. It is to be expected that such oscillating bandwidths should correspondingly affect the plasmon spectrum of the intra-Landau-band type, with the consequence that yet another type of oscillation will be present, also periodic in $1/B$, but with a different period and smaller amplitude, entirely similar to those discovered in the magnetoresistance[4] and capacitance[7] measurements.

The longitudinal dispersion relation given by Eq. (6) contains all the aforementioned plasma modes. However, the inter-Landau-band modes and the intra-Landau-band modes are, in general, coupled for arbitrary magnetic field strengths. A complete examination of Eq. (6) for all wave vectors, frequencies, and magnetic field regimes is only feasible numerically. In the present case of weak modulation ($V_0/E_F \ll 1$) and considering the long-wavelength limit, it is possible to solve Eq. (6) analytically for zero temperature. Expanding the coefficient $C_{nn'}(q^2/2m\omega_c)$ to lowest order in its argument, and the integrand in Eq. (6) to lowest order in $|V_n/(\omega - E_n + E_{n'})|$, the dispersion relation becomes

$$1 = \omega_{p,2D}^2/(\omega^2 - \omega_c^2) + \tilde{\omega}^2/\omega^2 , \quad (7)$$

where $\omega_{p,2D}^2 = 2\pi n_0 e^2 q/\kappa m$ is the ordinary 2D plasma frequency and

$$\tilde{\omega}^2 = (8m\omega_c e^2/\kappa\pi q)\sin^2(\pi x_0'/a) \times \sum_n |V_n|(1-\Delta_n^2)^{1/2}\Theta(1-\Delta_n) , \quad (8)$$

with $\Delta_n = |(E_F - E_n)/V_n|$, and $\Theta(x)$ the Heaviside unit step function. In obtaining Eq. (7) terms proportional to $|V_{n+1} - V_n|$ have been neglected, since they are in general much smaller than $|V_n|$. Equation (7) yields two normal-mode frequencies $\omega_1^2 = \omega_c^2 + \omega_{p,2D}^2$ and $\omega_2^2 = \tilde{\omega}^2$, with corrections of order $\tilde{\omega}^2/\omega_c^2$ and $\tilde{\omega}^2/\omega_{p,2D}^2$. ω_1 is the well-known local 2D principal magnetoplasma frequency [Bernstein modes are not represented because of the commitment to low wave numbers, but are described by Eq. (6) at higher wave numbers], and ω_2 is the intra-Landau-band plasma frequency. So long as $|V_n| < \omega_c$, mixing of these modes is small. In the frequency regime $\omega_c > \omega \sim |V_n|$, only the intra-Landau-band mode ($\tilde{\omega}$) will be excited. In deriving the expression for $\tilde{\omega}$ [Eq. (8)] we have employed the condition $\omega \gg |E_n(x_0 + x_0') - E_n(x_0)|$ as $x_0' \to 0$, which entails a relation between the frequency and the Landau level broadening:

$$\omega \gg |2V_n \sin(\pi x_0'/a)\sin[(2\pi/a)(x_0 + x_0'/2)]| .$$

This ensures that $\mathrm{Im}\,\pi_0(q_x,q_y,\omega)$ vanishes identically, and the intra-Landau-band magnetoplasmons are undamped. For a given V_n, this can invariably be achieved with a small but nonzero q_y (recall that $x_0' = -q_y/m\omega_c$).

The intra-Landau-band plasma frequency as given by Eq. (8) is shown graphically in Fig. 1, as a function of the inverse magnetic field, using parameters pertaining to the experiments of Ref. 4 (a 2DEG at a GaAs-AlGaAs heterojunction): $m = 0.07m_e$, $\kappa = 12.9$, $n_0 = 3.16\times10^{15}$ m^{-2}, $a = 382$ nm, and $V_0 = 1.0$ meV; also, we take $q_x = 0$ and $q_y = 0.01k_F$, with $k_F = (2\pi n_0)^{1/2}$ being the Fermi wave number of the unmodulated 2DEG in the absence of a magnetic field. The modulation-induced oscillations are clearly in evidence, superposed on the sharp SdH-type oscillations. As expected, the former amplitude modulation has a longer period and much reduced amplitude. Also shown in Fig. 1 (inset) are the two modes calculated exactly from Eq. (7), including the coupling between the inter-Landau-band mode and the intra-Landau-band mode. It is seen that the former has superimposed on it the SdH oscillations, while the latter appears with further reduced amplitude,

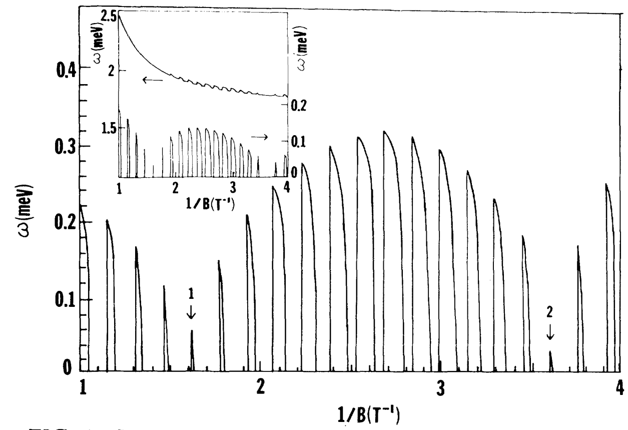

FIG. 1. Intra-Landau-band plasma frequency as a function of the inverse magnetic field. $q_x = 0$, $q_y = 0.01k_F$; $n_0 = 3.16\times10^{15}$ m^{-2}, $a = 382$ nm, $V_0 = 1.0$ meV. The arrows marked 1 and 2 correspond to the values of $j = 1$ and $j = 2$, respectively, in Eq. (1), with phase shifts of $\phi = -0.22$ for the former and $\phi = -0.21$ for the latter. Inset: The two roots of Eq. (7) calculated using the same parameters given above.

both resulting from the coupling. The two modes are nevertheless well resolved, separated by a finite gap.

A closer analytic examination of Eq. (8) reveals the origins of the two types of oscillations. With $\omega_c > |V_n|$, the Heaviside function vanishes for all but the highest occupied Landau band, corresponding, say, to the band index N. It follows that the sum over n is trivial, and the plasma frequency is simply given as $\tilde{\omega} \sim |V_N|^{1/2}(1 - \Delta_N^2)^{1/4}\Theta(1 - \Delta_N)$. The analytic structure primarily responsible for the sharp SdH-type oscillations is the function $\Theta(1 - \Delta_N)$, which jumps periodically from zero (when the Fermi level is above the highest occupied Landau band) to unity (when the Fermi level is contained within the highest occupied Landau band), with a period $\Delta_{\mathrm{SdH}}(1/B) = e/mE_F$. On the other hand, the periodic modulation of the amplitude of the SdH-type oscillation displayed in Fig. 1 is largely a consequence of the oscillatory nature of the factor $|V_N|^{1/2}$, which has been shown[4] to exhibit commensurability oscillations governed by Eq. (1). In view of this, it is not surprising that the period of the amplitude modulation in Fig. 1, $\Delta(1/B) = 2.07$ T^{-1}, is exactly the same as that predicted with Eq. (1), which is $\Delta(1/B) = ea/2k_F$, using $R_c = k_F/eB$ at the Fermi level.

Experimental verification of the commensurability oscillations in the magnetoplasmon spectrum should not in principal involve greater difficulty than that of the magnetoresistance[4] and capacitance[7] measurements. Typical frequencies of these plasmons are of the order of 0.1–1 meV, which should be measurable with available experimental means such as far-infrared spectroscopy. To clarify the regime in which our prediction may be pertinent to such experiments we discuss the simplifying assumptions involved in our considerations. First, we have neglected the collisional broadening of Landau levels.[24]

Such broadening (e.g., by impurity scattering) is essentially independent of wave vector. If it is further independent of the Landau-level index, as may be expected for the moderately low magnetic field considered, then the commensurability oscillations should not be affected in any fundamental way,[7] provided, of course, that the collisional broadening does not overwhelm the modulation broadening which may be estimated as ~0.5 meV. In comparison, the collisional broadening may be estimated as $\Gamma=(2\omega_c/\pi\tau)^{1/2}$, with $\tau=m\mu/e$, and μ being the low-temperature mobility at $B=0$. Setting an upper limit of collisional broadening as 0.5 meV, and taking $B=0.8$ T corresponds to a lower limit of mobility $\mu=5.6\times10^4$ cm^2/Vs. The samples investigated in the experiments of Refs. 4 and 7 typically have $\mu=2.4\times10^5$ cm^2/Vs, which leads to a collisional broadening of 0.25 meV for the same magnetic field, a value well below that of the modulation broadening. Second, to observe the commensurability oscillation unambiguously the coupling between the intra-Landau-band mode and the inter-Landau-band mode must be small. Since they involve different energy scales ($\omega>\omega_c$ for the former, and $\omega\sim|V_n|<\omega_c$ for the latter), mixing of modes can be minimized by controlling the degree of density modulation and by applying an appropriate magnetic field. It should be possible to observe these excitations separately, as is clear from Fig. 1, where the parameters employed are such that $|V_n|<0.5$ meV for $V_0=1$ meV, while the magnetic field strength ranges from 0.25 to 1 T, corresponding to 0.4 meV $<\omega_c<1.6$ meV. Thus, except at the low magnetic field end, the inequality $|V_n|<\omega_c$ is always satisfied. These realistic parameters, obtained directly from experiments,[4,7] may be used to guide an experimental search for the intra-Landau-band magnetoplasmons predicted here. Moreover, when substantial mode mixing occurs, it can be accounted for within the framework set forth here, in terms of the full dispersion relation Eq. (6).

To summarize, we find a novel oscillatory structure in the magnetoplasma spectrum of a two-dimensional electron gas subject to a one-dimensional density modulation. As with corresponding oscillatory structure involved in the magnetoresistance[4] and capacitance[7] of this system, the origin lies in the interplay of the two physical length scales of the system, i.e., the modulation period and the cyclotron diameter at the Fermi level. Experimental study of this new phenomenon should be as revealing as the dc magnetoresistance measurements, and perhaps even more rewarding since it bears directly on the many-body properties of the 2DEG, and its frequency-dependent transport and optical-response properties.

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