Beyond the Two-Fluid Model: Transition from Linear Behavior to a Velocity-Independent Force on a Moving Object in ^{3}He-B

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Using a simple one-dimensional model, we show that the existence of the energy gap for excitations in an isotropic BCS superfluid leads to a strongly nonlinear mechanical behavior of the liquid in the ballistic quasiparticle limit. The nonlinear damping of a vibrating wire in ^{3}He-B below 200 μK is explained, both in its velocity dependence and magnitude. At modest velocities ($v > kT/p_F$), the damping force on an object moving through the superfluid becomes *independent* of velocity, an unexpected result with several interesting implications.

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The quasiparticle excitations in superfluids have dispersion curves very unlike those of simple Newtonian objects, leading to, seemingly, nonintuitive behavior. For example, as pointed out by Andreev,[1] the fact that the energy minimum occurs at nonzero momentum allows the excitations to reverse their direction of motion with virtually no change in momentum or energy. Since the dispersion curve is distorted by superflow, a container of superfluid with a velocity gradient will have a range of excitation dispersion curves depending on the local velocity of the liquid (if the length scale is large enough for a dispersion relation to be meaningful). This has a number of interesting consequences for the dynamics of the liquid, which we discuss below in the context of vibrating-wire resonators.

Despite the fact that much of the experimental investigation of superfluid ^{3}He-B at the lowest temperatures has been made through the medium of wire resonators, the behavior of these devices has been a puzzle. Even at temperatures below $0.4T_c$, where long quasiparticle mean free paths ensure that the motion is ballistic and where we might expect the dynamics of the interaction of the condensate and quasiparticle gas with a moving wire to become very simple, the behavior is still complex.

Consider a Fermi gas of number density n and Fermi momentum p_F, interacting with a wire of radius a traveling at low velocity v. From simple kinetic arguments the damping force on the wire is given by

$$F = Aanp_Fv\ , \tag{1}$$

where A is a numerical constant of order unity, containing the geometrical factors of the scattering.[2] This approach is indeed found to give a good description of the low-temperature behavior for ^{3}He-^{4}He solutions, with n equal to the ^{3}He concentration. However, in the case of superfluid ^{3}He-B if n is set equal to the excitation density, Eq. (1) gives a force about 3 orders of magnitude smaller than that actually observed[3] at the lowest temperatures ($T \approx 0.1T_c$). Furthermore, since the quasiparticles exchange momentum of order p_F with the wire, while the quasiholes exchange of order $-p_F$, then for a symmetrical particle-hole distribution the force of Eq. (1) should vanish identically.

In fact, at low velocities the observed[3] force has the magnitude $F = Aa\rho_n v_g v$, where ρ_n is the normal-fluid density and v_g is the rms velocity of the quasiparticles. These two expressions for F differ by a very large factor, roughly E_F/kT. There has been much discussion of this discrepancy.

Recently, the damping force has been found[4] to become extremely nonlinear at higher velocities, the effective impedance of the liquid falling with increasing wire velocity until the pair-breaking velocity is reached.

In this paper we present a simplified one-dimensional (1D) argument which not only explains qualitatively the above features, but also fits the observed damping force to high accuracy up to the pair-breaking velocity. Moreover, the model predicts that the response of the superfluid to the motion of the wire will show several unusual features. Most importantly at high velocity ($v \gg kT/p_F$) the damping force on the wire should become independent of velocity.

Consider a simple one-dimensional analog of a wire resonator, shown schematically in the inset of Fig. 1. We assume the wire to be represented by a flat paddle of width $2a$, moving with constant velocity v in a 1D ^{3}He-B quasiparticle gas. The excitations have number density n, and v is small compared with the excitation group velocity v_g. The leading side of the paddle intercepts $na(v_g+v)$ excitations per unit time and the trailing side $na(v_g-v)$. We may note that in a normal Fermi fluid, particles scattered by the leading side would exchange momentum $+2p_F$ and those scattered by the trailing side $-2p_F$. Hence the net force on the paddle would be $F = 4anp_Fv$, essentially the linear damping of Eq. (1).

In a superfluid the situation is more complicated. The paddle scatters excitations so as to keep the excitation energy constant in its own rest frame. Thus the dispersion relation of the quasiparticle is best considered in that frame. The usual symmetrical dispersion curve re-

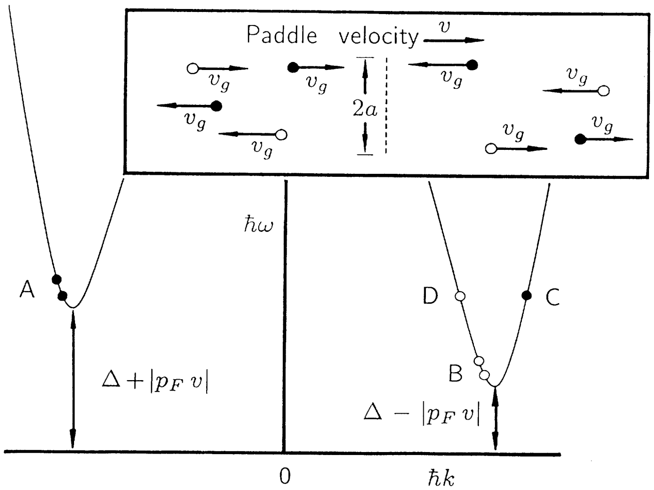

FIG. 1. Inset: The simple model; a superleak paddle moves at velocity v through a 1D gas of quasiparticles and quasiholes moving with group velocity v_g. Main figure: The dispersion curve for the excitations of the simple model as viewed in the frame of the paddle. Quasiparticles approaching the paddle from the front (A) may be normally scattered (to C) whereas quasiholes approaching from the front (B) cannot.

lates to the rest frame of the condensate, so we must also consider the superfluid flow. In 3D the condensate responds to the wire's passage by a pure potential backflow and thus the dispersion relation changes rapidly with position within a distance of order a from the wire. We can only simulate a backflow in 1D by subterfuge. Initially, however, we consider the paddle to be a perfect superleak, i.e., completely transparent to the condensate. This allows the condensate to remain everywhere at rest with respect to the distant container walls. We shall show that this model gives a very good representation of the behavior and the addition of a pseudobackflow only changes the behavior in detail.

The dispersion curve for quasiparticles in the paddle frame is shown in Fig. 1. Since normal elastic scattering processes in this frame maintain constant energy, only a restricted class of normal scattering processes is possible. Quasiparticles approaching the paddle from in front are found at A whereas quasiholes approaching from the front are found at B. The quasiparticles may be normally scattered to states on the other side of the Fermi surface at C but the quasiholes may not, as there are no states on the other side of the distribution for them to scatter into. From the rearward direction only quasiholes may be normally scattered since now the quasiparticles have no states to scatter into. The excitations which are not allowed to scatter normally by the above arguments are forced to undergo Andreev reflection. However, since Andreev processes exchange momentum of order only $p_F\Delta/E_F$ with the paddle, they may be ignored in comparison with normal processes. Normal processes exchange momentum $\pm 2p_F$ with the paddle.

If we further assume that any normal scattering process will occur if it is energetically possible, then the force on the paddle may be written as the sum of two terms, the momentum change per unit time from quasiparticles striking the front surface and that from quasiholes striking the rear surface, i.e., $F=nap_F(v_g+v)+nap_F\times(v_g-v)$. The momentum change from the two terms has the same sign since quasiparticles scattered from the front and quasiholes scattered from the rear both retard the paddle. The paddle velocity cancels to give $F=2nap_Fv_g$. This very surprising result suggests that the force on the paddle is *independent* of paddle velocity.

The above argument is appropriate to low temperatures ($kT \ll 2p_Fv < \Delta$). At higher temperatures we may assume that the incident ballistic excitations have a thermal distribution appropriate to the stationary walls of the vessel. In this case, not only are quasiparticles normally reflected from the front side of the paddle (as before), but also those quasiholes whose energy is greater than $2p_Fv$ above the bottom of the dispersion minimum (at D in Fig. 1). Similarly, the more energetic of the quasiparticles incident upon the rear side are also reflected. The force is now calculated from an integral over the appropriate states in each branch. The density of state per branch is $g(E)$. We integrate the momentum transfer from $E=\Delta$ on the $-p$ branches with a contribution of opposite sign from $E=\Delta+2p_Fv$ on the $+p$ branches. The force F then becomes

$$F=-\int_{\Delta}^{\Delta+2p_Fv} 8p_Fag(E)\exp(-E/kT)v_g\,dE\,. \quad (2)$$

In one dimension $g(E)=(2/h)\partial p/\partial E$. Since the group velocity is $\partial E/\partial p$, a convenient cancellation occurs, giving

$$F=-\int_{\Delta}^{\Delta+2p_Fv}(16ap_F/h)\exp(-E/kT)dE\,. \quad (3)$$

This yields

$$F=(16ap_FkT/h)\exp(-\Delta/kT)[1-\exp(-2p_Fv/kT)]\,. \quad (4)$$

This force as a function of velocity is plotted in Fig. 2. At low velocities ($p_Fv \ll kT$) the force is proportional to velocity, $F=(32ap_F^2/h)\exp(-\Delta/kT)v$, but, as already seen above, the force becomes independent of velocity at high velocity, with value $F=(16ap_FkT/h)\exp(-\Delta/kT)$.

How can we modify these simple arguments to take into account a solid paddle with a backflow of superfluid? Near a "real" paddle the condensate is moving with the same velocity as the paddle (normal to the surface), whereas at a distance the condensate is at rest with respect to the walls. Such a velocity distribution is not possible in a purely 1D system but for present purposes we may consider the liquid to flow around the paddle in some further dimension perpendicular to the motion, which we (conveniently) ignore. The dispersion

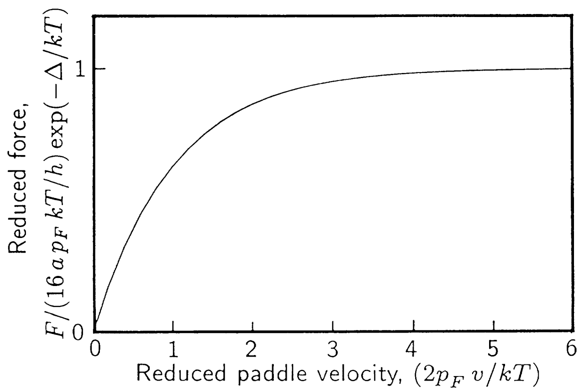

FIG. 2. The force of expression (4) plotted as a function of paddle velocity.

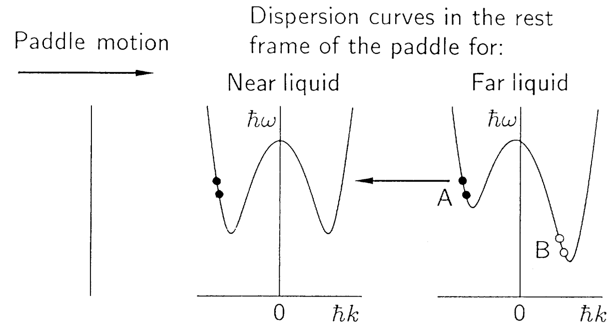

FIG. 3. The 1D model with backflow: liquid near the paddle travels at the paddle velocity; far liquid is at rest. The appropriate dispersion curves in the frame of the paddle are shown. Quasiparticles (at A) can reach the paddle from the front side but quasiholes (B) cannot.

relations in the paddle frame for near and far condensate are as shown in Fig. 3.

We can use the same argument as that leading to Eq. (4) above. Particles approaching the paddle from the forward side (at A) can reach the surface and be reflected normally where incoming holes (at B) cannot unless their energy is above $\Delta + p_F v$, and vice versa for the rear side. Thus we calculate the force with the same integral as in Eq. (2) but with the upper limits of $\Delta + p_F v$ rather than $\Delta + 2p_F v$. This gives

$$F = (16ap_F kT/h)\exp(-\Delta/kT)[1-\exp(-p_F v/kT)] , \qquad (5)$$

the same as (4) except for the loss of a factor of 2 in the final exponential term.

We may reexpress this force in terms of the two-fluid model. Both the normal-fluid density and the group velocity involve integrals over the density of states and are usually expressed in terms of the appropriate Yosida functions. However, the product $\rho_n |v_g|$ for each excitation, integrated over the whole distribution, $\langle \rho_n v_g \rangle$, shows the same cancellation we noted above in arriving at Eq. (3) and we readily find for 1D, $\langle \rho_n v_g \rangle = (8p_F^2/h) \times \exp(-\Delta/kT)$. Hence the low-velocity value of F from Eq. (5) may be rewritten $F = 2a\langle \rho_n v_g \rangle v$, which is essentially the long-known, but hitherto unjustified, experimental result.

We may now interpret the observed quasiparticle damping force on a vibrating wire in ^{3}He-B. In the absence of a full 3D theory we may expect the damping to have the same general form as Eqs. (4) and (5), namely

$$F = F_0 \exp(-\Delta/kT)[1-\exp(-\lambda p_F v/kT)] , \qquad (6)$$

with λ a constant of order unity.

The data we consider here comprise a series of quasiparticle damping measures taken with a 4.5-μm-diam NbTi filament. The data extend from 124 to 204 μK and from zero velocity to beyond the onset of pair breaking. We use the 0-bar results, since this set is fully discussed in Ref. 4. However, comparison with measurements at other pressures is essentially similar.

The force of Eq. (6) is that expected for a constant uniform velocity, whereas the resonator impedance involves a time-averaged and spatially averaged velocity. We need to convert this double average into an equivalent steady velocity to compare with (6). The spatial average over the geometry of the loop and its behavior under deformation is not too simple, but a reliable factor can be estimated[3] to convert the measured voltage from the moving loop to the maximum velocity v_m of the wire. Fortunately the time averaging is straightforward since the resonators are always operated in a high-Q regime ($Q = 100$ to 5000) and despite the nonlinearity of the damping the motion remains simple harmonic.

We thus extract the force as a function of maximum velocity. Earlier[4] the force was found to be strongly nonlinear with velocity and apparently scaled with the Landau velocity (Δ/p_F). Comparison with the ideas in the present Letter suggests the following: (i) The velocity scaling for the force should rather be in terms of a thermal velocity kT/p_F [Eq. (6)]; (ii) the universal curve for force should have the exponential form suggested by Eq. (6); and (iii) the prefactor F_0 should be given by the appropriate 3D generalization of Eqs. (4) and (5).

The experimental reduced force is plotted against reduced velocity in Fig. 4. The raw data[4] are shown in the inset. The temperatures were derived from the low-velocity damping which we can empirically transfer to earlier[5] measurements against Pt NMR, corrected to the currently accepted temperature scale.[6] The horizontal axis of Fig. 4 is $v_m p_F/kT$. We may note the following:

(i) The scaling with kT/p_F is excellent.

(ii) The solid line in Fig. 4 represents Eq. (6), with v simply replaced by v_m with $\lambda = 0.95$. The agreement is remarkable. One should not read too much into the value of λ because of the averaging problem. We have

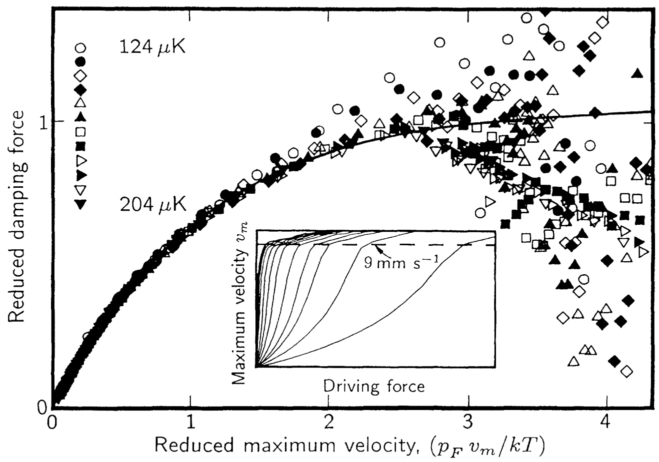

FIG. 4. The normalized force from quasiparticles scattering plotted against reduced maximum velocity $p_F v_m/kT$ for a wire moving in ^{3}He-B. The data are normalized to a slope of unity near the origin (i.e., the linear damping region). The solid curve represents $[1-\exp(-\lambda p_F v_m/kT)]$, cf. Eq. (6), with $\lambda=0.95$. Inset: The raw data. The onset of pair breaking at high velocity is indicated by the dashed line. The subtraction of this additional damping causes the scatter at the higher velocities on the main graph.

also performed a time average of the damping derived from Eq. (6), which shows the same type of variation, but now fits the data with $\lambda=1.15$.

(iii) Consideration of the magnitude of the force awaits a full 3D treatment. However, the simple replacement of $g(E)$ by the 3D density of states yields a value of F_0 which is only a factor of 5 greater than that observed. Since the average momentum transfer per collision in 3D will be much less than $\pm 2p_F$ assumed here, and since the flow around the wire extrema yields (by similar arguments) a force of opposite sign, this represents very reasonable agreement.

The universal nature of the observed force and the agreement with theory suggests that the present ideas are essentially correct, and that the damping force is determined by the accessibility of the wire to excitations for normal scattering processes.

The principal general conclusion we reach is that the force on a moving object in the excitation gas in ^{3}He-B becomes independent of velocity at velocities greater than kT/p_F, until the pair-breaking velocity is reached. At low velocities, however, the force is linear in velocity, the wire response being as if to a "normal fluid." Figure 2 shows the unique transition from this normal-fluid behavior at the low-v limit continuously to the velocity-independent force regime when $v \gg kT/p_F$. This must represent a general result for all Cooper-pair superfluids with isotropic gaps. We are not aware that such ideas have been at all widely discussed earlier, since it is only in ^{3}He-B in the very-low-temperature regime that velocities significantly greater than those appropriate to the two-fluid model can be reached in practice.

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