Stochastic Approach to Giant Dipole Resonances in Hot Rotating Nuclei

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A stochastic macroscopic approach to giant dipole resonances (GDR's) in hot rotating nuclei is presented. In the adiabatic limit the theory reduces exactly to a previous adiabatic model where the unitary invariant metric is used to calculate equilibrium averages. Nonadiabatic effects cause changes in the GDR cross section and motional narrowing. Comparisons with experiments where deviations from the adiabatic limit are substantial are shown and can be used to determine the damping of the quadrupole motion at finite temperature.

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Recent experiments are providing new information on the properties of highly excited nuclei created in heavy-ion fusion reactions. The major experimental probe is the giant dipole resonance (GDR) built on such hot nuclei. From these experiments one attempts to learn about the evolution of the nuclear shape with temperature and spin.[1-4]

We have developed a unified mean-field theory of hot rotating nuclei from which we have derived the universal features of the shape evolution.[5] The theory is based on the Landau theory of phase transitions where the quadrupole deformation parameters play the role of the order parameter. A macroscopic approach to the GDR at finite temperature was then developed by using a unified description of thermal quadrupole shape fluctuations.[6] With only two free parameters, determined by the measured ground-state (zero-temperature) GDR's, the theory reproduces well many of the experimental results at finite temperature and spin.[7] The importance of thermal shape fluctuations has been demonstrated by other authors as well.[8]

In several cases, however, the theory seems to overestimate the observed width of the giant resonance. It was recently suggested[9] that, in analogy with nuclear magnetic resonance in condensed matter systems and its application in rotational damping of nuclei,[10] the GDR could display motional narrowing. The basic idea is to assume the fluctuations in the quadrupole shape to be nonadiabatic so that the dipole vibration does not have enough time to probe separately each nuclear shape.

This idea was incorporated in a microscopic model of Ref. 9 by using a description of the compound-nucleus wave functions based on random-matrix theory. Although the model can explain motional narrowing, its adiabatic limit yields the Wigner semicircular distribution[9] when the Gaussian orthogonal ensemble of random matrices is applied and it does not reduce to the previously used adiabatic fluctuation theory.[6,8]

In this Letter we introduce a relatively simple macroscopic theory of time-dependent fluctuations which generalizes our previous theory[6,7] of the GDR to nonadiabatic situations. The quadrupole shape fluctuations are described by an equation of the Brownian-motion type in which the free energy plays the role of an external potential and the random force is generated by the coupling of the quadrupole shape degrees of freedom to all others. The giant dipole vibration is described by a damped harmonic oscillator which is coupled to the quadrupole fluctuations through the shape dependence of its frequency and damping width. Its cross section is found from the Fourier transform of its average time correlation function.[6]

When compared with the adiabatic theory of static fluctuations,[6] the present theory contains only one additional parameter which determines the degree of adiabaticity of the process. It spans the whole range between the adiabatic and the sudden limit. In the adiabatic limit it reproduces exactly the static fluctuation theory,[6,7] where thermal averages are done with the unitary invariant metric of Refs. 5 and 6. As the process becomes less adiabatic the GDR gets narrower. In the general case, we solve the above equations by Monte Carlo techniques. For those cases in which the experiment deviates from the adiabatic limit we determine the adiabaticity parameter from comparison with the experiment.

We begin by introducing the equations of motion which describe the evolution in time of the quadrupole shape parameters $\alpha_{2\mu}(t)$ in a frame which rotates with constant angular velocity $\boldsymbol{\omega}$. We assume that $\alpha_{2\mu}$ satisfy a Langevin equation[11] of the form

$$\dot{\alpha}_{2\mu} = -\chi^{-1}\partial F/\partial\alpha^*_{2\mu} + f_{2\mu}(t)\,, \tag{1}$$

where $F(T,\omega;\alpha_{2\mu})$ is the free-energy surface in the rotating frame at temperature T and angular velocity ω, and χ is a parameter. $-\chi^{-1}\partial F/\partial\alpha^*_{2\mu}$ plays the role of an external average driving force for $\alpha_{2\mu}$ while $f_{2\mu}(t)$ is a random force which causes statistical fluctuations in $\alpha_{2\mu}$ and makes the process stochastic. In the above equation we have assumed the quadrupole vibration to be overdamped so that a description in terms of a first-order equation (1) is feasible. The parameter χ is the proportional to the mean relaxation time of the quadrupole motion. The random force is assumed to be Gaussian and stationary, satisfying

$$\langle f_{2\mu}(t)\rangle=0\,,\quad \langle f_{2\mu}(t)f_{2\mu'}(t')\rangle=\xi\delta(t-t')\delta_{\mu\mu'}\,. \tag{2}$$

The stochastic process (1) determines an ensemble of solutions $\{\alpha_{2\mu}(t)\}$. One can construct the probability distribution function $P(\alpha_{2\mu},t)$ at any time t and show that it satisfies a Fokker-Planck equation,[11]

$$\frac{\partial P}{\partial t}=-\frac{\partial}{\partial\alpha_{2\mu}}\left(-\frac{1}{\chi}\frac{\partial F}{\partial\alpha_{2\mu}^*}P\right)+\frac{1}{2}\xi\frac{\partial^2 P}{\partial\alpha_{2\mu}\partial\alpha_{2\mu}^*}\,. \tag{3}$$

It can be shown that irrespective of the initial distribution, $P(\alpha_{2\mu},t)$ always converges in the limit $t\rightarrow\infty$ to a stationary distribution characterized by $\partial P_{\rm st}/\partial t=0$. We find

$$P_{\rm st}\propto e^{-(2/\chi\xi)F}\,. \tag{4}$$

Since it should coincide with the equilibrium distribution[6,8] $e^{-F/T}$, we conclude that

$$\xi=2T/\chi\,. \tag{5}$$

This is a fluctuation-dissipation theorem which determines the correlation amplitude ξ.

If the initial distribution (at $t=0$) is already the equilibrium distribution, it will stay so at any future time. The corresponding ensemble of solutions $\{\alpha_{2\mu}(t)\}$ is known as a stationary Markov process[11] and we shall use this process to describe the nuclear shape fluctuations at equilibrium.

The giant dipole vibrations are described by a three-dimensional damped harmonic oscillator which is rotating with angular velocity $\boldsymbol{\omega}$. The frequencies and damping widths depend on the shape $\alpha_{2\mu}$ exactly as in the adiabatic model. Denoting by $\mathbf{D}$ the dipole operator in the rotating frame and by $\mathbf{P}$ its conjugate momentum we have

$$\begin{aligned}\dot{\mathbf{D}}&=\mathbf{P}-\boldsymbol{\omega}\times\mathbf{D}-\tfrac{1}{2}\Gamma\mathbf{D}\,,\\ \dot{\mathbf{P}}&=-E^2\mathbf{D}-\boldsymbol{\omega}\times\mathbf{P}-\tfrac{1}{2}\Gamma\mathbf{P}\,,\end{aligned} \tag{6}$$

where E and Γ are frequency and damping matrices, respectively. E is the matrix which in the principal frame is diagonal with elements

$$E_j=E_0\exp\left[-\left(\frac{5}{4\pi}\right)^{1/2}\beta\cos\left(\gamma-\frac{2\pi}{3}j\right)\right] \tag{7}$$

as in the adiabatic model.[6,7] The matrix Γ is given by a power law[6,7]

$$\Gamma=\Gamma_0(E/E_0)^{\delta}\,. \tag{8}$$

Notice that the matrices E and Γ which enter Eqs. (6) should be expressed in terms of $\alpha_{2\mu}$ in the rotating frame. Usually[7] $\delta=1.6$ and E_0,Γ_0 (characterizing a spherical nucleus) are determined from the ground-state GDR.[7] Thus, the GDR equation of motion (6) is coupled to the quadrupole equation (1) through the α dependence of E^2 and Γ.

The set of stochastic equations (1) and (6) comprise our macroscopic model for the time-dependent fluctuations. The GDR absorption cross section is then calculated[6] from the Fourier transform of the dipole correlation function $\langle D_j(t)D_j(0)\rangle$, where the $\mathbf{D}$'s have been rotated back to the laboratory frame.

The degree of adiabaticity of the process is determined by the parameter χ. To see that we define the adiabaticity parameter η as the ratio between the frequency spread ΔE of the GDR due to variation in the static deformation and the mean relaxation rate λ of the quadrupole motion,

$$\eta\equiv\Delta E/\lambda\,. \tag{9}$$

We can estimate $\Delta E\approx(5/4\pi)^{1/2}E_0\Delta\beta$ and $\lambda\approx 5T/\chi\langle\beta^2\rangle$, so that

$$\eta\cong\frac{E_0}{(20\pi)^{1/2}}\frac{\chi}{T}\Delta\beta\langle\beta^2\rangle\,, \tag{10}$$

where $\langle\beta^2\rangle$ is the equilibrium average of β^2 and $(\Delta\beta)^2$ is the equilibrium variance of β. The parameter χ plays a role similar to that of Γ_μ in the microscopic models of Ref. 9. The adiabatic limit corresponds to $\eta\gg 1$ ($\chi\rightarrow\infty$) where the quadrupole deformation changes slowly enough for the GDR to feel these changes. In this limit we can assume the quadrupole deformation $\alpha_{2\mu}$ in the GDR equations (6) to be frozen at its initial value $\alpha_{2\mu}(0)$, so the Fourier transform of the dipole correlation function [for a fixed $\alpha_{2\mu}(0)$] is a superposition of Breit-Wigner curves similar to Eq. (3) of Ref. 6. The actual GDR absorption cross section then becomes the average over the initial distribution, i.e., the equilibrium distribution

$$\sigma_{\rm abs}(\epsilon;T,\omega)=\frac{\int D[\alpha]e^{-F/T}\sigma(\epsilon;\alpha_{2\mu})}{\int D[\alpha]e^{-F/T}}\,, \tag{11}$$

where $D[\alpha]=\Pi_\mu d\alpha_{2\mu}=\beta^4|\sin 3\gamma|\,d\beta\,d\gamma\,d\Omega$. Equation (11) is identical with our previous adiabatic model.[6,7] Furthermore, the above unitary metric emerges as the one which should be used in that limit if (1) describes the correct dynamical evolution of $\alpha_{2\mu}$.

To solve the above stochastic equations and to determine the dipole correlation function in the general case we proceed as follows:

(i) We choose an initial ensemble of quadrupole deformations $\{\alpha_{2\mu}(0)\}$ which is distributed according to the equilibrium ensemble $e^{-F/T}$.

(ii) For each $\alpha_{2\mu}(0)$ we solve (1) by Monte Carlo techniques. We use a second-order stochastic Runge-Kutta method such that[12]

$$\alpha_{2\mu}(t+\Delta t)=\alpha_{2\mu}(t)+g_{2\mu}\Delta t+(\Delta t\xi)^{1/2}Y_{2\mu}\,, \tag{12}$$

where $g_{2\mu}$ is an average of $-\chi^{-1}\partial F/\partial\alpha_{2\mu}^*$ and the real and imaginary parts of $Y_{2\mu}$ are five independent standard

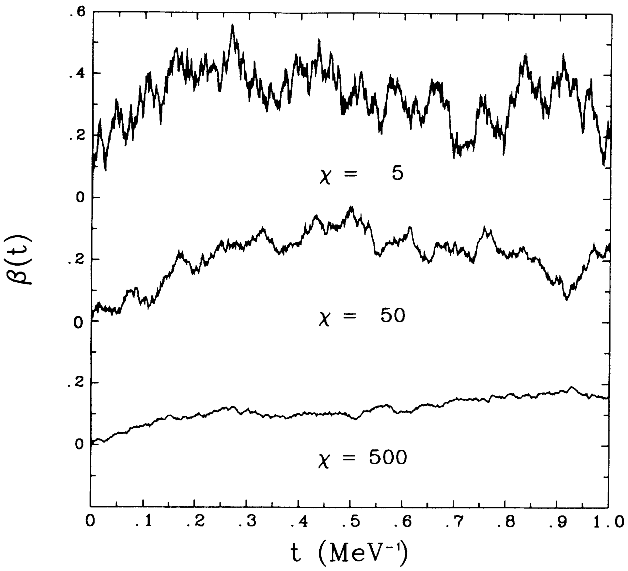

FIG. 1. Typical shape trajectories β vs time t for ^{112}Sn at $T=1.8$ MeV and $\omega=0.535$ MeV obtained from the solution of Eq. (12). The three trajectories shown correspond to various values of χ: $\chi=500$ (adiabatic limit), $\chi=50$, and $\chi=5$ (sudden limit). Note that as the process becomes more nonadiabatic the trajectory samples a larger fraction of the phase space at a given time interval.

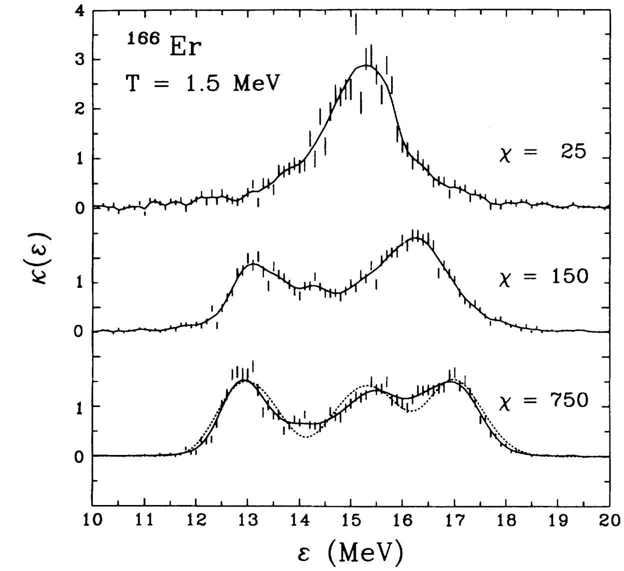

FIG. 2. The Fourier transforms of the dipole correlation function found from the solution of the stochastic equations (1) and (6) for various values of χ ($\chi=750$, 150, and 25). For the purpose of demonstrating the effects of nonadiabaticity we have chosen a hypothetical case where $\Gamma_0=0$ (i.e., no intrinsic damping of the dipole) for ^{166}Er at $T=1.5$ MeV. The bars show the statistical errors associated with the Monte Carlo calculations and the solid lines are a weighted fit. The dashed line presents the adiabatic model of Refs. 6 and 7. Notice that various peaks coalesce and get narrower as the process becomes less adiabatic (χ get smaller).

normal random variables. The derivatives of F above are calculated analytically using the Landau expansion in Ref. 5. We obtain an ensemble of "shape trajectories" $\{\alpha_{2\mu}(t)\}$ which is equilibrated at any time t.

In Fig. 1 we show three typical trajectories $\beta(t)$ for ^{112}Sn at $T=1.8$ MeV and $\omega=0.535$ MeV. The lower one corresponds to the adiabatic limit ($\eta \gg 1$) while the upper one is in the "sudden" limit ($\eta \ll 1$) where the quadrupole shape fluctuates rapidly. An intermediate situation ($\eta \sim 1$) is shown in the middle. We see that the adiabatic process is very "smooth" but as we move further away from the adiabatic limit the trajectories become more "erratic" (on the same time scale) and the changes in deformation are larger.

(iii) For each trajectory $\alpha_{2\mu}(t)$ we solve (6) for $\mathbf{D}(t)$ in terms of $\mathbf{D}(0)$ and $\mathbf{P}(0)$. The correlation $\langle \mathbf{D}(t)\mathbf{D}(0)\rangle$ is then calculated by averaging over the ensemble $\{\alpha_{2\mu}(t)\}$ using appropriate (quantum-mechanical) initial correlation functions which are consistent with the adiabatic model.[6,7]

Nonadiabatic effects are seen most clearly when one assumes a zero intrinsic width (i.e., $\Gamma_0=0$) so that broadening of the resonance comes only from the coupling to the quadrupole degrees of freedom. We thus consider such a hypothetical ^{166}Er nucleus at $T=1.5$ MeV and $\omega=0$ for which $(\Delta E)^{-1}=2$ MeV^{-1}. The solid lines in Fig. 2 are the fit to the Monte Carlo calculations (error bars) of the GDR absorption cross section and the dotted line is the adiabatic model. At $\chi=750$, $\lambda^{-1}=8.3$ MeV^{-1} so that we are close to the adiabatic limit where three peaks are seen. At smaller χ (such as $\chi=150$ where $\lambda^{-1}=1.7$ MeV^{-1}) the two peaks on the right coalesce and get narrower. Then the left-hand peak starts to move to the right while disappearing, and in the sudden limit we have a single narrow Lorentzian. Thus though the general effect is that of motional narrowing as discussed in Ref. 9, the detailed shape of the resonance is also sensitive to η. Note that as the process becomes more sudden it is necessary to take a smaller time step Δt since the quadrupole fluctuations are more erratic. Thus in order that the results will be independent of the time step we have to choose $\Delta t \lesssim 0.006$ MeV^{-1} in the adiabatic case but $\Delta t \lesssim 0.0002$ MeV^{-1} in the sudden limit.

A realistic calculation ($\Gamma_0 \neq 0$) is shown in Fig. 3 for ^{112}Sn at $T=1.8$ MeV and $\omega=0.535$ MeV for which $(\Delta E)^{-1}=1.1$ MeV^{-1}, where our adiabatic model ($\chi=500$) overestimates the experimental width (dotted line). We used $E_0=15.2$ MeV, $\Gamma_0=3.76$ MeV, and $\delta=1.6$ and calculate the GDR absorption cross section for several values of χ ($\chi=5$, 30, and 500). As χ decreases the resonance gets narrower and its structure

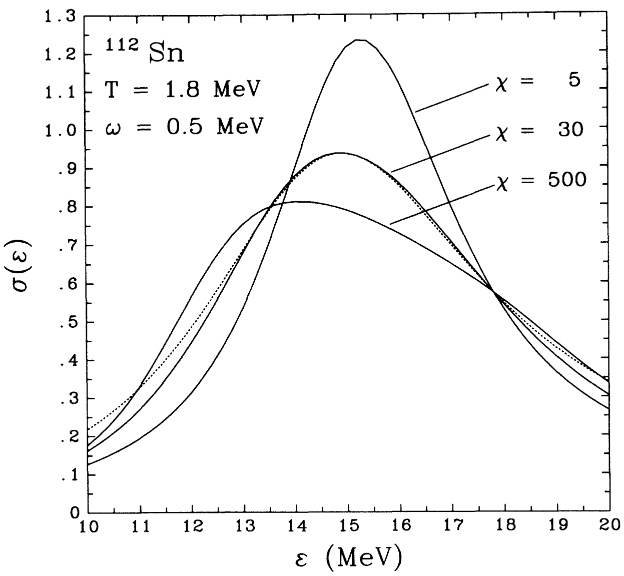

FIG. 3. The GDR cross section for ^{112}Sn at $T=1.8$ MeV and $\omega=0.535$ MeV. The dotted line is a CASCADE fit to the experiment and the solid lines are the calculations of the stochastic model for $\chi=500$, 30, and 5. The cross section for $\chi=30$ seems to give good agreement with the experiment and suggests that the process is intermediate between adiabatic and sudden.

changes. The value which fits the experiment the closest is $\chi=30$ which corresponds to $\lambda^{-1}=0.36\ \mathrm{MeV}^{-1}$ and therefore to an intermediate η ($\eta=0.3$).

In the sudden limit $\eta \ll 1$ ($\chi \rightarrow 0$) it is possible to reduce the stochastic equation (6) to an equation of motion for $\langle \mathbf{D} \rangle$ which is basically that of a damped rotating oscillator with some effective frequency and damping width. In the absence of intrinsic damping ($\Gamma_0=0$) the effective damping $\tilde{\Gamma}$ is estimated to be

$$\tilde{\Gamma} \approx (\Delta E)^2/\lambda = \eta \Delta E \ll \Delta E \,, \tag{13}$$

which is narrower by a factor η than the width in the adiabatic limit. This is exactly the motional-narrowing effect discussed in Refs. 9 and 10. In realistic applications Γ_0 has to be added to the right-hand side of (13).

To conclude, it seems that a macroscopic time-dependent fluctuation theory based on a Landau theory is successful in describing the observed GDR even in nonadiabatic situations. The model contains a relaxation parameter χ whose dependence on temperature and the specific nucleus under consideration can be inferred from experimental data. It will be interesting to determine this parameter from a theoretical model for a damping of the quadrupole motion at finite temperature or from independent experimental data such as fission.

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