Topological Extensions of the Supersymmetry Algebra for Extended Objects

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We show that the supersymmetry algebra for supersymmetric extended objects can contain, depending on the topology of space, topological charges, the origin of which is the Wess-Zumino term in the effective action. We exhibit the consequences of these charges for the phenomenon of partial breaking of supersymmetry.

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There has been a recent renewal of interest in the Green-Schwarz (GS) action for the superstring because, unlike the Neveu-Schwarz-Ramond action, it has a *manifest* super Poincaré invariance. The GS action can be thought of as one example, the $p=1$ case, of a class of super "p-brane" actions for supersymmetric p-dimensional extended objects defined on the $(p+1)$-dimensional world volume W. The $p=2$ case is the supermembrane, which has received recent attention, and for some purposes it is instructive to consider the general case. Given the manifest super Poincaré invariance of these actions, one might expect the charges Q_A, corresponding to the associated Noether currents j_A, to generate the super Poincaré algebra. The first aim of this Letter is to point out that this is not necessarily so: *An additional topological charge can appear in the algebra.*

Let (flat) superspace Σ be parametrized, as usual, by the coordinates $Z^M=(X^\mu,\theta^\alpha)$, $\mu=0,1,\ldots,d-1$, where, for simplicity of presentation, the anticommuting spinor θ^α is assumed to be Majorana. Given an immersion $\phi: W\rightarrow\Sigma$ of the $(p+1)$-dimensional world volume W of a p-dimensional extended object [coordinates $\xi^i=(t,\sigma)$, $i=0,1,\ldots,p$] into superspace Σ, we can construct the *identically* conserved current density

$$j_T^{i\mu_1\cdots\mu_p}(\xi)=\epsilon^{ij_1\cdots j_p}\partial_{j_1}X^{\mu_1}(\xi)\cdots\partial_{j_p}X^{\mu_p}(\xi). \quad (1)$$

If, at a fixed time, the extended object defines a nontrivial p-cycle in space in the sense of de Rham, then the corresponding charge $T^{\mu_1\cdots\mu_p}=\int d^p\sigma\, j_T^{0\mu_1\cdots\mu_p}(\xi)$ will be nonzero. It is this charge which appears in the supersymmetry algebra via the anticommutator

$$\{Q_\alpha,Q_\beta\}=2(\Gamma^\mu)_{\alpha\beta}P_\mu+2T(\Gamma_{\mu_1\cdots\mu_p})_{\alpha\beta}T^{\mu_1\cdots\mu_p}, \quad (2)$$

where $\Gamma_{\mu_1\cdots\mu_p}$ is the antisymmetrized product of Dirac matrices. The factor T is the p-volume tension of the extended object with dimensions $(c=1)$ $[T]=ML^{-p}$ (in the limiting pointlike case of $p=0$, T reduces to the particle mass m). We adopt the convention that C is used to raise and lower spinor indices ($\bar\theta_\beta\equiv\theta^\alpha C_{\alpha\beta}$). If $(\Gamma_\mu)^\alpha{}_\beta$ are the entries of the Dirac matrices satisfying $\{\Gamma^\mu,\Gamma^\nu\}=2\eta^{\mu\nu}$, then $(\Gamma_\mu)_{\alpha\beta}$ are the entries of the symmetric matrices $C\Gamma_\mu$. For the allowed values of d and p [1] it follows that the matrices $C\Gamma_{\mu_1\cdots\mu_p}$ are also symmetric. Moreover, we choose $C=\Gamma_0$ (θ real).

The second aim of this Letter is to show that the origin of this topological term in the supersymmetry algebra is the Wess-Zumino (WZ) term in the action, which takes the form[1-4]

$$S=\int_W d^{p+1}\xi(\mathcal{L}_1+\mathcal{L}_{\rm WZ})\equiv S_1+S_{\rm WZ}, \quad (3)$$

where $\mathcal{L}_1(\xi)=T[-\det\Pi_i^\mu(\xi)\Pi_j^\nu(\xi)\eta_{\mu\nu}]^{1/2}$ is the obvious supersymmetric invariant extension of the bosonic Lagrangian density $T[-\det\partial_iX^\mu(\xi)\partial_jX^\nu(\xi)\eta_{\mu\nu}]^{1/2}$, and $\eta_{\mu\nu}$ is the ("mostly plus") metric of Minkowski spacetime. Let $\Pi^A=(\Pi^\mu,\Pi^\alpha)\equiv(dX^\mu-i\bar\theta\Gamma^\mu d\theta, d\theta^\alpha)$ denote the left-invariant (LI) one-forms on Σ, considered as the supertranslation group. The pullback of Π^A by the above map ϕ induces the one-forms $(\phi^*\Pi^A)(\xi)=d\xi^i\times\Pi_i^A(\xi)$ on W, the coordinates of which,

$$\Pi_i^\mu=\partial_iX^\mu-i\bar\theta\Gamma^\mu\partial_i\theta,\quad \Pi_i^\alpha=\partial_i\theta^\alpha, \quad (4)$$

appear in $\mathcal{L}$. The WZ term in the action is

$$S_{\rm WZ}=T\int_W\phi^*b, \quad (5)$$

where b is a $(p+1)$-form potential for a super Poincaré invariant $(p+2)$-form h on Σ, i.e., $h=db$. Since $\delta h=0$ (super Poincaré invariance), and the variation δ and the exterior derivative d commute, $\delta b=d\Delta$ for some p-form Δ and therefore

$$T\delta(\phi^*b)=d(\phi^*\Delta)\equiv\partial_i\Delta^i dt\wedge d\sigma^1\wedge d\sigma^2\wedge\cdots\wedge d\sigma^p. \quad (6)$$

A crucial feature of h is that δb, and hence Δ, does not vanish. [This is equivalent to the statement[5] that h is a nontrivial $(p+2)$-cocycle of of the **R**-valued Chevalley-Eilenberg equivariant cohomology of the supertranslation group; in other words, the closed LI form h does *not* come from a LI potential. This restricts p to $p\le 5$.] It is the nonvanishing of Δ (a phenomenon which is of course not restricted to this case[6]) that is responsible for the modification of the supersymmetry algebra.

To see how such a modification can arise, consider a Lagrangian $\mathcal{L}(Z^M,\partial_iZ^M)$ invariant under the transformation $Z^M\rightarrow Z^M+k^A\delta_AZ^M$, for constant parameters k^A [i.e., $\delta_A\mathcal{L}=0$, cf. (9) below]. The associated conserved Noether current is then given by

$$j^i_A = \delta_A Z^M \frac{\partial \mathcal{L}}{\partial(\partial_i Z^M)}, \quad i=0,1,\ldots,p. \qquad (7)$$

The charge densities j^0_A can be written as $\delta_A Z^M \Pi_M$, where Π_M is the momentum variable conjugate to Z^M. As a result, they satisfy the equal-time Poisson-bracket (PB) algebra

$$\{j^0_A(t,\sigma), j^0_B(t,\sigma')\}_{\rm PB} = \delta^p(\sigma-\sigma') f_{AB}{}^C j^0_C(t,\sigma), \qquad (8)$$

where $f_{AB}{}^C$ are the structure constants of the (super)-symmetry algebra. If, however, $\mathcal{L}$ is not invariant but only *quasi-invariant*, i.e., if

$$\delta_A \mathcal{L} = \partial_i \Delta^i_A, \qquad (9)$$

then the conserved currents $\tilde{j}^i_A$ contain an "anomalous" extra piece Δ^i_A,

$$\tilde{j}^i_A = \delta_A Z^M \frac{\partial \mathcal{L}}{\partial(\partial_i Z^M)} - \Delta^i_A. \qquad (10)$$

Consequently, the charge densities $\tilde{j}^0_A$ now satisfy the local PB algebra

$$\{\tilde{j}^0_A(t,\sigma), \tilde{j}^0_B(t,\sigma')\}_{\rm PB} = \delta^p(\sigma-\sigma') f_{AB}{}^C \tilde{j}^0_C(t,\sigma) - A_{AB}(t;\sigma,\sigma'), \qquad (11)$$

where the "anomalous" term $A_{AB}(t;\sigma,\sigma')$ is given by the expression

$$\{j^0_A(t,\sigma), \Delta^0_B(t,\sigma')\}_{\rm PB} - (-1)^{ab}\{j^0_B(t,\sigma'), \Delta^0_A(t,\sigma)\}_{\rm PB} - \delta^p(\sigma-\sigma') f_{AB}{}^C \Delta^0_C(t,\sigma) \qquad (12)$$

[$(-1)^{ab}=-1$ if A and B are fermionic symmetries and $+1$ otherwise]. When it is well defined, a double integration of (11) leads to the charge algebra

$$\{Q_A, Q_B\}_{\rm PB} = f_{AB}{}^C Q_C - A_{AB}, \qquad (13)$$

where

$$A_{AB} = \int d^p\sigma [\delta_A \Delta^0_B(t,\sigma) - (-1)^{ab}\delta_B \Delta^0_A(t,\sigma) - f_{AB}{}^C \Delta^0_C(t,\sigma)]. \qquad (14)$$

A similar result of course holds in the quantum theory upon replacing Poisson brackets by (anti)commutators. Note that, because of (13) and the conservation of $\tilde{j}^i_A$, A_{AB} is time independent.

A trivial example of a quasi-invariant Lagrangian is one of the form $\mathcal{L} = \mathcal{L}_1 + \partial_i \alpha^i$, where $\mathcal{L}_1$ is invariant. In this case $\Delta^0_A(\sigma) = \delta_A \alpha^0$ and the anomalous term A_{AB} vanishes since, obviously, $\delta_A\delta_B - (-1)^{ab}\delta_B\delta_A = f_{AB}{}^C \delta_C$.

As the simplest nontrivial example for which A_{AB} does not vanish we take the action for the massive superparticle[7] where $\mathcal{L}_{\rm WZ} = m(\phi^* b)$. For $d=9$, for example, for which Γ^0 is symmetric and imaginary, b is the one-form potential for $h = d\bar{\theta} d\theta = d(\bar{\theta} d\theta) \equiv db$. Under a supersymmetry transformation we have $\delta_\alpha b = d(\theta_\alpha)$ and we take $\Delta_\alpha = m\theta_\alpha$, all other vanishing. In this example (14) reduces to $A_{AB} = \delta_A \Delta_B - (-1)^{ab}\delta_B \Delta_A - f_{AB}{}^C \Delta_C$, and therefore

$$A_{\alpha\beta} = 2mC_{\alpha\beta}, \quad A_{\mu\nu} = A_{\mu\alpha} = 0. \qquad (15)$$

Thus, for the massive superparticle ($p=0$) Δ_α depends only on t and the current algebra (11) reduces to (13), which is the usual central extension, by the mass m, of the algebra of supersymmetry charges.

As our first example for which the WZ term produces a modification of the supersymmetry algebra *not* of the standard form we take the closed $N=1$ GS superstring ($p=1$). In this case there is a charge-*density* algebra (8) which is modified by the WZ term to (11). Explicitly, the relevant forms on Σ are

$$h = i(d\bar{\theta}\Gamma_\mu d\theta)\Pi^\mu, \quad h = db,$$

$$b = i\Pi^\mu(d\bar{\theta}\Gamma_\mu\theta), \quad T\delta_\alpha b = d\Delta_\alpha, \qquad (16)$$

$$T^{-1}\Delta_\alpha = i[dX^\mu - \tfrac{1}{3} i(\bar{\theta}\Gamma^\mu d\theta)](\Gamma_\mu\theta)_\alpha,$$

where here and henceforth the wedge product $\wedge$ is to be understood. From (6) we find the induced form on W,

$$T^{-1}\Delta^0_\alpha = i\left[\frac{\partial X^\mu}{\partial\sigma} - \frac{i}{3}\left(\bar{\theta}\Gamma^\mu \frac{\partial\theta}{\partial\sigma}\right)\right](\Gamma_\mu\theta)_\alpha. \qquad (17)$$

A direct calculation yields

$$T^{-1}\int d\sigma' A_{\alpha\beta}(t;\sigma,\sigma') = 2i(\Gamma_\mu)_{\alpha\beta}\frac{\partial X^\mu}{\partial\sigma} + \frac{4}{3}\frac{\partial}{\partial\sigma}[(\Gamma^\mu\theta)_\alpha(\Gamma_\mu\theta)_\beta]. \qquad (18)$$

To obtain this result we have used the identity $(\Gamma_\mu)_{(\alpha\beta}(\Gamma^\mu)_{\gamma\delta)} \equiv 0$, valid for $d=3,4,10$ (and for $d=6$ with appropriate modifications for complex spinors). Further integration gives

$$A_{\alpha\beta} = 2iT(\Gamma_\mu)_{\alpha\beta}\oint \frac{\partial X^\mu}{\partial\sigma} d\sigma \qquad (19)$$

since, because θ is periodic in σ, the second term in (18) does not contribute to the loop integral (19). If the loop is contractible then the first term in (18) will also fail to contribute to the integral and $A_{\alpha\beta} \equiv 0$, but if the loop is noncontractible $A_{\alpha\beta}$ will not vanish. For example, if we take spacetime to be $S^1 \times M_{d-1}$ where M_{d-1} is $(d-1)$-dimensional Minkowski spacetime, then, for a closed string (with coordinate $\sigma \in [0,2\pi]$) which wraps n times around the S^1, we may take $X^1 = n\sigma R + Y(t,\sigma)$ with Y and all the other components of X^μ periodic in σ, and R the radius of S^1. Then $\oint d\sigma = 2n\pi$ and

$$A_{\alpha\beta} = 2i(\Gamma_1)_{\alpha\beta}(2\pi n R T). \qquad (20)$$

For a two-dimensional field theory for which space has infinite length (corresponding in the present context to an infinite string) such a central term in the charge algebra is clearly not possible since (19) would be given by an open line integral and consequently would be infinite. In this case, of course, one can consider the algebra of the currents for which there is a corresponding modification. Specifically, the expression (18) appears in the *once-integrated* current algebra. That such an extension for the current algebra for the GS superstring should be present was first pointed out by Hughes and Polchinski[8] and it has recently been confirmed by Gauntlett[9] for the $d=3$ GS superstring by an explicit calculation in a "physical gauge" ($X^i=\sigma^i$). The gauge-fixed action may be identified as a two-dimensional field theory with a partially broken rigid supersymmetry (PBRS). From the *usual* supersymmetry current algebra one can deduce that this phenomena is not possible. Hence, *the significance of the topological term in the current algebra is that it allows the possibility of PBRS.* Similar considerations apply to higher-dimensional extended objects[3,10,11] but we postpone further discussion of PBRS until the end.

Our next example, which sets the pattern for the general case, is the supermembrane for which

$$
\begin{aligned}
&h=i(d\bar\theta\Gamma_{\mu\nu}d\theta)\Pi^\mu\Pi^\nu, \quad h=db\,,\\
&b=i(\bar\theta\Gamma_{\mu\nu}d\theta)[\Pi^\mu\Pi^\nu+i\Pi^\mu(\bar\theta\Gamma^\nu d\theta)-\tfrac{1}{3}(\bar\theta\Gamma^\mu d\theta)(\bar\theta\Gamma^\nu d\theta)], \quad T\delta_\alpha b=d\Delta_\alpha\,,\\
&T^{-1}\Delta_\alpha=i(\Gamma_{\mu\nu}\theta)_\alpha\Pi^\mu\Pi^\nu-\tfrac{5}{3}(\Gamma_{\mu\nu}\theta)_\alpha(\bar\theta\Gamma^\mu d\theta)\Pi^\nu+\tfrac{1}{3}(\Gamma^\mu\theta)_\alpha(\bar\theta\Gamma_{\mu\nu}d\theta)\Pi^\nu\\
&\qquad -\tfrac{11}{15}i(\Gamma_{\mu\nu}\theta)_\alpha(\bar\theta\Gamma^\mu d\theta)(\bar\theta\Gamma^\nu d\theta)-\tfrac{4}{15}i(\Gamma^\nu\theta)_\alpha(\bar\theta\Gamma_{\mu\nu}d\theta)(\bar\theta\Gamma^\mu d\theta)\,.
\end{aligned}
\tag{21}
$$

Using the identity $(\Gamma_\mu)_{(\alpha\beta}(\Gamma^{\mu\nu})_{\gamma\delta)}\equiv 0$ valid for $d=4,11$ (and for $d=5,7$ with appropriate modifications for complex spinors) we now obtain, after some calculation, the once-integrated expression

$$
\begin{aligned}
T^{-1}\int d^2\sigma' A_{\alpha\beta}(t;\sigma,\sigma') &= 2i(\Gamma_{\mu\nu})_{\alpha\beta}\epsilon^{0ij}\partial_i X^\mu\partial_j X^\nu\\
&\quad+\epsilon^{0ij}\partial_i[\tfrac{4}{3}(\Gamma_{\mu\nu}\theta)_\alpha(\Gamma^\nu\theta)_\beta\partial_j X^\mu+\tfrac{4}{3}(\Gamma^\nu\theta)_\alpha(\Gamma_{\mu\nu}\theta)_\beta\partial_j X^\mu\\
&\quad+\tfrac{6}{15}i(\Gamma_{\mu\nu}\theta)_\alpha(\Gamma^\mu\theta)_\beta\bar\theta\Gamma^\nu\partial_j\theta+\tfrac{2}{15}i(\Gamma^\nu\theta)_\alpha(\Gamma_{\mu\nu}\theta)_\beta\bar\theta\Gamma^\nu\partial_j\theta\\
&\quad-\tfrac{4}{15}i(\Gamma^\mu\theta)_\alpha(\Gamma^\nu\theta)_\beta(\bar\theta\Gamma_{\mu\nu}\partial_j\theta)]\,.
\end{aligned}
\tag{22}
$$

Since θ is single valued we find, upon further integration, that

$$
\begin{aligned}
A_{\alpha\beta}&=2iT(\Gamma_{\mu\nu})_{\alpha\beta}\int d^2\sigma\,\epsilon^{0ij}\partial_i X^\mu\partial_j X^\nu\\
&=2iT(\Gamma_{\mu\nu})_{\alpha\beta}\int d^2\sigma\, j_T^{0\mu\nu}\,,
\end{aligned}
\tag{23}
$$

where $j_T^{0\mu\nu}$ is the time component of the topological current (1) for $p=2$.

For the general supersymmetric p-extended object the relevant θ-dependent terms generalizing those in (22) are quite complicated and we shall not give them here. The relevant first term is always present, however, and generalizes in the obvious way so that

$$A_{\alpha\beta}=2iT(\Gamma_{\mu_1\cdots\mu_p})_{\alpha\beta}\int d^p\sigma\, j_T^{0\mu_1\cdots\mu_p} \tag{24}$$

[one can always find a translationally invariant $(p+1)$-form b,[12] in which case the last term in (14) does not contribute to A_{AB}; a calculation then reveals that the first two terms are a total spatial derivative and the only contribution is (24)]. Passing to the quantum (anti)commutators, by adding a factor i in (24), the announced result (2) is established.

For a closed p-cycle of p-volume V_p this can be written as $A_{\alpha\beta}=2iT(\Gamma_{12\cdots p})_{\alpha\beta}V_p$. Since we have been supposing that spacetime is flat it follows that the only p-cycle is a p-torus. However, the action may be generalized to a curved spacetime although, in general, the supersymmetry of the action will be lost. Nevertheless, it may happen that some of the supersymmetry is preserved in which case we can again ask whether the supersymmetry algebra will be modified. Since the WZ term is independent of the supervielbein of Σ, the previous analysis will still hold, but now the topology of space may be such that there are p-cycles that are *not* p-tori. A nice example of this is the eleven-dimensional supermembrane with a background spacetime $K_3\times M_7$, where K_3 is the compact four-dimensional Ricci-flat surface with holonomy SU(2) and M_7 is seven-dimensional Minkowski spacetime. In this case half of the original eleven-dimensional supersymmetry is preserved.[13] (Note also that this is a solution of the eleven-dimensional supergravity field equations, which is required for consistency of the $d=11$ supermembrane action.[1]) As the second Betti number (the number of independent two-cycles) of K_3 is 22, any one of 22 topological charges may appear in the supermembrane supersymmetry algebra.

We return to the question of PBRS, which we may now discuss in the general context of p-dimensional extended objects. For extended objects of *finite* p-volume we need only examine the consequences of the charge algebra (2). For a standard static configuration of p-

volume V_p, we have $P_\mu = (TV_p, \mathbf{0})$. The algebra (2) becomes

$$\{Q_\alpha, Q_\beta\} = 2V_p T(\Gamma^0)_{\alpha\gamma}[1+\lambda(\Gamma_{012\cdots p})]^\gamma{}_\beta\,, \qquad (25)$$

where λ is a dimensionless coefficient which, for present purposes, we now introduce as a multiplicative factor for the WZ term (5). Note $(\Gamma_{012\cdots p})^2 = (-1)^{(p-2)(p-5)/2}$. Strictly speaking our conventions here apply only if $d=3$, 4, or 11 (or $d=10$ if we were to include chirality projection operators). Only $p=1,2,5$, are possible for these values of d, in which case $(\Gamma_{012\cdots p})^2 = 1$. Although this fact is essential for the argument to follow, which therefore only applies for $p=1,2,5$, the final conclusion is general. (The $d=9$ superparticle case is incorporated by adding an additional factor of i required for imaginary Γ's.) It follows from $(\Gamma_{012\cdots p})^2 = 1$ that for $\lambda = \pm 1$ the matrix $\frac{1}{2}(1 \pm \Gamma_{012\cdots p})$ is a projection operator. Since $\mathrm{Tr}(\Gamma_{012\cdots p}) = 0$ we further conclude that *half* of the eigenvalues of $1 \pm \Gamma_{012\cdots p}$ vanish. Thus for the "critical" values $\lambda = \pm 1$ the configuration leading to the algebra (25) breaks half of the supersymmetry. Without loss of generality we can choose $\lambda = 1$, and it is precisely in this case that the inclusion of the WZ term (5) in the action leads to the presence of an additional fermionic "κ symmetry." This is not a coincidence as one can show by other means that κ symmetry implies a partial breaking of supersymmetry[3,9–11] (which also applies to the massive superparticle[7]).

In this Letter we have shown how the WZ term in the action of a supersymmetric extended object leads to a topological charge in the supersymmetry algebra. Since this action may be considered as the effective action for extended-object solutions of supersymmetric field theories, we expect that the same result could be obtained directly from the d-dimensional field theory by paing attention to total-derivative terms in the anticommutator of the *field-theory* supersymmetry charges. Indeed, it has already been verified[11] that for the superparticle, at least, the modified supersymmetry algebra is the same whether one obtains it from the d-dimensional field theory[14] or from the effective particle action. Note, however, that in the context of the d-dimensional field theory one should *not* think of the topological charge as allowing a partial breaking of supersymmetry because the particlelike configuration leading to the modified algebra is not the ground state of the theory. In contrast, such a configuration *is* the ground state of the effective $(p+1)$-dimensional theory, and PBRS occurs as a result of the topological charge.

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[1]E. Bergshoeff, E. Sezgin, and P. K. Townsend, Phys. Lett. **189B**, 75 (1987); A. Achúcarro, J. Evans, P. K. Townsend, and D. L. Wiltshire, Phys. Lett. **198B**, 441 (1987).

[2]M. B. Green and J. H. Schwarz, Phys. Lett. **136B**, 367 (1984); Nucl. Phys. **B243**, 285 (1984); M. Henneaux and L. Mezincescu, Phys. Lett. **152B**, 340 (1985).

[3]J. Hughes, J. Liu, and J. Polchinski, Phys. Lett. **180B**, 370 (1986).

[4]For further details of our conventions, see E. Bergshoeff, E. Sezgin, and P. K. Townsend, Ann. Phys. (N.Y.) **185**, 330 (1988); E. Bergshoeff, E. Sezgin, Y. Tanii, and P. K. Townsend, "Super p-branes as gauge theories of volume-preserving diffeomorphisms" (to be published).

[5]J. A. de Azcárraga and P. K. Townsend, Phys. Rev. Lett. **62**, 2579 (1989). [Note that the sentence in parentheses in (ii), p. 2580, "This . . . group," should be deleted.]

[6]J. A. de Azcárraga, J. M. Izquierdo, and A. J. Macfarlane, University of Cambridge Report No. DAMTP-89-18 (to be published).

[7]J. A. de Azcárraga and J. Lukierski, Phys. Lett. **113B**, 170 (1982); Phys. Rev. D **38**, 509 (1988).

[8]J. Hughes and J. Polchinski, Nucl. Phys. **B278**, 147 (1986).

[9]J. P. Gauntlett, "Current algebra of the $d=3$ superstring and partial breaking of global supersymmetry" (to be published).

[10]P. K. Townsend, Phys. Lett. B **202**, 53 (1985).

[11]A. Achúcarro, J. P. Gauntlett, K. Itoh, and P. K. Townsend, Nucl. Phys. **B314**, 129 (1989).

[12]J. Evans, Classical Quantum Gravity **5**, L87 (1988).

[13]M. J. Duff, B. E. W. Nilsson, and C. N. Pope, Phys. Lett. **129B**, 39 (1983).

[14]E. Witten and D. Olive, Phys. Lett. **78B**, 97 (1978).