# Pinning and Thermal Fluctuations of a Flux Line in High-Temperature Superconductors

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Using a simple model of single vortex depinning, we show that the temperature dependence of both resistivity and ac susceptibility in the superconducting state in moderate magnetic fields can be understood in terms of the activation of vortices from homogeneous and densely distributed pinning centers. Furthermore, the field dependence of the activation energy is explained by including the interaction of vortices above $H \sim 1$ T, below which the distribution of pinning strengths becomes important.

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Considerable effort has gone into the study of the dynamics of flux lines in high-temperature (high-$T_c$) superconductors. However, it is not yet clear what exactly are the relevant mechanisms involved in order to explain the experimental results. Because the effects of disorder can be quite important, it is advantageous to focus on experiments which have a good chance of probing the *equilibrium* state of the system; here, we concentrate on flux-flow resistivity and ac susceptibility.

The resistivity of high-$T_c$ superconductors[1] in moderate magnetic fields has an Arrhenius dependence on temperature for $T \ll T_c$, suggesting thermally activated flux creep. However, we lack clear understanding of the deviation from Arrhenius behavior seen as the temperature is raised, and the "prefactor" of the exponential extracted from the fit at low temperatures seems, anomalously, both large and magnetic field independent. In addition, the imaginary part of the ac susceptibility shows a peak at a frequency roughly given by $\omega_{\text{peak}} = \omega_0 \times \exp(-U/k_B T)$, where the activation energy $U$ involved is consistent with that in resistivity, and the prefactor $\omega_0$ is field independent.[2] There have been some theoretical studies regarding the dynamics of a flux lattice addressing some of these points,[3,4] but these works have not made connection with experiments in a comprehensive manner.

We propose that the apparently anomalous behavior of resistivity can be explained by conventional means if we assume that the pinning sites are uniformly and densely distributed throughout the sample. Such a view is supported by the recent decoration of Dolan *et al.*[5] in which an apparently well-ordered flux lattice is seen at low temperature, even after the external field is removed.

Another important aspect of this problem is the unusual softness of a flux lattice for localized deformations, which has been demonstrated for large $\lambda$ and anisotropy.[6] This enables us, at the simplest level, to treat vortices independent of each other, with interaction included as a correction to the single vortex dynamics. In the same spirit, we will not consider the possibility of collective pinning[7] in this paper.

We first define the "effective" pinning barrier associated with a single vortex depinning from a single pinning site in two dimensions. This will facilitate the later treatment of multiple pinning sites. Consider that the interaction potential between a vortex and a pinning site is proportional to $|\psi|$, where $\psi$ is the Ginzburg-Landau superconducting order parameter around a vortex[8] $|\psi| \simeq |\psi_\infty| \tanh(\nu r/\xi_c)$. Here, $\xi_c$ is the coherence length and $\nu$ is a constant of order unity. Then the potential is given in the form $(v>0)$ $v(r) \simeq V_0 \tanh(\nu r/\xi_c)$. For simplicity, we approximate this potential by

$$v(r) \approx \begin{cases} V_0 r/r_c, & r<r_c, \\ V_0, & r>r_c, \end{cases} \tag{1}$$

with $r_c \approx \xi_c$. The effective escape rate out of this potential well can be estimated using Kramer's method,[9] i.e., by solving the diffusion equation with an absorbing boundary condition at large distances. We suppose that, since $v(r)$ is constant for $r > r_c$, a vortex is escaped from the well only if it flows outside of some radius $r_b$ $(\sim n_i^{-1/3}$, where $n_i$ is the concentration of pinning sites). Then we write the escape rate (integrated over the perimeter) as

$$\Gamma(T) \simeq \frac{2\pi D_0 \int_0^{r_b} d^2r\, p(r) \exp[v(r)/k_B T]}{\int_0^{r_b} d^2r\, p(r) \int_0^{r_b} d^2r \int_r^{r_b} dr'\, r'^{-1} \exp[v(r')/k_B T]}, \tag{2}$$

where $p(r) = p_0 \exp[-v(r)/k_B T]$ is the probability distribution and $D_0$ is the "bare" diffusion constant. One can then define the effective (temperature dependent) barrier $\bar{v}$ by introducing the relation $\exp(-\bar{v}/k_B T) = \Gamma(T)/\Gamma(T \to \infty)$. At low temperatures, we have $\bar{v} \approx V_0 - k_B T \ln(\alpha\beta^2/2)$ for $\beta \gg 1$, where $\alpha = (r_b/r_c)^2$ $(>1)$ and $\beta = V_0/k_B T$. To further simplify, we write, neglecting the weak temperature dependence in the logarithm,

$$\bar{v} \approx V_0 - \mu k_B T, \tag{3}$$

where $\mu$ is roughly constant. Note that this expression fails as soon as there are multiple pinning sites within the range of interaction $r_c$ $(\approx \xi_c)$.

We next estimate the effect of *multiple* pinning centers on vortex diffusion. In order to keep the treatment simple, we take the position of pinning sites $\mathbf{R}_n$ to be periodic in one-dimensional space and approximate it by a sinusoidal form ($|\mathbf{K}|=2\pi/l_{\rm imp}$, where $l_{\rm imp}$ is the mean distance between pinning sites)

$$V=-\sum_n v(|\mathbf{r}-\mathbf{R}_n|)\approx-\tfrac{1}{2}\bar{v}\cos(\mathbf{K}\cdot\mathbf{r})\,. \qquad (4)$$

The resulting diffusion equation in a sinusoidal potential has been solved by Ambegaokar and Halperin[10] (in the context of Josephson junctions) to yield an effective diffusion constant. Using the Einstein relation and Eq. (3), we obtain the resistivity as a function of temperature

$$\rho=\rho_0[I_0(\bar{v}/2k_BT)]^{-2}\simeq\rho_0[I_0(V_0/2k_BT-\mu/2)]^{-2}\,, \qquad (5)$$

where $I_0$ is the modified Bessel function and Eq. (3) has been used to make the temperature dependence more explicit. The prefactor $\rho_0$ is expected to depend on the external field in that it should reflect the density of vortices in a sample, approaching zero as external field is reduced to zero. A simple argument by Bardeen and Stephen[11] gives $\rho_0/\rho_n\simeq B/H_{c2}$, where $\rho_n$ is the normal-state resistivity. Although the form (5) is very crude, it has the essential features of the experiments; a large enhancement of the prefactor at low $T$ and the crossover to constant resistivity (flux flow) as $\bar{v}$ becomes comparable to $k_BT$. Note that this crossover will occur at a temperature below the mean field $T_c(H)$.

A similar method can be used to interpret the ac susceptibility. Consider the overdamped equation of motion of a *pinned* vortex,

$$-\eta\dot{\mathbf{u}}+\frac{\phi_0 d}{c}(\mathbf{j}\times\mathbf{e}_z)-\frac{\delta E_{\rm pin}}{\delta\mathbf{u}}=0\,, \qquad (6)$$

where $\eta=\phi_0H\sigma d/c^2$ is the damping constant for a vortex with $\sigma$ the conductivity, $\mathbf{u}$ is the displacement of a vortex from its equilibrium position, $d$ is the typical length scale for deformations along a flux line (taken to be in the $z$ direction), and $E_{\rm pin}$ is the pinning energy. Although the length $d$ cannot be determined accurately, we expect it to be of the same order as $l_{\rm imp}$ at low temperatures where the impurity energy dominates the elastic stiffness of the vortex line. (An estimate will be made below.) We take $E_{\rm pin}$ to be approximately $\sim\frac{1}{2}\kappa u^2$, where $\kappa$ is constant for $T\to 0$ but crosses over to $\sim\bar{v}/l_0^2$ with $l_0$ the characteristic length scale for the pinning, expected to be of the scale of $\xi_c$. Then it is straightforward to derive the relation[12]

$$\delta\mathbf{j}=-\frac{c\kappa}{\phi_0 Hd}\delta\mathbf{A}+\frac{\eta c^2}{\phi_0 Hd}\delta\mathbf{E}=\frac{c}{\phi_0 Hd}(i\omega\eta-\kappa)\delta\mathbf{A}\,, \qquad (7)$$

which defines the effective penetration depth

$$\lambda_{\rm eff}^{-2}=\frac{4\pi\kappa}{\phi_0 Hd}\ \text{for}\ T\ll T_c\,. \qquad (8)$$

Consequently, $\lambda_{\rm eff}^{-2}$ measures the strength of the underlying pinning potential, and the temperature at which $\lambda_{\rm eff}$ diverges is then the point where $\bar{v}$ becomes comparable to $k_BT$.

The peak in the response $\delta A/\delta j$ is found from Eq. (7) to be at

$$\omega_{\rm peak}\simeq\frac{c^2\rho(T)}{4\pi\lambda_{\rm eff}^2}=\frac{c^2\rho_0}{4\pi\lambda_{\rm eff}^2\{I_0[\bar{v}(T)/2k_BT]\}^2}\,. \qquad (9)$$

Therefore, apart from temperatures near $T_c$, the temperature dependence of $\omega_{\rm peak}$ is dominated by that of $\rho(T)$, which also implies that we expect to see the deviation from activated behavior similar to the one found in resistivity, as measurement is extended to higher temperature and frequency. Furthermore, the prefactor $\omega_0$ should be field independent.

These simple results can be compared with available experimental data. We show in Fig. 1 the fit of (5) to the resistivity data of $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_{8+\delta}$;[13] the general shape of experimental curve is well described by our form. In particular, the observed "plateau" in the resistivity is reproduced for $H=10$ T, indicating the crossover from thermally activated creep regime to flow regime. We have also fitted data using the full expression of $\bar{v}$ (see definition of $\bar{v}$) for $H=1$, 3, and 10 T, and found that the quality of the fit improves slightly at higher temperatures.

The parameter $\mu$, which originates from the correction

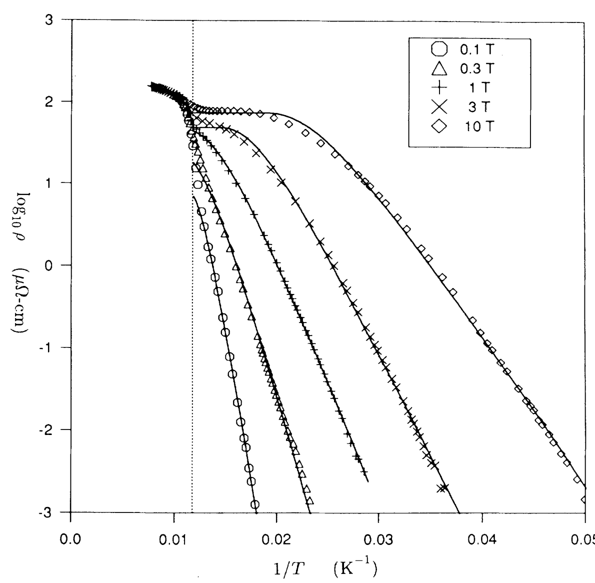


FIG. 1. Theoretical fit of Eq. (5) to the resistivity data of $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_{8+\delta}$ vs inverse temperature. Fits are shown by solid lines. Fitting parameters are, in sequence of increasing field, $\rho_0=8$, 23, 46, 49, and 74 $\mu\Omega$, $V_0=1890$, 1090, 760, 630, and 460 K, and $\mu=21.26$, 11.44, 8.48, 9.16, and 8.84. The $T_c(H=0)=84$ K is indicated by the dashed line. The values of $\rho$ and $\mu$ become difficult to fix at low fields where the crossover from activated to flow regime is smeared out close to $T_c$.

to the pinning barrier due to the thermal fluctuations, is found to be roughly independent of magnetic field for $H \gtrsim 1$ T and is about 8–9. This value is somewhat larger than the estimate $\mu \sim 6$ with $\alpha = (r_b/r_c)^2 \sim 10$ and $V_0/k_B T \sim 6$. However, it is clear that the value of $\mu$ depends on the detailed form of the pinning potential, and our crude calculation is only semiquantitative. More significant is the prefactor $\rho_0$ corresponding to the vortex flow resistence at $T \rightarrow \infty$. It is of the same order as the normal-state resistance, unlike the value obtained by fitting a simple exponential form, and we find $\rho_0$ to depend on the external magnetic field, approaching zero as the field is reduced, in qualitative agreement with our expectations.[11] However, the activation energy at $T=0$ is also found to be field dependent (increasing as the field is decreased). Such field dependence cannot be understood within the framework of single vortex depinning, and it is an indication that the interaction between vortices cannot be entirely ignored. We will comment on this point below.

We do not attempt to deduce the field dependence of $\rho_0$ on $H$ since our calculation is based on independent vortices with London approximation without any temperature or field dependence of microscopic parameters. This must break down as we approach $T_c$ or $H_{c2}$. For the same reason, the fluctuations are too large near $T_c$ to fix fitting parameters precisely and we are not successful in making meaningful fits to the available resistivity data for $YBa_2Cu_3O_{7-y}$. We should point out, however, that the observed "glitch" in the resistivity data of $YBa_2Cu_3O_{7-y}$ (the point where the data start to deviate from Arrhenius form, called $T_x$ by Batlogg *et al.*[14]) is indicative of the crossover from creep to flow, like the ones seen in $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_{8+\delta}$ data.

The comparison of our model and ac magnetic susceptibility of $YBa_2Cu_3O_{7-y}$ is also useful. As mentioned above, the effective penetration depth is a measure of the temperature-dependent pinning strength. According to Hebard *et al.*,[12] the effective penetration depth follows the form for $H \gtrsim 1$ T,

$$\lambda_{\text{eff}} \sim \frac{\lambda_0 H^{1/2}}{1-(T/T_c')^2}, \qquad (10)$$

where $T_c'$ here is the temperature where $\lambda_{\text{eff}}$ is extrapolated to diverge. Equation (10) shows that $\lambda_{\text{eff}}^{-2}$ stays roughly constant at low temperatures and then decreases as temperature is increased. This is in qualitative agreement with our analysis at low temperatures based on Eq. (8). The observed $H^{1/2}$ field dependence of $\lambda_{\text{eff}}$ indicates that an independent vortex model is appropriate, and the comparison of the full form of $\bar{v}$ near the temperature of diverging $\lambda_{\text{eff}}$ is consistent with experiment in that we find a wide temperature range where $(\bar{v})^{1/2}$ decreases roughly linearly with temperature until $\bar{v}$ becomes comparable to $k_B T$. In addition, a characteristic length scale $l_0$ can be estimated from this comparison. Using $\bar{v} \sim 10^4$ K near $T=0$, we obtain $dl_0^2 \sim 10^5$ Å$^3$, which is consistent with $d \sim 100$ Å and $l_0 \sim 30$ Å $\sim \xi_c$.

The low-temperature activation energies observed by Gammel *et al.*[2] are of the same order as those obtained from the resistivity of $YBa_2Cu_3O_{7-y}$, as expected from Eq. (9). In fact, it would be interesting, as already mentioned, to see if the significant deviation from activated behavior can be observed by increasing experimental frequency and temperature.

The field dependence of the $T=0$ activation energy requires a somewhat more elaborate model than the single vortex depinning picture. The intervortex interaction superimposed to the periodic potential Eq. (4) actually increases the diffusion rate, thus reducing the measured activation energy. A simple estimate can be made in the limit of $T \rightarrow 0$ as follows: Suppose that $x=0$ is the equilibrium position of a vortex in the absence of impurity pinning. Then we have the effective potential for this vortex in the form $\frac{1}{2}\bar{v}\cos K(x-x_0)+\gamma x^2$. Considering hopping out of the *minimum* of this potential in the limit of $\gamma \ll \bar{v}K^2$ and $T \rightarrow 0$, we find that the average over the phase $x_0$ is dominated by the minimum value of the barrier encountered. Hence the zero-temperature barrier is modified from $\bar{v}$ to (replacing $\bar{v}$ by $V_0$) $V_0 - \pi^2\gamma/K^2$. This expression is only valid when there is a single (field independent) value of $V_0$.

At low enough fields, however, the use of a single $V_0$ becomes unjustified because of the distribution of pinning strengths, and the observed $V_0$ becomes strongly field dependent at low vortex density.[15] Choosing an exponential distribution[16] $\{\sim\exp[-(V-V_0)/\sigma]\}$ and assuming that vortices "fill" the strongest pinning sites first, we find the field dependence of the "typical" barrier to be $V_0+\sigma\ln(H_0/H)$, where $H_0$ is some constant.

Thus we have two types of field dependence separated by a crossover field $H_1$,

$$V_{\text{barrier}} \approx \begin{cases} V_0+\sigma\ln(H_0/H)\,, & H \ll H_1\,, \\ V_0-\pi^2\gamma/K^2\,, & H \gg H_1\,. \end{cases} \qquad (11)$$

In Fig. 2, we show that Eq. (11) gives a consistent fit to the $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_{8+\delta}$ data; we use the expression (we neglect the compression modulus) $\gamma \approx d\phi_0 H/128\pi^3\lambda^2$, where $d$ is the coherence length in the $z$ direction as before. The coefficient of the linear term gives for the volume $dl_{\text{imp}}^2$ approximately $2\times10^6$ Å$^3$, a reasonable value for $d \sim l_{\text{imp}} \sim 100$ Å. Since the field dependence becomes small above the crossover ($\sim$2 T), our analysis has the same qualitative features as the observations of Hebard *et al.* [cf. Eqs. (8) and (10)]. Our fit clearly is not a rigorous test of our ideas because of the accuracy and small number of data points involved. However, it demonstrates that the *values* of these parameters are consistent with other experiments.

In summary, we have made an interpretation of the recent experiments on resistivity and ac susceptibility in terms of densely populated pinning sites. We find that

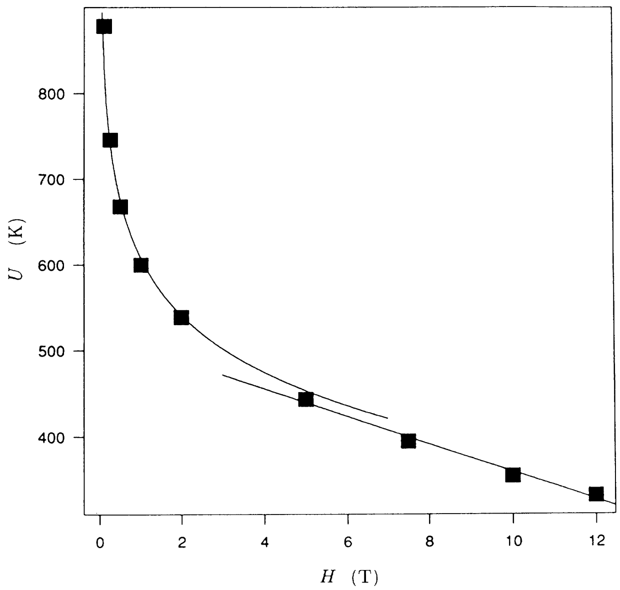


FIG. 2. Fit to the field-dependent zero-temperature activation energy as calculated from resistivity data (Ref. 1). The solid line indicates the fit of (11). The parameters are $V_0=520$ K, $\sigma=220$ K, $H_0=2.5$ T, and $\pi^2\gamma/K^2=16$ K/T. The crossover $H_1$ is about 2 T.

the temperature dependence of these experiments can be explained with a very simple single vortex depinning picture. The additional effects due to the intervortex interaction can be included to obtain the field dependence consistent with available experimental data.

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