Bifurcation Theory of Poloidal Rotation in Tokamaks: A Model for the *L*-*H* Transition

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It is shown that the poloidal momentum balance equation in tokamaks has bifurcated solutions. The poloidal flow velocity U_p can suddenly become more positive when the ion collisionality decreases. The corresponding radial electric field E_r becomes more negative and hence suppresses the turbulent fluctuations. Thus, plasma confinement is improved. The theory is employed to explain the *L*-*H* transition observed in tokamaks.

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It was suggested that the onset of the transition from the *L* mode to the *H* mode in tokamaks is triggered by a sudden change of the radial electric field, E_r, to a more negative value, which subsequently suppresses the turbulent fluctuations.[1-3] Plasma confinement dominated by the anomalous transport process is thus improved in the *H* mode. The conclusion that a more negative value of E_r or a more positive value of dE_r/dr can suppress the fluctuation amplitudes resulted from a kinematic argument.[2,3] This conclusion was later demonstrated both numerically and analytically for specific turbulence models.[3,4] Besides the *H* mode, other enhanced-confinement regimes observed in tokamaks and stellarators could also be attributed to a change of E_r to a more negative value.[1-3] A model for the *L*-*H* transition based on the suppression of the turbulent fluctuations with a more negative value of E_r was proposed to describe the change of E_r when the ion collisionality decreases.[2,3] A difficulty with the model is that the equation for E_r has no bifurcated solutions. Thus, E_r becomes continually more negative as ion collisionality decreases. The time scale of the transition from the *L* mode to the *H* mode is of the order of the energy confinement time in the edge region. This seems to contradict the extremely short time scales observed in the experiments.[5,6]

Recently, the value of E_r at the onset of the *L*-*H* transition was inferred from measurements of plasma rotational speeds in the Continuous Current Tokamak (CCT) and DIII-D and directly measured on CCT.[6,7] It was found that, indeed, E_r became more negative at the onset of the transition. The power threshold of the transition was lower for the cases with counterinjected neutral beams than for those with coinjected beams. The measured fluctuation spectrum in the *H* mode also showed that the fluctuation amplitudes are reduced in the high-frequency, short-wavelength regions.[8] All these qualitative behaviors are in agreement with the original predictions in Refs. 1 and 2. Itoh and Itoh have proposed a different model for the *L*-*H* transition based on the change of E_r, in which it is assumed that diffusive transport processes are not intrinsically ambipolar.[9] They concluded that a more positive value of E_r improves plasma confinement.

Here, we develop a theory based on the original proposition: The transition is triggered by a sudden change of E_r to a more negative value and/or of dE_r/dr to a more positive value, which in turn suppresses turbulent fluctuations by modifying the decorrelation time through the shear of the angular velocity and improves plasma confinement. The model to be discussed has bifurcated states of E_r that remove the difficulty of the original model. We choose, however, the poloidal flow speed U_p and the toroidal flow speed U_t as independent variables, rather than U_p and E_r, as used in the original model. This choice is made so that the comparison with the experimental observations will be easier, since the data are given in terms of U_p and U_t.[6] Here, we assume that U_t remains unchanged at the transition. This assumption is approximately valid, as can be seen from the data given in Ref. 6. In an extended version of the model, this assumption will be removed.

We first discuss the relationship between the sign of E_r and that of U_p. The plasma flow velocity $\mathbf{U}$ can be written as $\mathbf{U}=U_\parallel\hat{\mathbf{n}}+\mathbf{U}_\perp$, where $U_\parallel$ is the parallel (to the magnetic field $\mathbf{B}$) flow speed, $\hat{\mathbf{n}}=\mathbf{B}/B$, and the perpendicular flow velocity is $\mathbf{U}_\perp=(cE_r\hat{\mathbf{r}}\times\mathbf{B}/B^2)+c\mathbf{B}\times\nabla P/NeB^2$. Here, $\hat{\mathbf{r}}$ is the unit vector in the r direction, N is the plasma density, $P=NT$ is the ion pressure, T is the ion temperature, c is the speed of light, and e is the electric charge of the ions. Taking the dot products of $\mathbf{U}$ with $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\zeta}}$, the unit vectors in the poloidal $\boldsymbol{\theta}$ and toroidal $\boldsymbol{\zeta}$ directions, respectively, and eliminating the $U_\parallel$ dependence, we obtain

$$U_p=U_t(B_p/B)-(cE_r/B)+(c/NeB)dP/dr$$

by neglecting terms of the order of B_p^2/B^2, where B_p denotes the poloidal magnetic field strength. We have thus shown that U_p becomes more positive if E_r becomes more negative for fixed values of dP/dr and U_t. In the DIII-D experiment, dP/dr is also changing during the transition. However, because dP/dr is small compared with U_p, the change of U_p is related to the change of E_r.[6] Furthermore, if the dP/dr term becomes more negative at the transition, E_r will become even more negative for a fixed amount of the change of U_p to a more positive value. In an extended version of the model, the coupling of the momentum balance equations to the pressure evolution equation will be included.

To determine U_p, we solve the poloidal momentum equation. In standard neoclassical theory, the poloidal momentum is damped by the poloidal viscosity $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ because there is neither a momentum source nor a momentum sink. Here, $\boldsymbol{\pi}$ is the ion viscosity, and the angular brackets denote the flux-surface average. However, in the edge region the poloidal rotation is driven by the torque associated with the ion-orbit loss. To model the transition of U_p, we need to extend the validity of the poloidal viscosity to the regime where $U_p B/v_t B_p \sim 1$. where $v_t = (2T/M)^{1/2}$, with M the ion mass. This can be achieved by solving the drift kinetic equation with mass flow velocity[10] by the standard method. The result is[11]

$$\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle = \frac{\sqrt{\pi}}{4} \left(\frac{\epsilon^2}{r} \right) N M v_t B (I_p U_p + I_T U_{p0}) , \qquad (1)$$

where M is the ion mass, $\epsilon = r/R$, R is the major radius, $U_{p0} = -\rho_i v_t (dT/dr)/2T$, and ρ_i is the ion gyroradius. The integrals I_p and I_T are defined as

$$\begin{Bmatrix} I_p \\ I_T \end{Bmatrix} = \frac{1}{\pi} \int_0^{\nu_{*i}^{1/2}} dx \begin{Bmatrix} 1 \\ \frac{5}{2} - x \end{Bmatrix} x^2 e^{-x} \left[\tan^{-1} \left(\frac{1 + U_{p,m}/\sqrt{x}}{\nu_{*i} \epsilon^{3/2}/x^2} \right) - \tan^{-1} \left(\frac{-1 + U_{p,m}/\sqrt{x}}{\nu_{*i} \epsilon^{3/2}/x^2} \right) \right] ,$$

where $U_{p,m} = U_p B/v_t B_p + \lambda_p/2$, $\lambda_p = -\rho_{pi} (dP/dr)/P$, $\nu_{*i} = \nu R q / v_t \epsilon^{3/2}$, $q = \epsilon B/B_p$, ρ_{pi} is the ion poloidal gyroradius, and ν is the ion-ion collision frequency. The expression in Eq. (1) is valid for $\nu_{*i}(v) \gtrsim 1$. Note that $\nu_{*i}(v)$ is defined to have the same form as ν_{*i} except ν is evaluated at speed v, and v_t is replaced by v. The $\lambda_p/2$ term in $U_{p,m}$ is to cancel the diamagnetic flow term in U_p so that only $\mathbf{E} \times \mathbf{B}$ drift and parallel flow $U_\parallel \hat{\mathbf{n}}$ appear in the convective term of the drift kinetic equation. It is straightforward to show that, as $U_{p,m}$ approaches zero, Eq. (1) reduces to the expression of the standard neoclassical viscosity to within a numerical factor of the order of unity. The fact that $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ has a local maximum in U_p and that $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ decreases with ν_{*i} are important for the poloidal momentum balance equation to have bifurcated solutions of U_p. An expression for $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ similar to Eq. (1) valid in the plateau regime was obtained in Ref. 12.

In the edge region (about one poloidal ion gyroradius away from the boundary) of a tokamak, the ion orbit can either intersect the limiter or cross the separatrix. The ion-orbit loss region is roughly determined by the resonance between the parallel particle speed $v_\parallel$ and the poloidal $\mathbf{E} \times \mathbf{B}$ drift velocity. This relation gives rise to the resonance condition

$$v_\parallel / v = U_{p,m} v_t / v , \qquad (2)$$

where v is the particle speed. Because $|v_\parallel / v| \le 1$, Eq. (2) can be satisfied if $v/v_t > |U_{p,m}|$. Indeed, Eq. (2) is only a condition to form a banana orbit. The width of the banana orbit depends on $dU_{p,m}/dr$. The minimum energy of a loss orbit estimated from detailed consideration of the orbit width is of the form $(v/v_t)^2 > \beta (r-a)^2 (dU_{p,m}/dr)^2$, where β is a numerical constant and a is the minor radius.[13,14] Instead of solving a differential equation for $U_{p,m}$, we express $(dU_{p,m}/dr)^2 \times (r-a)^2$ in terms of $U_{p,m}$. This procedure is appropriate because $\Delta r/a \ll 1$, where $\Delta r \sim \rho_{pi}$ is the characteristic width over which ion-orbit losses are important. One can always relate $dU_{p,m}/dr$ to $U_{p,m}$ with a Taylor-series expansion. The minimum energy of a loss orbit is therefore $(v/v_t)^2 > (\alpha U_{p,m})^2$, where α, a numerical constant, accounts for the orbit shape, such as orbit squeezing,[13-15] and the profile of E_r. The value of α can be calculated for a given magnetic equilibrium. From the results of Refs. 14 and 15, we obtain $\alpha = \sqrt{A}/2[2(1+\epsilon)]^{1/2}$ with $A = 1/\epsilon$. For $\epsilon = 0.25$, $\alpha = 0.63$; for $\epsilon = 0.33$, $\alpha = 0.53$. Because ion-orbit loss is important only in the banana regime where $\nu_{*i}(v) < 1$, the speed of the loss ions must satisfy $v/v_t > \nu_{*i}^{1/4}$. With these two constraints on v/v_t, we can estimate the nonambipolar ion-orbit loss rate to be[2,3]

$$\left(\frac{\partial N}{\partial t} \right)_{\text{orbit}} = -N \nu \frac{G}{[\nu_{*i} + (\alpha U_{p,m})^4]^{1/2}} \times \exp\{ -[\nu_{*i} + (\alpha U_{p,m})^4]^{1/2} \} , \qquad (3)$$

where G is a geometric factor of the order of unity that depends on the detailed shape of the loss cone in phase space. The derivation of Eq. (3) is based on the calculation of the particle loss rate in the presence of a velocity-space loss cone in a mirror machine.[16] In that calculation, it is found that the particle loss rate can be estimated to be

$$(\partial N_j / \partial t)_{\text{loss}} = -N_j \nu_j \mathcal{R} \exp(-2E_c / M_j v_{tj}^2) / (2E_c / M_j v_{tj}^2) .$$

Here, $\mathcal{R}$ is a numerical constant that depends on the mirror ratio, E_c is the critical energy above which the loss cone exists, and the subscript j indicates the plasma species. In our case, the critical energy E_c depends on two factors: (a) the resonance condition shown in Eq. (2), which gives rise to $2E_c^R / M v_t^2 = (\alpha U_{p,m})^2$, and (b) the collisionality constraint, which leads to $2E_c^c / M v_t^2 = \nu_{*i}^{1/2}$. We combine these two constraints on E_c to obtain the overall $E_c^T = [(E_c^c)^2 + (E_c^R)^2]^{1/2}$. Other possible choices of E_c^T lead to different dependences on ν_{*i} in the ion-orbit loss rate. The parameter range over which the model can have bifurcated solutions for these other choices is wider than the one we employ here. We note that both $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ and $(\partial N/\partial t)_{\text{orbit}}$ depend only on U_p. Because of the toroidal symmetry, the toroidal flow velocity U_t does not appear in any classical or neoclassical process to lowest order in the gyroradius expansion.

The poloidal momentum balance equation is

$$\frac{d}{dt}(NM\langle B_p U_p \rangle)$$

$$=\frac{1}{c}\langle \mathbf{J}\times\mathbf{B}\cdot\mathbf{B}_p\rangle-\langle \mathbf{B}_p\cdot\nabla\cdot\boldsymbol{\pi}\rangle-\frac{1}{c}e\boldsymbol{\Gamma}_{\text{orbit}}\times\mathbf{B}\cdot\mathbf{B}_p\,, \qquad (4)$$

where $\boldsymbol{\Gamma}_{\text{orbit}}$ is the particle flux associated with the ion-orbit loss and $\mathbf{J}$ is the plasma current, which depends on $\partial E_r/\partial t$. We have neglected poloidal momentum, poloidal viscosity, and orbit loss of the electrons in Eq. (4). At steady state, we have

$$-\frac{e}{c}\boldsymbol{\Gamma}_{\text{orbit}}\times\mathbf{B}\cdot\mathbf{B}_p=\langle \mathbf{B}_p\cdot\nabla\cdot\boldsymbol{\pi}\rangle\,. \qquad (5)$$

Substituting Eqs. (1) and (3) into Eq. (5) and assuming $\boldsymbol{\Gamma}_{\text{orbit}} \simeq -\hat{\mathbf{r}}(\Delta r)(\partial N/\partial t)_{\text{orbit}}$, we obtain an algebraic equation for U_p,

$$G\frac{\nu_{*i}}{[\nu_{*i}+(\alpha U_{p,m})^4]^{1/2}}\exp\{-[\nu_{*i}+(\alpha U_{p,m})^4]^{1/2}\}$$

$$=\frac{\sqrt{\pi}}{4}\left(\frac{\sqrt{\epsilon}\rho_{pi}}{\Delta r}\right)\left(I_p\frac{U_pB}{v_tB_p}+I_T\frac{U_{p0}B}{v_tB_p}\right). \qquad (6)$$

For a given particle distribution function, some particles are in the banana regime and some are in the plateau-Pfirsch-Schlüter regime. Exactly which regime a particle is in depends on its energy. If a particle has a speed v such that $\nu_{*i}(v)$ is less than unity, it is in the banana regime. Otherwise, it is in the plateau-Pfirsch-Schlüter regime. If a particle is in the banana regime, it contributes to the ion-orbit loss according to Eq. (3). If it is in the plateau-Pfirsch-Schlüter regime, it contributes to $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ in Eq. (1). The appropriate weighting in the energy space for each term in Eq. (6) is treated with the energy integrals I_p and I_T for $\langle \mathbf{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ and with the constraint on E_c for ion-orbit loss. A simple physical interpretation of Eq. (6) is that high-energy collisionless particles contribute to the ion-orbit loss and drive a poloidal torque, while low-energy collisional particles contribute to the poloidal viscosity and resist poloidal rotation.

The solution of Eq. (6) can be obtained graphically by examining the intersections of two functions, $Y_1(U_{p,m})$ [the left side of Eq. (6)] and $Y_2(U_{p,m})$ [the right side of Eq. (6)] for a given set of parameters. As an example, we assume $G=1$, $(\pi\epsilon)^{1/2}\rho_{pi}/2\Delta r=2.78$, $\epsilon=\frac{1}{4}$, $\alpha=0.5$ for simplicity, $\lambda_p=0.2$, and $U_{p0}B/v_tB_p=0.2$. For this set of parameters, the bifurcated solution exists as long as $\alpha \lesssim 1.1$. We choose ν_{*i} as the control parameter. As can be seen from Fig. 1, at $\nu_{*i}=3.5$, there is only one solution of U_p, which is $\sim U_{p0}I_T/I_p$ and is the continuation of the solution of the neoclassical theory. We call

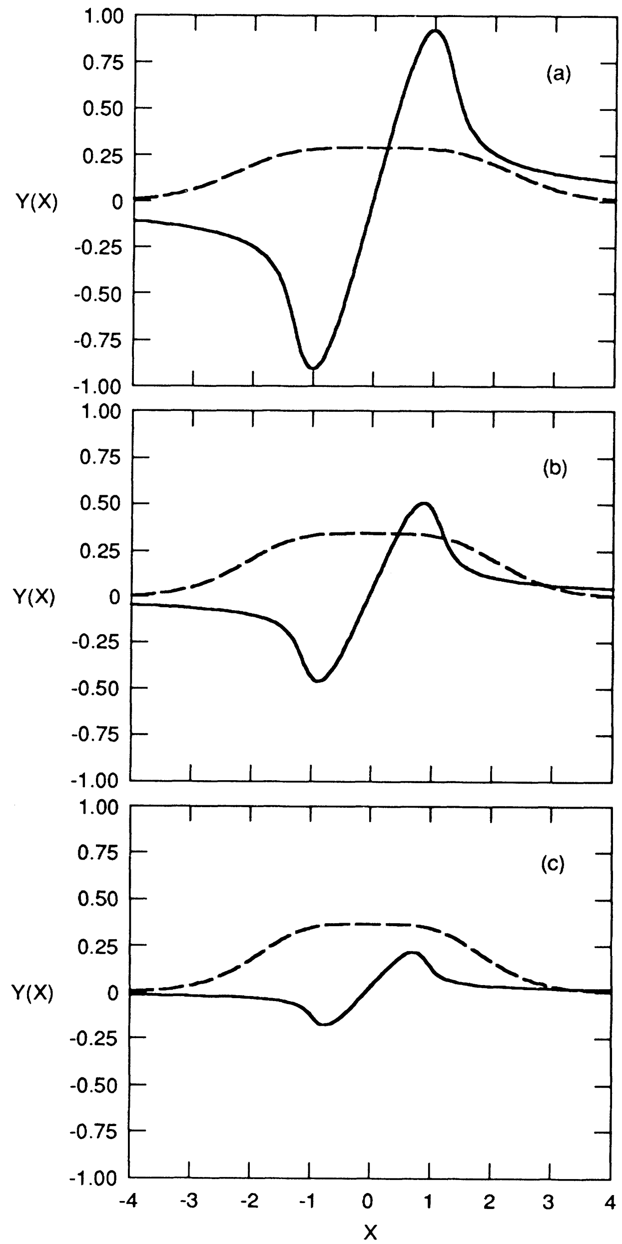

FIG. 1. The transition of $U_{p,m} \equiv X$ from (a) the L root to (b) the multiple-root state and finally to (c) the H root as ν_{*i} decreases. The dashed lines are $Y_1(X)$, and the solid lines are $Y_2(X)$.

this solution the L root. At $\nu_{*i}=2$, there are three solutions of U_p. The two outside solutions are stable, and the one in the middle is unstable. The solution close to the origin is the continuation of the L root; the more-positive stable solution is the new root (we call it the H root), which does not exist if there is no ion-orbit loss. When ν_{*i} decreases further, only the H root exists. In Fig. 2, we show $U_{p,m}$ as a function of ν_{*i} for the same parame-

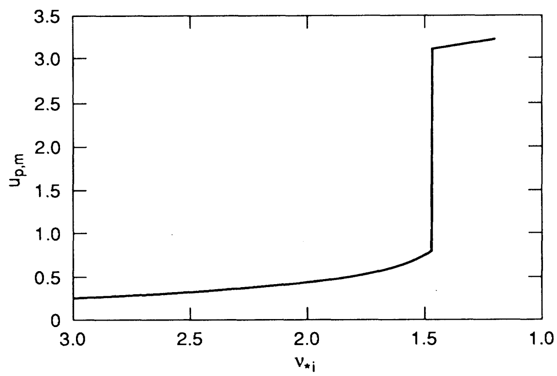

FIG. 2. The change of $U_{p,m}$ as v_{*i} decreases.

ters as in Fig. 1. The critical v_{*i} at which $U_{p,m}$ makes a sudden change can be either greater than or less than unity, depending on the parameters. The transition from the L root to the H root depends on the fluctuation level of U_p. If it is very low, U_p will stay on the L root as v_{*i} decreases and will not make the transition to the H root until the L root disappears. (We employ this rule in obtaining Fig. 2.) If, on the other hand, the fluctuation level of U_p is high, the transition is determined by the global minimum of a function that is proportional to the rate of entropy production.[17] The characteristic time scale involved at the transition is of order ν^{-1}.

When U_p becomes more positive, the corresponding E_r becomes more negative if U_t and dP/dr are fixed. As discussed in Refs. 1-3, a more negative E_r can suppress the turbulent fluctuation level and thereby improve confinement. The phenomenology of the L-H transition based on the model discussed here can be summarized as follows:

(i) Because of the plasma hearing, v_{*i} in the edge region decreases.

(ii) At a critical value of v_{*i} that depends on the details of the device, U_p makes a transition from the L root to the H root and becomes more positive. The corresponding value of E_r becomes more negative. An example of the change of the U_p at the transition is shown in Fig. 2. The time scale of the transition is of the order of ν^{-1}.

(iii) Because E_r becomes more negative, the level of the turbulent fluctuations decreases, and the plasma confinement thereby improves.

(iv) The intensity of the H_α emission decreases because of the confinement improvement.

Detailed descriptions of the model and its extended version, to include the determination of the toroidal flow speed U_t, will be presented in a separate article.

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