Larger Higgs-Boson-Exchange Terms in the Neutron Electric Dipole Moment

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The neutron electric dipole moment (d_n) due to Higgs-boson exchange is reconsidered, now without assuming that Higgs-boson exchange is solely responsible for $K_L^0 \to 2\pi$. The dominant contribution to d_n arises from a three-gluon operator, produced in integrating out top quarks and neutral Higgs bosons. The estimated result together with current experimental bounds on d_n show, even for the largest plausible Higgs-boson masses, that CP is not maximally violated in neutral-Higgs-boson exchange.

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We are still uncertain about the mechanism or mechanisms responsible for CP violation.[1] Some years ago, I proposed a model[2] in which CP violation is caused solely by Higgs-boson exchange. The model seemed at the time quite realistic, but since then it has run into difficulties. Although it is possible to get the observed ϵ'/ϵ ration in K_L^0 decay,[3] the model tends to give too large a value.[4] Also, in order to get ϵ large enough, one of the Higgs bosons must be given an uncomfortably small mass,[5] and the predicted neutron electric dipole moment then appears to be somewhat too large.[6] Above all, we now know that there is a third quark generation, that has only small mixing angles with the first two, so the complex phase in the Kobayashi-Maskawa (KM) matrix provides a natural mechanism for a small but nonzero violation of CP conservation. Indeed, with three quark generations, it would be unnatural for the KM phase *not* to contribute to CP nonconservation.[7]

On the other hand, unless the Higgs sector is extremely simple, it would also be unnatural for Higgs-boson exchange not to contribute to CP nonconservation. CP nonconservation can occur in charged-Higgs-boson exchange if there are at least three scalar doublets. This possibility was stressed in Ref. 2 because it is charged-Higgs-boson exchange that is most relevant to a flavor-changing process like $K_L^0 \to 2\pi$. Subsequently, Deshpande and Ma[8] emphasized that theories with CP violation in charged-Higgs-boson exchange are also likely to exhibit CP violation in neutral-Higgs-boson exchange. Indeed, it is somewhat more likely for CP violation to occur in this way. For instance, suppose there were just two scalar doublets ϕ_1, ϕ_2 (and any number of singlets). In unitarity gauge there would be only one physical charged scalar field $\phi^-(x)$, and thus no way for CP-violating phases to show up in the propagator $\langle T\{\phi^-, \phi^{-*}\}\rangle_0$. However, in unitarity gauge the two complex neutral fields ϕ_1^0, ϕ_2^0 are subject to only one relation, that $\lambda_1^* \phi_1^0 + \lambda_2^* \phi_2^0$ be real (where $\lambda_i \equiv \langle \phi_i \rangle_0$), and CP nonconservation show up in any or all of the propagators $\langle T\{\phi_1^0, \phi_2^{0*}\}\rangle_0$, $\langle T\{\phi_1^0, \phi_2^0\}\rangle_0$, $\langle T\{\phi_1^0, \phi_1^0\}\rangle_0$, and $\langle T\{\phi_2^0, \phi_2^0\}\rangle_0$. CP nonconservation actually does arise in this way if any scalar singlets mix with ϕ_1 and ϕ_2. In particular, in the "minimally nonminimal" supersymmetric model of Ellis *et al.*,[9] CP invariance was imposed by requiring that the couplings satisfy a certain phase relation; for general couplings the model would violate CP conservation, though only in the neutral Higgs sector.

These considerations suggest that we should carefully consider the possibility that CP nonconservation occurs everywhere it can, both through a phase in the KM matrix *and* in neutral- as well as charged-Higgs-boson exchange. If none of the Higgs particles are very light (say, mass less than 15 GeV) then the KM phase would dominate in K_L^0 decay, as seems to be indicated by the ϵ'/ϵ ratio. On the other hand, the KM phase gives only an unobservably small neutron electric dipole moment,[10] and here it is Higgs-boson exchange that should dominate.

With this in mind, I have recently undertaken a survey[11] of the various ways that Higgs-boson exchange can contribute to a neutron electric dipole moment, but now without assuming that Higgs-boson exchange has anything to do with $K_L^0 \to 2\pi$. In the course of this work, it has become apparent that neutral-Higgs-boson exchange can make a remarkably large contribution through a class of Feynman graphs that does not seem to have been previously considered. The balance of this paper will deal with this contribution.

The first step in any calculation of the neutron dipole moment is to calculate the CP-violating operators, involving light quarks, gluons, and/or photons, that appear in the effective weak-interaction Lagrangian, by integrating out heavy quarks and Higgs bosons. One would expect the dominant operators to be those of minimum dimensionality, because their coefficients will have the smallest number of heavy particle masses in the denominator. Operators of dimension 4 or less all automatically conserve CP (except for those whose CP violation is removed by the Peccei-Quinn mechanism). There are CP-violating operators of dimension 5, such as the light-quark electric dipole operator itself and the analogous light-quark color electric dipole operators, but chirality requires them to contain at least one light-quark mass factor, so these operators are effectively of dimension 6, and are further suppressed by either two more light-quark masses (divided by the Higgs-boson mass) or by two small mixing angles. There are also a large number of four-quark operators of dimension 6, but these are all suppressed by two factors of the ratio of the light-quark to the Higgs mass.

However, there is one P- and T-nonconserving operator of dimension 6 whose coefficient involves neither light-quark masses nor small mixing angles. It is purely gluonic operator[12]

$$\mathcal{O} = -\tfrac{1}{6} C f_{\alpha\beta\gamma} G_{\alpha\mu\rho} G_{\beta\nu}{}^{\rho} G_{\gamma\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} . \tag{1}$$

(Here $G_{\alpha\mu\nu}$ is the gluon field-strength tensor, $f_{\alpha\beta\gamma}$ is the totally antisymmetric Gell-Mann coefficient, and $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric quantity with $\epsilon^{0123} \equiv +1$.) The Feynman diagram that produces the dominant contribution to C is one in which three gluons are attached to a top-quark loop, with a neutral Higgs boson emitted and reabsorbed from the quark, anywhere within the loop (see Fig. 1). Under the assumption[13] that only one scalar doublet (say, ϕ_2) couples to quarks of charge $\frac{2}{3}$, and approximating its propagator by[14]

$$(\lambda_2)^{-2} \int \langle T\{\phi_2(x), \phi_2(0)\}\rangle_0 e^{iq\cdot x} d^4x \simeq \sqrt{2} G_F Z_2/(q^2+m_H^2) , \tag{2}$$

we expect the coefficient C in Eq. (1) to take a value

$$C = \sqrt{2} G_F \, \mathrm{Im} Z_2 (4\pi)^{-4} g_s^3 h(m_t/m_H) , \tag{3}$$

where g_s is the QCD coupling constant, and h is a dimensionless quantity depending only on the ratio of the top-quark and neutral-Higgs-boson masses. This is a two-loop graph, but it is not so hard to show that in the limit $m_H \gg m_t$,

$$h\left(\frac{m_t}{m_H}\right) \to \frac{1}{2} \frac{m_t^2}{m_H^2} \ln\left(\frac{m_H^2}{m_t^2}\right) . \tag{4}$$

It is noteworthy that in the opposite limit, $m_t \gg m_H$, the function h approaches a mass-independent constant of order unity. These results have been confirmed and extended to general values of m_t/m_H in a full two-loop calculation by Dicus.[15] He finds that, in general,

$$h(\sigma) = \frac{\sigma^4}{2} \int_0^1 dx \int_0^1 du \frac{u^3 x^3 (1-x)}{[\sigma^2 x(1-ux) + (1-u)(1-x)]^2} . \tag{5}$$

In particular, in the limit $m_t \gg m_H$, this takes the value $h(\infty) = \frac{1}{8}$.

For this use of perturbation theory to be justified, it is necessary that the operator GGG in Eq. (1) be defined at a renormalization scale λ of the order of m_H and m_t; the coupling g_s in Eq. (3) is then taken to be $g_s(\lambda)$, defined at this high-energy scale. However, in order to calculate a low-energy quantity like the neutron electric dipole moment, we need to express $\mathcal{O}$ in terms of an operator renormalized at a low-energy scale μ, where g_s is larger. Part of this renormalization factor just comes from the renormalization of the individual operators $G_{\alpha\mu\nu}$ in (1). If this were the whole story, then the factor needed in order to convert the operator in (1) into one defined at a running scale μ would be supplied by simply changing g_s in Eq. (3) from $g_s(\lambda)$ to $g_s(\mu)$. For instance, this is well known to be the case (to one-loop order) for the operator $G_{\alpha\mu\nu} G_\alpha^{\mu\nu}$. Matters are more complicated for the operator in (1), which undergoes a renormalization of its own, apart from the renormalization of the factors $G_{\alpha\mu\nu}$. Detailed calculations[16] give this extra renormalization factor as $[g_s(\mu)/g_s(\lambda)]^{108/23}$. This factor is to be inserted in Eq. (3), with both $g_s(\mu)$ in (3) and the GGG operator in (1) now taken at the running scale μ.

The really difficult part of the problem is in calculating the effect of an operator like (1), that appears in the effective Lagrangian at energies above the chiral-symmetry-breaking scale, on a low-energy hadronic parameter like the neutron electric dipole moment. No one knows how to do this with any precision, but at least there is a rule known as "naive dimensional analysis" for keeping track of factors of 4π and mass scales.[17] This rule can be expressed most simply by introducing dimensionless "reduced" coupling constants. For a coupling constant g appearing in an interaction of dimensionality

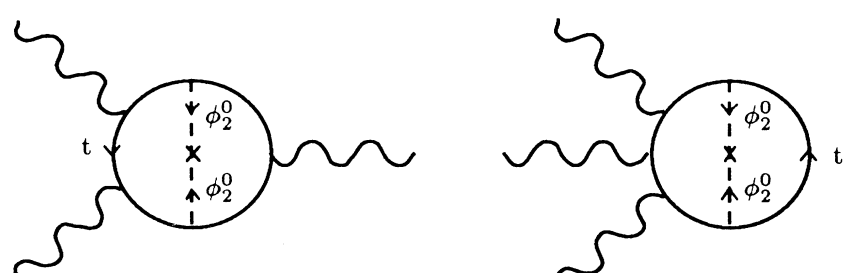

FIG. 1. Graphs contributing to the operator (1). (Wavy lines are gluons.)

$[\text{mass}]^{\mathcal{D}}$ and containing $\mathcal{N}$ field operators, the reduced coupling is $(4\pi)^{2-\mathcal{N}}M^{\mathcal{D}-4}g$, where $M=2\pi F_\pi \simeq 1190$ MeV is the chiral-symmetry-breaking scale. The rule is that the reduced coupling of any term in the effective hadronic theory at energies below M is given to within an order of magnitude by the product of the reduced couplings of the operators, appearing in the effective Lagrangian at energies above M, that produce this term. The neutron electric dipole moment operator given by $d_n \bar{n}\gamma_5\sigma_{\mu\nu}nF^{\mu\nu}$ has reduced coupling $d_nM/4\pi$, while the quark electromagnetic coupling has reduced coupling $e/4\pi$, so naive dimensional analysis suggests that d_n is of the order of e/M times the reduced coupling of whatever operator is responsible for CP violation. For instance, a CP-violating quark mass term $\theta m_q\bar{q}\gamma_5 q$ would have reduced coupling $\theta m_q/M$, so it should give a neutron electric dipole moment of order $e\theta m_q/M^2$, or $10^{-16}e\theta$ cm for $m_q=7$ MeV. This compares well with more detailed estimates[18] of d_n, which range from $0.4\times 10^{-16}e\theta$ cm to $2\times 10^{-15}e\theta$ cm.

There is also the question of the relevant scale at which to evaluate parameters like the strong coupling $g_s(\mu)$. We may guess that in evaluating hadronic matrix elements, the relevant value of μ is that which makes graphs of all orders equally important, i.e. [judging from the first two terms in the renormalization-group equation for $g_s(\mu)$], $g_s^2(\mu)/16\pi^2 \simeq 1/6$. For example, the previously mentioned operator $\epsilon g_s^2 G_{\alpha\mu\nu}G_\alpha^{\mu\nu}$ (with ϵ infinitesimal) is known[19] to produce a shift $-2(4\pi)^2 m_N\epsilon/9$ in the nucleon mass, in good agreement with what would be expected from naive dimensional analysis with $g_s \simeq 4\pi/\sqrt{6}$. (This example incidentally also shows that there is no necessity for light-quark masses to appear in the contribution of a purely gluonic operator to a γ_5-noninvariant hadronic operator.) With $g_s(\mu)$ given this value, and $g_s(\lambda)/4\pi \simeq 0.1$ (corresponding to taking $\lambda \simeq 100$ GeV and $\Lambda_{\rm QCD} \simeq 150$ MeV between m_b and λ), the μ-dependent factors in C have the value

$$\zeta \equiv \left(\frac{g_s(\mu)}{g_s(\lambda)}\right)^{108/23}\left(\frac{g_s(\mu)}{4\pi}\right)^3 \simeq 50\,. \qquad (6)$$

For the CP-violating operator (1), the reduced coupling is $M^2C/4\pi$, so the electric dipole moment may be estimated as of order

$$d_n \simeq eMC/4\pi = e\zeta\sqrt{2}G_F M\,{\rm Im}Z_2(4\pi)^{-2}h(m_t/m_H)$$
$$= 10^{-19}e\,{\rm Im}Z_2 h(m_t/m_H)\ {\rm cm}\,. \qquad (7)$$

This is very large compared with other contributions, and potentially in conflict with the experimental results for d_n, $(-14\pm 6)\times 10^{-26}$ $e$cm from Leningrad[20] and $(-3\pm 5)\times 10^{-26}$ $e$cm from Grenoble.[21] We do not know m_H or m_t, but the experimental lower bound on m_t is rapidly increasing, and it is hard to imagine that m_H could be larger than $10m_t$. This gives[15] $h > 0.015$. The experimental bound[21] $|d_n| < 1.2\times 10^{-25}$ $e$cm thus requires that $|{\rm Im}Z_2| < 8\times 10^{-5}$. Our conclusion is that CP is not maximally violated in the neutral Higgs sector.[14] The only way that I can see for this to be natural is for the Higgs sector to be very simple: no more than two doublets, and with two doublets, no mixing with any scalar singlets.

It is instructive to compare these results with those of Anselm *et al.*[22] They considered the neutron electric dipole moment produced when a neutron emits and reabsorbs a neutral Higgs boson, with the photon attached to the virtual neutron line. For the Higgs-boson–neutron coupling, they used the results of Shifman, Vainshtein, and Zakharov,[19] in which the Higgs-boson line is attached to a heavy quark loop, which is attached to the neutron by a pair of gluons. In effect (though they did not put in this way) they were considering the effect of an operator

$$G_{\alpha\mu\nu}G_\alpha^{\mu\nu}G_{\beta\rho\sigma}G_{\beta\kappa\lambda}\epsilon^{\rho\sigma\kappa\lambda} \qquad (8)$$

produced in integrating out Higgs bosons and heavy quarks. (The Feynman diagram is one in which the two pairs of gluon lines are attached to two different heavy quark loops, which are attached to each other by a neutral-Higgs-boson line.) As noted in Ref. 22, the coefficient of this operator is not suppressed by light-quark masses or small mixing angles, and so for a Higgs-boson mass that is small enough (say, a few GeV) to give the observed rate for $K_L^0 \to 2\pi$, this effect gives too large a neutron electric dipole moment. However, the operator (8) is of dimensionality 8, and its effect is therefore suppressed relative to (1) by a factor M^2/m_H^2, so its contribution to d_n is within experimental limits if $m_H \gtrsim 100$ GeV. Only a dimension-6 operator like (1) can threaten to produce an excessive d_n for any reasonable Higgs-boson mass.

The same operator may dominate CP violation produced by mechanisms other than Higgs-boson exchange. For instance, in supersymmetric theories there is a graph in which a gluino or Higgsino is emitted and reabsorbed from a top-quark–squark loop, with three gluons attached anywhere on the quark, squark, or gluino lines, that also makes a contribution to (1). This is estimated[23] to increase the neutron electric dipole moment produced by a given CP-violating gluino mass by at least an order of magnitude over earlier results,[24] thus strengthening the conclusion that CP is not maximally violated by the gluino mass operator. If a neutron electric dipole moment shows up in the next round of experiments, it will not necessarily be due to Higgs-boson exchange, but it will almost certainly appear through the operator (1).

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[1]For recent discussions, see *CP Violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989).

[2]S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).

[3]J.-M. Frere, J. Hagelin, and A. Sanda, Phys. Lett. **151B**, 161 (1985); G. C. Branco, A. J. Buras, and J. M. Gérard, Nucl. Phys. **B259**, 306 (1985); H.-Y. Cheng, Phys. Rev. D **34**, 1397 (1986).

[4]A. I. Sanda, Phys. Rev. D **23**, 2647 (1981); N.-G. Deshpande, Phys. Rev. D **23**, 2654 (1981).

[5]The value estimated in Ref. 2 was 15 GeV. Other estimates include 2–3 GeV, by A. A. Anselm and N. G. Ural'tsev, Yad. Fiz. **30**, 465 (1979) [Sov. J. Nucl. Phys. **30**, 240 (1979)], and 15 GeV, by M. J. Booth, R. Briere, and R. G. Sachs, Phys. Rev. D (to be published).

[6]For improvements on the calculation of d_n in Ref. 2, see A. A. Anselm and D. I. D'yankonov, Nucl. Phys. **B145**, 271 (1978); A. R. Zhitnitski and I. B. Khriplovich, Yad. Fiz. **34**, 167 (1981) [Sov. J. Nucl. Phys. **34**, 95 (1981)]; G. Beall and N. G. Deshpande, Phys. Lett. **132B**, 427 (1983); I. I. Bigi and A. Sanda, Phys. Rev. Lett. **58**, 1604 (1987); and articles cited in Ref. 5. These calculations generally indicate that the neutron electric dipole moment inferred from the rate of $K_L \to 2\pi$ is close to or a little larger than the present experimental bound, but that the predicted values become considerably larger if the strange-quark operator $\bar{s}\gamma_5\sigma_{\mu\nu}s$ is assumed to have an appreciable expectation value in the neutron.

[7]The only way that this might be regarded natural is for *CP* conservation to be only spontaneously (or softly) broken, as originally proposed by T. D. Lee, Phys. Rep. **9C**, 144 (1974); also see G. Branco, Phys. Rev. Lett. **44**, 504 (1980). The model of Ref. 2 left it an open question whether *CP* conservation is broken spontaneously, or intrinsically in the scalar polynomial. A spontaneous breaking of *CP* could produce troublesome cosmological domain boundaries; see I. Yu. Kobsarev, L. B. Okun, and Ya. B. Zeldovich, Phys. Lett. **50B**, 340 (1974).

[8]N. G. Deshpande and E. Ma, Phys. Rev. D **16**, 1583 (1977).

[9]J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, SLAC Report No. SLAC-PUB-4565, 1988 (to be published).

[10]D. V. Nanopoulos, A. Yildiz, and P. H. Cox, Ann. Phys. (N.Y.) **127**, 126 (1980); H. Georgi and L. Randall, Nucl. Phys. **B276**, 241 (1986); C. Jarlskog, Phys. Rev. D **35**, 1685 (1987); B. H. J. McKellar, S. R. Choudhury, X.-G. He, and S. Pakvasa, Phys. Lett. B **197**, 556 (1987).

[11]S. Weinberg, in Proceedings of the July 1989 Erice Workshop on Higgs Particles (unpublished); (to be published).

[12]Other purely gluonic *P*- and *T*-odd operators of dimension 6, such as $D_\rho G_{a\mu\nu} D^\rho G_{a\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma}$, are all proportional to (1). [This remark is due to J. Polchinski (private communication).]

[13]S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).

[14]Note that for two doublets, the Fermi coupling constant G_F is given by $\sqrt{2}G_F = (|\lambda_1|^2 + |\lambda_2|^2)^{-1}$, so with maximum *CP* violation, both ImZ_2 and ReZ_2 would be expected to be of order unity, or (if $|\lambda_1| \gg |\lambda_2|$) greater.

[15]D. Dicus (private communication); University of Texas, Center for Particle Theory, report (to be published). Charged-Higgs-boson exchange also makes a contribution to *C*.

[16]J. Dai and H. Dykstra, University of Texas report (to be published).

[17]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); Georgi and Randall, Ref. 10.

[18]For a survey, see H.-Y. Cheng, Phys. Rep. **158**, 1 (1988).

[19]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **78B**, 443 (1978).

[20]I. S. Altarev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 360 (1986) [JETP Lett. **44**, 460 (1986)].

[21]K. F. Smith *et al.* (to be published).

[22]A. A. Anselm, V. E. Bunakov, V. P. Gudkov, and N. G. Uraltsev, Phys. Lett. **152B**, 116 (1985).

[23]W. Fischler and J. Polchinski (private communication).

[24]J. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. **114B**, 231 (1982); J. Polchinski and M. B. Wise, Phys. Lett. **125B**, 392 (1983).