Comment on "DNA Plasmon"

Van Zandt and Saxena[1] have recently shown that a long charged molecule, such as DNA, in the presence of a fluid containing oppositely charged ions will possess an acoustic plasma excitation. As it is quite unusual for a system to have an acoustic plasma mode, I feel that it is be useful to study a simpler model which illustrates the basic physics which may be giving rise to such modes.

To this end, I will present the results of a calculation of the plasma mode of a charged cylinder of length large compared to its width which is also assumed to contain (within the cylinder) a mobile oppositely charged fluid of total charge equal in magnitude to that of the cylinder. This cylinder represents the region of space in the solution, containing both the DNA molecule and the surrounding positive counter-ion cloud which neutralizes it. Although, in principle, the counter-ion cloud extends infinitely far out, since the ion-density falls off exponentially and since the Debye screening length is small compared to the length of the DNA molecule, it is reasonable to assume that the ions are contained in a cylinder of radius small compared to its length. As illustrated in Ref. 2, the zero-wave-vector plasma oscillations of a three-dimensional solid produce an oscillating polarization charge on two opposite surfaces of the solid. This surface polarization charge results in a uniform electric field which exerts a restoring force on the mobile charges in the solid, resulting in nonzero frequency plasma oscillations. Similar zero-wave-vector charge oscillations along the axis of the cylinder considered here will again produce oscillating surface polarization charge on the two ends of the cylinder. Because the size of the two ends is small compared to the length of the cylinder, however, these may be treated as point charges. As the cylinder is taken to be arbitrarily long, the Coulomb's law electric field of the charges on the ends will become vanishingly small. Therefore, unlike the three-dimensional example discussed above, the zero-wave-vector plasma mode has zero frequency for the case of a long cylinder. The advantage of studying this simplified model is that it allows one to understand the generality of the results presented in Ref. 1. For example, we see that all that is needed to have such an acoustic plasma mode is a chain of charge surrounded by mobile charges of opposite sign localized within a radius small compared to both the length of the chain and the wavelength of the excitation. Furthermore, we see that the mode is basically electrostatic in origin.

To study the case of nonzero wave vector, consider the equation of motion for the displacement u along the axis of a segment of length Δx of mobile charges:

$$A n m \ddot{u} \Delta x = n e A E \Delta x \,, \tag{1}$$

where A is the cross-sectional area of the cylinder, n is the mean number of mobile charges of charge e per unit volume, and E is the electric field resulting from the charge oscillations. The resulting polarization vector along the cylinder axis is given by

$$P = neu = neu_0 \cos(kz - \omega t) \,, \tag{2}$$

where we have assumed sinusoidal oscillation of the mobile charge along the axis. The resulting electrostatic potential is given in mks units by

$$\Phi = (4\pi\epsilon_0)^{-1} \int [-\nabla' \cdot \mathbf{P}(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|] d^3 r' = (2\epsilon_0)^{-1} neu_0 k \sin(kz - \omega t) \int_0^a \rho \, d\rho \int [v^2 + (kr)^2]^{-0.5} \cos v \, dv \,, \tag{3}$$

where $v = kx$ and a is the cylinder radius. Performing the integrals on v and r in the small-k limit, we obtain

$$\Phi \approx (2\epsilon_0)^{-1} neku_0 \sin(kx - \omega t)(a^2/2)[-\ln(ka)] \,. \tag{4}$$

Calculating E from Φ and substituting in Eq. (1) yields

$$\omega^2 \approx (ne^2/4\epsilon_0 m) k^2 a^2 [-\ln(ka)] \,. \tag{5}$$

This is the plasma mode of Ref. 1. This is the plasma mode which occurs for a one-dimensional charge distribution with a three-dimensional (i.e., r^{-1}) Coulomb potential. Without the logarithmic factor, which does not vary too much with k, the square of the velocity is given by $ne^2a^2/4\epsilon_0 m$, from which the velocity of the plasmon is estimated to be 6.2×10^3 m/s (about $\frac{1}{3}$ of the value of Ref. 1 which is reasonable considering the crude nature of this model) if we take n to be 2×10^{27} m^{-3}, the number of electron charges per base pair on the DNA molecule, a equal to 10^{-9} m, and m the mass of a sodium ion.

Of course, as pointed out in Ref. 1, in order to have these unusual plasma oscillations, the spacing of the molecules must be large compared to their length (otherwise we will just get the usual plasma modes for a three-dimensional solid), and the DNA molecule must be straight. Even if the molecule is curved, however, the arguments for the occurrence of an acoustic plasma mode presented here and in Ref. 1 would still be valid for wavelengths long compared to the radius of the counter-ion screening cloud of a DNA molecule but small compared to the radius of curvature of the molecule.

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