# Theory of Collective Flux Creep

M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin,[(a)] and V. M. Vinokur[(b)]
*Landau Institute for Theoretical Physics, Moscow, U.S.S.R.*
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The nature of flux-creep phenomena in the case of collective pinning by weak disorder is discussed. The Anderson concept of flux bundle is explored and developed. The dependence of the bundle activation barrier $U$ on current $j$ is studied and is shown to be of power-law type: $U(j) \propto j^{-\alpha}$. The values of exponent $\alpha$ for the different regimes of collective creep are found.

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There are a number of experimental results confirming the existence of a giant thermal flux creep in high-$T_c$ superconductors.[1,2] It seems probable that pinning of vortices in these materials is due to weak randomly distributed defects (e.g., oxygen vacancies). These defects induce elastic distortions of the vortex line lattice, whose free energy $F$ associated with distortion is

$$F=\int d^3r\left[(C_{11}-C_{66})\frac{(\mathrm{div}\mathbf{u})^2}{2}+C_{66}\frac{(\nabla_\perp \mathbf{u})^2}{2}+C_{44}\frac{(\partial \mathbf{u}/\partial z)^2}{2}+U_{\mathrm{pin}}(\mathbf{u},\mathbf{r})\right]. \quad (1)$$

Here, the two-dimensional vector $\mathbf{u}(\mathbf{r})$ describes a local displacement of the flux-line lattice; $C_{11}$, $C_{44}$, and $C_{66}$ are, respectively, bulk, tilt, and shear elasticity modules; $U_{\mathrm{pin}}(\mathbf{u},\mathbf{r})$ is the random potential describing the lattice interaction with defects. This random potential is supposed to be short-range correlated:

$$\langle U_{\mathrm{pin}}(\mathbf{u},\mathbf{r})U_{\mathrm{pin}}(\mathbf{u}',\mathbf{r}')\rangle = K(|\mathbf{u}-\mathbf{u}'|,|\mathbf{r}-\mathbf{r}'|),$$

where $K(x,y)$ decreases rapidly at $x,y$ larger than some characteristic length $r_p$ ($r_p \simeq \xi$ if the size of defects is smaller than the coherence length $\xi$). The random potential leads to destruction of the long-range order in the vortex line lattice.[3] It has been shown in the theory of collective pinning[4,5] that the critical current density $j_c$ is determined by the pinning lengths $R_c$ and $L_c$ as $j_c \simeq (W/V_c)^{1/2}B^{-1} \simeq C_{66}\xi/R_c^2B$, where $W$ is the mean-square value of random force produced by defects:

$$W=\int [\partial^2 K(u,r)/\partial u^2]_{u\sim\xi}\, d^3r .$$

Here, we define a pinning volume $V_c = R_c^2 L$ as a volume of the lattice region where the elastic distortions of the lattice $|\mathbf{u}(\mathbf{r})| \lesssim \xi$. Longitudinal ($L_c$) and transverse ($R_c$) (with respect to the magnetic field $\mathbf{B}$ direction) sizes of this region are $L_c \simeq R_c(C_{44}/C_{66})^{1/2}$, $R_c \simeq C_{44}^{1/2} \times C_{66}^{3/2}\xi^2/W$.

At low currents $j<j_c$ the vortex lattice is in some metastable state. Transitions between different metastable states are due to thermal activation through free-energy barriers whose characteristic scale is $U(j)$. If $j \to j_c$, then $U(j)\to 0$. Our main goal is to obtain the $U(j)$ dependence.

In the case of collective pinning $U(j)$ is of the order of elastic energy of the hopping flux bundle (introduced by Anderson[6]). We start from the simplest case $B\sim H_{c1}$ (where $C_{11}\sim C_{44}\sim C_{66}\sim C$) and consider the current density $j\sim j_c$. In this case the volume of the bundle is of the order of $V_c$, while the hopping distance $u_{\mathrm{hop}}\sim\xi$, so the activation energy is $U_c \sim C(\xi/R_c)^2 V_c \simeq C^3\xi^4/W$.

If $H \gg H_{c1}$, $C_{11}=C_{44}\gg C_{66}$, then $R_c$, $L_c$, and $j_c$ are determined by shear and tilt deformations and are independent of $C_{11}$. Nevertheless, shifting of a large region of the lattice by distance $u_{\mathrm{hop}}\sim\xi$ induces considerable compression deformation. The energies of shear and compression deformations should be of the same order of magnitude. For the bundle with the sizes $L$, $R_\parallel$, and $R_\perp$ (along the magnetic field, in the direction of hopping, and in the transverse direction, respectively) compression, shear, and tilt deformations are $|\mathrm{div}\mathbf{u}| \sim \xi/R_\parallel$, $|(\nabla_\perp \mathbf{u})|\sim\xi/R_\perp$, $|\partial\mathbf{u}/\partial z|\sim\xi/L$. since $C_{11} \gg C_{66}$, then $R_\parallel$ should be much larger than $R_\perp$. Considering the energy of these deformations one obtains

$$V_B C_{11}\xi^2/R_\parallel^2 \sim V_B C_{66}\xi^2/R_\perp^2 \sim V_B C_{44}\xi^2/L^2 \sim j_c V_B B\xi ,$$

where $V_B \simeq R_\parallel R_\perp L$ is the bundle volume and $j_c V_B B\xi$ is an estimate of the energy gain due to hopping under the action of Lorentz force. Therefore $R_\perp \sim R_c$, $R_\parallel \sim L \sim L_c \sim R_c(C_{11}/C_{66})^{1/2}$, and the energy barrier $U_c \sim C_{66} \times(\xi/R_c)^2 R_c L_c^2 \sim C_{66}^{3/2}C_{11}^{3/2}\xi^4/W$. This value is by the factor $(C_{11}/C_{66})^{1/2}$ larger than the energy of elastic distortions in the pinning volume $V_c \simeq R_c^2 L_c$. This is due to the fact that the flux bundle is made up of a large number [$\sim(C_{11}/C_{66})^{1/2}$] of subbundles of the volume $V_c$, these subbundles being formed independently from each other by competition betwen shear (and tilt) elastic energies and disorder potential. These subbundles hop all together as one bundle because the large value of $C_{11} \gg C_{66}$ prohibits independent hopping of subbundles. The energy barrier $U_c$ for such correlated hop is roughly the sum of energy barriers of the subbundles [which are of the order of $C_{66}(\xi/R_c)^2R_c^2L_c$].

So far we have considered the case $j\sim j_c$. However, it is also quite natural to explore the region $j \ll j_c$, where the bundle volume $V_B(j)$ proves to be even larger than the above estimate for $V_B$. In conventional superconductors the activation energy is large, the temperature is low, and the creep is weak, so the creep measurements are performed in the region of current $j_c - j \ll j_c$. However, in high-$T_c$ superconductors the creep rate is much larger and usually one measures currents which are considerably lower than $j_c$. When a magnetic field $B > H_{c1}$ is applied to a sample of superconductor, the Bean's critical state[7] is formed at short time scales $t_0$. Then, after a time $t_{\rm obs}$ current density decreases due to flux creep down to some value $j(t_{\rm obs})$ which is determined by the following relation:[8]

$$U(j(t_{\rm obs})) = T\ln(t_{\rm obs}/t_0)\,, \tag{2}$$

where $T$ is the temperature and $\ln(t_{\rm obs}/t_0)$ is usually of the order 10–30. It is usually assumed in the theory of flux creep[6,9] that $j_c - j \ll j_c$ and that in this case $U(j) \simeq U_c(1 - j/j_c)$ which leads immediately to the well-known result $j(t) = j_c[1-(T/U_c)\ln(t/t_0)]$. Kes *et al.*[10] have supposed that at $j \ll j_c$ the energy barrier $U(j)$ tends to some constant $U_0 \sim U_c$ and have developed the theory of thermally activated flux flow (TAFF) where the flux relaxation is governed by the conventional linear diffusion equation with diffusion constant $D \propto \exp(-U_c/T)$. Below we shall show that in the case of elastic deformations of the flux-line lattice the energy $U(j)$ grows at $j \ll j_c$ as $j^{-\alpha}$, which means the absence of usual linear diffusion ($D \to 0$ at $j \to 0$).

The origin of this unusual behavior can be understood as follows.[11,12] In the absence of an external current the lattice is in some local most favorable metastable state. Under the action of the external current $j$ some other metastable states become more preferable. These states are determined by the condition that the energy gain due to the external Lorentz force is of the order of the elastic and pinning energies. At $j \sim j_c$ this condition is fulfilled for neighboring states which differ by a bundle shift by the distance $u_{\rm hop}(j_c) \sim \xi$. At $j \ll j_c$ the hopping distance $u_{\rm hop}(j)$ should be much larger and is determined by the following estimate:

$$jBu_{\rm hop}(j) \sim C_{66}u_{\rm hop}^2(j)/R_\perp^2(j)\,, \tag{3}$$

where $R_\perp(j)$ is the size of the bundle in the direction of the vector $[\mathbf{B}\times\mathbf{u}_{\rm hop}]$.

To find the $u_{\rm hop}(j)$ dependence one needs some additional relation between $u_{\rm hop}(j)$ and $R_\perp(j)$. We shall show below that in most favorable metastable states of elastic media interacting with the random potential, fluctuations of the displacement field $u$ at the distance $R$ increase as

$$u(R) = \langle |\mathbf{u}(\mathbf{r}) - \mathbf{u}(\mathbf{r}+\mathbf{R})|^2\rangle^{1/2} \propto R^\zeta\,, \tag{4}$$

where the positive exponent $\zeta$ will be found below. The same estimate (4) also describes the relation between the amplitude of hopping distance $u_{\rm hop}(j)$ and the size of the respective bundle $R_\perp(j)$. Substitution of Eq. (4) into Eq. (3) leads to the following estimates:

$$u(j) \propto j^{-\zeta/(2-\zeta)},\quad R_\perp(j) \propto j^{-1/(2-\zeta)}\,,$$
$$R_\parallel \simeq L \simeq (C_{11}/C_{66})^{1/2}R_\perp,\quad U(j) \propto j^{-\alpha}\,, \tag{5}$$

where $\alpha = (d-2+2\zeta)/(2-\zeta)$, $d$ is the dimsionality of elastic lattice.

Now we turn to the determination of the exponent $\zeta$. An analogous problem has been solved exactly[11,13] for the case of a one-dimensional elastic line in two-dimensional random media, with the result $\zeta = \frac{2}{3}$. This is a particular example of a general problem of $d$-dimensional elastic media whose position in the random potential is characterized by an $n$-dimensional vector $\mathbf{u}(\mathbf{r})$. We shall denote the exponent $\zeta$ for this problem as $\zeta_{d,n}$. The behavior of the three-dimensional flux-line lattice is described then by an exponent $\zeta_{3,2}$. The value of $\zeta_{1,2}$ (elastic line in three-dimensional media) was numerically found in the interval 0.6–0.65.[14,15] Now we show how to relate values of $\zeta_{d,n}$ with the same $n$, but different $d$. We suppose that the random potential is a short-range correlated potential with the characteristic length $\xi$. As above we define $V_c \simeq R_c^d$ as a volume of region where elastic distortion of the media $|\mathbf{u}| \lesssim \xi$. Equating once more elastic energy and the energy of the interaction with defects, we obtain $C(\xi^2/R_c^2)V_c \simeq (V_cW)^{1/2}\xi$ (here $C$ and $W$ are the elasticity modules of the media and the mean-squared value of the random force). Thus, finally $R_c \simeq (C^2\xi^2/W)^{1/(4-d)}$ and the energy is $U_c \simeq W^{1/2}\times\xi(C^2\xi^2/W)^{d/2(4-d)}$. Fluctuation of the energy in a volume $V > V_c$ is of order $\Delta E_{\rm el} \sim C(u^2/R^2)V$. We propose that the wandering exponent $\zeta(u \propto R^\zeta)$ is determined from the equality between fluctuation of the elastic energy $\Delta E_{\rm el}$ and fluctuation of the random potential energy (pinning energy) $\Delta E_{\rm pin}$. Now we face the problem of estimating fluctuations of the random potential energy. The key point of our analysis is the suggestion that $\Delta E_{\rm pin}$ is *independent of elastic module* but does depend only on the fluctuating volume ($V$) involved, its displacement ($u$), and quantities $W$ and $\xi$ which characterize the random potential. Then writing down the expression for fluctuation of the energy $\Delta E_{\rm pin}$ in the form $\Delta E_{\rm pin} \simeq U_c(V/V_c)^\delta(u/\xi)^{-\beta/2}$ and making use of the independence of $\Delta E_{\rm pin}$ on $C$ we find straightforwardly the value of exponent $\delta$ and express $\beta$ in terms of wandering exponent $\zeta$ [note that $\delta$ and $\beta$ may depend on dimensionality $n$ of displacement vector $\mathbf{u}(\mathbf{r})$].

Namely, since $U_c \propto C^{d/(4-d)}$ and $V_c \propto C^{d/(4-d)}$ we find that the exponent $\delta = \frac{1}{2}$ and the fluctuation of the pinning energy is

$$\Delta E_{\rm pin} \sim (WV)^{1/2}\xi(\xi/u)^{\beta(n)/2}. \tag{6}$$

From the condition $\Delta E_{\rm pin} \sim \Delta E_{\rm el}$ we get $\zeta_{d,n} = (4-d)/[4+\beta(n)]$. For instance, if $\zeta_{1,2} = 0.6$ (the vortex in the three-dimensional space), then the exponent for the case of the vortex lattice is $\zeta_{3,2} = \frac{1}{3}\zeta_{1,2} = \frac{1}{5}$. One can interpret Eq. (6) as follows. The factor $\sqrt{V}$ describes the usual square-root dependence of the fluctuations of the pinning energy in the volume $V$ on the number of pinning centers in this volume. If the lattice cannot wander (the absolutely rigid lattice), then $\Delta E_{\rm pin} \sim (WV)^{1/2}\xi$. If permitted to wander, the lattice adjusts itself to the random potential, so fluctuations of $\Delta E_{\rm pin}$ decrease by a factor $f_n = (\xi/u)^{\beta/2}$. The factor $f_n$ is associated with the number of metastable states which lattice encounters when shifting over $u$. The number of metastable states is in turn determined by the volume $V_n \propto u^n$ spanned by the displacement vectors $\mathbf{u}$. So it seems plausible that $f_n = f_1^n$, $\beta(n) = \beta(1)n$. Making use of the exact result $\zeta_{1,1} = \frac{2}{3} \Longrightarrow \beta(1) = \frac{1}{2}$,[11,13] we conclude immediately that $\beta(n) = n/2$. The same result has been found by Halpin-Healy[16] by means of functional nonlinear renormalization-group analysis. Based on the results of Ref. 17, Natterman has obtained the wandering exponent in the form

$$\zeta_{d,n} = 2(4-d)/(8+n). \tag{7}$$

which is in accordance with our qualitative considerations.

Now let us return to the case of flux-line lattice in superconductors. The flux-line lattice is a rather complex object because it is characterized by different lengths: the vortex core length $\xi$ (which characterizes the vortex interaction with small defects), the lattice constant $a \simeq (\phi_0/B)^{1/2}$, and the London penetration depth $\lambda$ ($a$ and $\lambda$ characterize elastic properties of the lattice). The elasticity modules $C_{44}$ and $C_{11}$ have strong spatial dispersion on wave vectors $K > 1/\lambda$.[18] If $K < 1/a$, then $C_{11} = C_{44} = B^2/4\pi(\lambda^2K^2+1)$, $C_{66} = \phi_0 B/(8\pi\lambda)^2$. When $K$ is of order of $1/a$ all modules $C_{11}$, $C_{44}$, and $C_{66}$ have the same order of magnitude. Depending upon the magnetic field and temperature range several qualitatively different types of collective pinning and creep are possible. We concentrate our attention on one of them which, as it seems to us, is realized in high-$T_c$ superconductors at relatively low temperature. In these superconductors the critical current density $j_c$ is independent of magnetic field at low temperatures. In terms of the collective pinning theory it means that $R_c$ is smaller than the lattice constant $a$ therefore the critical current is determined by the collective pinning of single vortices. So the pinning is characterized by one length $L_c$ which can be easily found:[5] $L_c = \pi[\phi_0^4\xi^2/(2\pi)^4a^2W\lambda^4]^{1/3}$ ($W$ is proportional to $B$ therefore $L_c$ does not depend on magnetic field). Our case of collective pinning of single vortex corresponds to a region $\xi < L_c < a$. In this region the estimates of the critical current density $j_c$ and characteristic energy $U_c$ yield, $j_c \sim j_0(\xi/L_c)^2$, $U_c \sim H_c^2\xi^3(\xi/L_c)$, $j_0$ is a depairing current $j_0 \sim H_c/\lambda$.

In the region of the current

$$j_c > j > j_1 \sim j_c(L_c/a)^{2-\zeta_{1,2}} = j_c(L_c/a)^{7/5}$$

the bundle is in fact a segment of vortex line of length $L_c < L < a$, and $d=1$ in Eqs. (5) and (7), so we obtain the energy of the creep of vortex lines (see also Ref. 12):

$$U(j) = U_c(j_c/j)^{(2\zeta_{1,2}-1)/(2-\zeta_{1,2})} \simeq U_c(j_c/j)^{1/7}. \tag{8}$$

Inserting this energy into Eq. (2) we obtain

$$j(t) \simeq j_c\left[\frac{U_c}{T\ln(t/t_0)}\right]^7. \tag{9}$$

When $j \lesssim j_1$, then the characteristic size of the hopping vortex $L \gtrsim a$ and for this length the interaction of vortices (elasticity modula of the flux-line lattice) affects the wandering and the activation barrier in the creep. In this region fluctuation of the displacement can be written in the following form:

$$u = \langle |\mathbf{u}(\mathbf{r}) - \mathbf{u}(\mathbf{r}+\mathbf{R})|^2\rangle^{1/2} \simeq \xi^{1-2\zeta_{3,2}} u_{\rm LO}^{2\zeta_{3,2}} = \xi^{3/5} u_{\rm LO}^{2/5},$$

where $u_{\rm LO}$ is Larkin-Ovchinnikov formula which is valid for $u < \xi$:

$$u_{\rm LO}(R,L) \simeq \xi\left(\frac{a}{L_c}\right)^{3/2}\left[\frac{(R^2+a^2L^2/\lambda^2)^{1/2}}{\lambda} + \ln\left(1+\frac{R^4+a^2L^2}{a^4}\right)\right]^{1/2}.$$

(In the original paper[5] there was a mistake in the last term of this formula, which was corrected in Refs. 19 and 20.) In the region $j_1 > j > j_2 = j_1(a/\lambda)^2$ the creep is determined by the hopping of a small bundle with a transverse size $R_\perp < \lambda$. This is the region of strong spatial dispersion of elasticitiy modules, where $L(j)$ and $R_\parallel(j)$ scale as $R_\perp^2(j)$, and $\zeta = 0$ [cf. Eq. (5) which is valid in the local limit $R_\perp \gg \lambda$]. Neglecting logarithmic factors we obtain

$$U(j) \sim C_{66}\frac{u^2(j)}{R_\perp^2(j)}R_\perp(j)R_\parallel(j)L(j) \sim U_1\left(\frac{j_1}{j}\right)^{3/2}, \quad U_1 \simeq U_c\left(\frac{a}{L_c}\right)^{1/5}. \tag{10}$$

From Eq. (2) we obtain

$$j(t) \sim j_1\left[\frac{U_1}{T\ln(t/t_0)}\right]^{2/3} \tag{11}$$

when the current $j < j_2$, then the transverse size $R_j > \lambda$; therefore there is no dispersion of elasticity modules and we obtain a result analogous to Eq. (5):

$$U(j) \sim U_2 \left[\frac{j_2}{j}\right]^{(2\zeta_{3,2}+1)/(2-\zeta_{3,2})} = U_2 \left[\frac{j_2}{j}\right]^{7/9}, \quad U_2 \simeq U_1 \left[\frac{\lambda}{a}\right]^3, \quad j(t) \simeq j_2 \left[\frac{U_2}{T\ln(t/t_0)}\right]^{9/7}. \tag{12}$$

We believe that Eqs. (9), (11), and (12) which demonstrate rapid decrease of $j(t)$ with temperature $T$ and time {as $[T\ln(t/t_0)]^{-\alpha}$} may provide the explanation of the results of the magnetic measurements of the critical currents where the fast drop of $j(t,T)$ with $T$ increasing and large relaxation was found.

The recent decoration experiments[20,21] have demonstrated that even in superconductors with comparatively high values of critical current (i.e., in superconductors with considerable disorder) one can observe well-pronounced flux lattice until rather large distances. Possible explanation of this fact is as follows. The correlation radius of the destruction of long-range order by the random potential is defined by the relation

$$R(a) \sim \lambda (L_c/\xi)^3 (a/\xi)^{1/\zeta_{3,2}-3} \simeq \lambda (L_c/\xi)^3 (a/\xi)^2$$

and one sees that $R(a) \gg \lambda$. So even for high magnitudes of critical current (if $L_c \sim \xi$, then $j \sim j_0$) the long-range order is destroyed on very large distances.

The main result of our paper is that creep activation energy depends strongly on the current at $j \ll j_c$. Note that there exists a direct way to measure this dependence. Namely, one should measure magnetic relaxation at different values of current but the same values of magnetic field and temperature. Different small currents may be obtained by heating the sample up to the temperature where the relaxation is strong and the current rapidly decreases and subsequent cooling down to the measurement temperature.

For the sake of simplicity we do not consider the effect of anisotropy of superconducting properties and temperature dependence of the parameter $W$ (see Refs. 4, 5, and 12). The detailed results of this study will be published elsewhere.

At high temperature the current rapidly decreases during the experiment and as follows from the result obtained, the activation energy of creep becomes very high. So another mechanism of creep (for instance, the plastic hopping of the extended defects of flux-line lattice, e.g., dislocation) becomes essential. The activation energy of such processes appears to depend weakly upon current but can be considerably higher than the energy of collective flux creep at low temperature and large current. Our results, which will be presented elsewhere, show that the results of resistive measurements[22] could be plausibly explained by taking into consideration the process of motion of dislocations in the flux-line lattice.

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[a]Also at Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106.

[b]Permanent address: Institute of Solid State Physics, Chernogolovka, Moscow Region, U.S.S.R.

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