# Model for Large Transition Magnetic Moment of the Electron Neutrino

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We propose a simple extension of the standard model by adding an $SU(2)_H$ horizontal gauge symmetry in the leptonic sector, which leads to a large transition magnetic moment of the electron neutrino while keeping the neutrino mass naturally small. This model can provide a solution to the solar neutrino puzzle while at the same time avoiding the SN 1987A bound on the neutrino magnetic moment.

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An interesting resolution of the deficit of solar neutrinos observed by Davis and collaborators[1] compared to the prediction of the standard solar model[2] is to assume that the neutrino has a magnetic moment $\mu_{\nu_e} \simeq 10^{-10}\mu_B$, where $\mu_B$ is the Bohr magneton. Such a large magnetic moment will cause the helicity of the neutrinos emitted in the solar core to flip in the magnetic field ($B \simeq 10^3$ G) present in the convective zone of the Sun, converting a large fraction of them into right-handed neutrinos which are sterile with respect to conventional weak interactions.[3] If the present hint of an anticorrelation of the observed solar neutrino flux with solar activity (i.e., number of sunspots) is borne out by future experiments, it will provide a dramatic confirmation of this proposal.

There are, however, indications that such a large value of $\mu_{\nu_e}$ may be in conflict with SN 1987A observations which require[4] $\mu_{\nu_e} \lesssim (10^{-12}\text{–}10^{-14})\mu_B$. Although it is possible to evade this restriction by postulating exotic Higgs interactions of the neutrinos,[5] which are necessary anyway to generate a large magnetic moment theoretically,[6] understanding the small mass of the neutrino naturally in these models requires several additional (and by no means compelling) symmetry assumptions.[7,8] The role of the exotic Higgs interactions is to cause trapping of the right-handed neutrinos in the supernova core, thereby reducing their emission temperature (due to thermalization in the neutrino sphere) and their contribution to the energy loss from the supernova.

An alternative suggestion similar in spirit is the proposal that instead of the direct magnetic-moment interaction $\bar{\nu}_{eL}\sigma_{\mu\nu}\nu_{eR}F^{\mu\nu}$, the neutrino deficit may be due to a transition-magnetic-moment interaction of the type $\bar{\nu}_e\sigma_{\mu\nu}\nu_\mu^c F^{\mu\nu}$, which connects two different neutrino flavors, both of which participate in the usual Fermi-strength weak interaction.[3,9] Since the $\nu_\mu^c$ energy is below the threshold of the weak interaction, they well escape detection, thus accounting for the solar neutrino deficit. This interaction easily avoids the supernova bounds since the $\nu_\mu^c$'s generated become automatically trapped and do not add new channels to the energy-loss mechanism.[10]

Since neutrinos of two different flavors need not be degenerate in energy (unlike the two helicity states of a Dirac neutrino), for the transition-magnetic-moment mechanism to be relevant in the solar interior, their mass difference should satisfy $\Delta m^2 \lesssim 10^{-7}$ eV$^2$. However, it has been shown recently by Lim and Marciano and by Akhmedov[9] that in the presence of matter, a resonant enhancement of the flavor-changing spin precession can occur, for a wide range of neutrino masses satisfying $10^{-8} \lesssim \Delta m^2 \lesssim 10^{-4}$ eV$^2$, in analogy with the widely discussed Mikheyev-Smirnov-Wolfenstein mechanism.[11]

The purpose of this Letter is to propose a model based on a very minimal extension of the standard model, which leads to a large $\nu_e$-$\nu_\mu$ transition magnetic moment while keeping the neutrino mass naturally small. In our discussion a crucial role is played by an $SU(2)_H$ horizontal symmetry between the electron and muon generations. The basic observation is that the $\nu_e$-$\nu_\mu$ transition magnetic moment is a singlet under $SU(2)_H$, whereas the mass term is a triplet, and therefore is forbidden in the $SU(2)_H$-symmetric limit.

*The model.*— We consider the gauge group to be $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_H$, where $SU(2)_H$ is the local horizontal symmetry. We assume the leptons to transform as

$$\begin{aligned} &\Psi_L = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}_L, \quad (1,2,-1,2)\,, \\ &\Psi_R = \begin{pmatrix} e & \mu \end{pmatrix}_R, \quad (1,1,-2,2)\,, \\ &\Psi_{3L} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad (1,2,-1,1)\,, \\ &\tau_R, \quad (1,1,-2,1)\,. \end{aligned} \tag{1}$$

We choose the following set of Higgs multiplets:[12]

$$\begin{aligned} &\Phi = \begin{pmatrix} \phi_1^+ & \phi_2^+ \\ \phi_1^0 & \phi_2^0 \end{pmatrix}, \quad (1,2,1,2)\,, \\ &\phi_s = \begin{pmatrix} \phi_s^+ \\ \phi_s^0 \end{pmatrix}, \quad (1,2,1,1)\,, \\ &\eta = \begin{pmatrix} \eta_1^+ & \eta_2^+ \end{pmatrix}, \quad (1,1,2,2)\,, \\ &\sigma_a = \begin{pmatrix} \sigma_1^0 & \sigma_2^0 & \sigma_3^0 \end{pmatrix}_a, \quad (1,1,0,3), \quad a=1,2\,. \end{aligned} \tag{2}$$

Our main results are independent of the details of the quark sector. We therefore do not discuss it in any detail. If the quarks are assumed to be neutral under $\mathrm{SU}(2)_H$, the model is automatically free from triangle anomalies without the need for any exotic fermions.

The most general Yukawa coupling for our model is given by

$$\mathcal{L}_Y = h_1 \mathrm{Tr}(\bar{\Psi}_L \phi_s \Psi_R) + h_2 \bar{\Psi}_{3L} \phi_s \tau_R + h_3 \bar{\Psi}_{3L} \Phi i \tau_2 \Psi_R^T + f \eta \tau_2 (\Psi_L^T \tau_2 C^{-1} \Psi_{3L}) + f' \mathrm{Tr}(\bar{\Psi}_L \Phi) \tau_R + \mathrm{H.c.} \tag{3}$$

The Higgs potential, in addition to the usual terms [viz., $\mathrm{Tr}(\Phi^\dagger \Phi)$, $\phi_s^\dagger \phi_s$, $\eta\eta^\dagger$, $\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_a$, and their quartic products] contains the following interlocking terms:

$$\begin{aligned} V' = {} & \lambda_1 |\mathrm{Tr}(\Phi^T \tau_2 \Phi \tau_2)|^2 + \lambda_2 |\Phi^\dagger \phi_s|^2 + \lambda_3 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)^2 \\ & + \{\lambda_4 \phi_s^T i \tau_2 \Phi \boldsymbol{\tau}^T \eta^\dagger \cdot \boldsymbol{\sigma}_1 + \mu_1 \eta \Phi^\dagger i \tau_2 \phi_s^* + \mu_2 \eta^* \boldsymbol{\tau} \eta^T \cdot \boldsymbol{\sigma}_1 + \mu_3 \mathrm{Tr}(\Phi \boldsymbol{\tau}^T \Phi^\dagger) \cdot \boldsymbol{\sigma}_1 + \mathrm{H.c.}\} . \end{aligned} \tag{4}$$

We have imposed a $\sigma_2 \to -\sigma_2$ symmetry in the above. The potential is minimized by the following vacuum expectation values:

$$\langle \sigma_1 \rangle = \begin{pmatrix} 0 & 0 & v_1 \end{pmatrix}, \quad \langle \sigma_2 \rangle = \begin{pmatrix} 0 & v_2 & 0 \end{pmatrix}, \qquad \langle \Phi \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa \end{pmatrix}, \quad \langle \phi_s \rangle = \begin{pmatrix} 0 \\ \kappa_s \end{pmatrix}. \tag{5}$$

Now, it would appear that in general both $\langle \phi_1^0 \rangle$ and $\langle \phi_2^0 \rangle$ could be nonzero, but if we parametrize $\langle \phi_1^0 \rangle = r\cos\theta$, $\langle \phi_2^0 \rangle = r\sin\theta$, minimization of the potential [Eq. (4)] with respect to $\theta$ yields $\theta = 0$ or $\pi/2$. Similarly, among the second and third components of $\langle \sigma_2 \rangle$, only one can be nonzero.[13] This is a welcome result, since in this case, the mixing between the generation-changing horizontal gauge boson $V_\pm$ and the generation-diagonal $V_3$ vanishes at the tree level. Such mixing, if present, would set a stringent lower limit on the scale of horizontal-symmetry beaking from $\mu \to 3e$ decay. We will choose $v_1, v_2 \gg \kappa, \kappa_s$, making the horizontal gauge bosons much heavier than the $W$ and $Z$ bosons. We shall see later that the present experimental constraints require $m_{V_\pm}$ to be in the TeV range.

Note that Eqs. (3) and (4) taken together implies violation of lepton number,[14] which is necessary for the generation of transition magnetic moments (as well as Majorana neutrino masses). The full Lagrangian, however, respects $L_e - L_\mu$ before spontaneous symmetry breaking. Furthermore, in the absence of the $h_3$ (or $f'$) term in Eq. (3), the $\tau$ lepton number $L_\tau$ is also conserved separately. The vacuum expectation values $\langle \sigma_i \rangle$, $\langle \Phi \rangle$, and $\langle \phi_s \rangle$ leave $L_e - L_\mu - L_\tau$ unbroken, while $\langle \sigma_2 \rangle$ breaks this symmetry.[15] These considerations will turn out to imply that the neutrinos in our model are pseudo Dirac particles.[16]

Before addressing the question of neutrino masses and magnetic moments, let us analyze the situation with the charged-lepton masses. We shall work in the limit $h_1 = 0$ from now on. (This can be achieved naturally by a discrete symmetry without changing the rest of the Lagrangian, e.g., $\Psi_R \to -\Psi_R$, $\Psi_{3L} \to i\Psi_{3L}$, $\tau_R \to i\tau_R$, $\Phi \to -i\Phi$, $\eta \to -i\eta$.) As a result, the electron is massless at the tree level, but will pick up a radiative mass at the one-loop level. The $\tau$ and $\mu$ have tree-level masses

$$m_\tau \simeq h_2 \kappa_s, \quad m_\mu \simeq \frac{f' h_3 \kappa^2}{h_2 \kappa_s} . \tag{6}$$

The presence of a tree-level mass term $\phi_1^0 \phi_1^0$ leads to the generation of electron mass at one loop:

$$m_e \simeq \frac{f' h_3}{16\pi^2} \frac{\lambda_3 \mu_3^2 \kappa^2}{m_\phi^4} m_\tau . \tag{7}$$

Turning to the discussion of neutrino masses and magnetic moments, it is clear that the neutrinos are massless at the tree level. After electroweak-symmetry breaking triggered by $\langle \phi_s^0 \rangle = \kappa_s$, the $\mu_1$ term of Eq. (4) generates the following mixing between the Higgs bosons:

$$\mathcal{L}_{\eta\phi} = \mu_1 \kappa_s (\eta_1^+ \phi_1^- + \eta_2^+ \phi_2^-) + \mathrm{H.c.} \tag{8}$$

This mixing will induce a mass connecting $\nu_e$ and $\nu_\mu$ at the one-loop level via the $\eta$-$\phi$ exchange of Fig. 1. However, it follows from Eqs. (3) and (4) that as long as $\eta_1$ and $\eta_2$ are degenerate as are $\phi_1$ and $\phi_2$ [which is indeed the case in the $\mathrm{SU}(2)_H$-symmetric limit], the two diagrams cancel, leading to zero mass for the neutrino. On the other hand, to obtain the magnetic moment of the neutrino, we must insert a photon line, which due to different electric charges of the scalar bosons in the loop causes both the diagrams to add, leading to a nonzero $\mu_{\nu_e \nu_\mu}$ given by

$$\mu_{\nu_e \nu_\mu} \simeq 2e \frac{f f'}{16\pi^2} m_\tau \frac{\mu_1 \kappa_s}{m_\eta^2 - m_\phi^2} \left( \frac{1}{m_\eta^2} - \frac{1}{m_\phi^2} \right) . \tag{9}$$

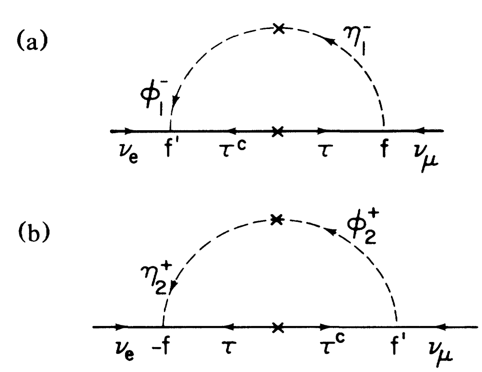


FIG. 1. One-loop graphs that can lead to nonvanishing $m_\nu$. Attaching a photon line to the internal lines of these graphs leads to the large transition magnetic moment between $\nu_e$ and $\nu_\mu$.

As shown in Ref. 6, $\mu_{\nu_e\nu_\mu}$ of Eq. (9) can be as large as $10^{-10}\mu_B$ without violating any of the experimental constraints. Thus, as a consequence of the horizontal symmetry, one can have a large $\nu_e\nu_\mu$ transition magnetic moment while at the same time having a vanishing neutrino mass.

Once the horizontal-symmetry breaking is turned on by $\langle\sigma_1\rangle,\langle\sigma_2\rangle\neq 0$, the $\eta_1$-$\eta_2$ masses split as do the $\phi_1$-$\phi_2$ masses and a nonvanishing $m_{\nu_e\nu_\mu}$ results. We obtain

$$m_{\nu_e\nu_\mu}\simeq\frac{ff'}{16\pi^2}m_\tau\mu_1\kappa_s\left[\frac{1}{m_{\phi_1}^2-m_{\eta_1}^2}\ln\frac{m_{\phi_1}^2}{m_{\eta_1}^2}-\frac{1}{m_{\phi_2}^2-m_{\eta_2}^2}\ln\frac{m_{\phi_2}^2}{m_{\eta_2}^2}\right]. \tag{10}$$

Note that the above expression vanishes in the $SU(2)_H$-symmetric limit, whence $m_{\eta_1}^2=m_{\eta_2}^2$ (and similarly for $\phi$). For quasidegenerate $\eta$ (as well as $\phi$) fields, Eq. (9) can be rewritten as

$$\mu_{\nu_e\nu_\mu}\simeq 2e\frac{m_{\nu_e\nu_\mu}}{\Delta}\left[\frac{1}{m_\eta^2}-\frac{1}{m_\phi^2}\right], \tag{11}$$

where

$$\Delta=\frac{\Delta m_\phi^2}{m_\phi^2}-\frac{\Delta m_\eta^2}{m_\eta^2}-\frac{\Delta m_\phi^2-\Delta m_\eta^2}{m_\phi^2-m_\eta^2}\ln\frac{m_\phi^2}{m_\eta^2}, \tag{12}$$

with $\Delta m_\eta^2=m_{\eta_2}^2-m_{\eta_1}^2\ll m_\eta^2$ (and similarly for $\phi$). Consider as an example the case where $\Delta m_\phi^2/m_\phi^2\simeq\Delta m_\eta^2/m_\eta^2$, when Eq. (11) simplifies to

$$\mu_{\nu_e\nu_\mu}\simeq 2e\frac{m_{\nu_e\nu_\mu}}{\Delta m_\eta^2}\left[\frac{m_\eta^2}{m_\phi^2}-1\right]\ln\frac{m_\eta^2}{m_\phi^2}. \tag{13}$$

Demanding $m_{\nu_e}\leq 20$ eV, and $\mu_{\nu_e\nu_\mu}\geq 10^{-11}\mu_B$, we obtain from the above the naturalness condition (for $m_\eta\simeq 3m_\phi$)

$$\Delta m_\eta^2\leq 200\ \mathrm{GeV}^2. \tag{14}$$

In the general case, this bound can be easily relaxed by a factor of 4 or so without any fine tuning.

The value of $\Delta m_\eta^2$ is given by $\mu_2 v_1$ and therefore depends on the scale of $SU(2)_H$ breaking. One might therefore think that Eq. (14) will impose a naturalness upper bound on the horizontal-gauge-boson masses $m_{V_\pm}$, $m_{V_3}$ as in Ref. 8. But unlike Ref. 8, the $\mu_2$ term in Eq. (4) responsible for $\eta_1$-$\eta_2$ splitting is a soft term and receives only finite radiative corrections, proportional to $\mu_2$ itself. Similarly, the $\mu_3$ term of Eq. (4) which splits the masses of $\phi_1$ and $\phi_2$ is also soft. Therefore arbitrarily small values of $\mu_{2,3}$ are radiatively unaltered. As a consequence, there is no upper limit on the horizontal-gauge-boson masses.[17] Note also that in the limit $\mu_3\to 0$, the potential has an extra U(1) symmetry. This means that one of the neutral Higgs bosons has a mass proportional to $\mu_3 v_1$. Thus the model predicts a light Higgs particle with a mass less than 30 GeV or so.

There are also diagrams analogous to Fig. 1 connecting $\nu_e$ and $\nu_\tau$ and thus contributing to $m_{\nu_e\nu_\tau}$. These diagrams are proportional to the $SU(2)_H$-breaking vacuum expectation value $\kappa$ and therefore vanish in the $SU(2)_H$-symmetric limit. Since the coupling $h_1$ has been set to zero, all entries of the neutrino mass matrix other than $\nu_e\nu_\mu$ and $\nu_e\nu_\tau$ are zero at the one-loop level. These entries will become nonvanishing once higher-order corrections are taken into account. This means that the neutrino is a pseudo Dirac particle in our model. Thus $\Delta m^2\leq 10^{-7}$ eV$^2$ can be easily satisfied. Demanding that $m_{\nu_e\nu_\tau}\leq 20$ eV leads to the conditions $h_2\kappa/f'\kappa_s\leq 10^{-3}$ and $h_3^2\kappa/h_2 f'\kappa_s\leq 10^{-3}$, both of which can be satisfied naturally by choosing $\kappa\ll\kappa_s$.

The horizontal gauge bosons, if light enough, can have dramatic effects on low-energy weak processes, which we shall address below.

(a) The generation-changing horizontal gauge bosons $V_\pm$ add new channels to $\mu$ decay. The amplitude is modified to

$$A=\frac{G_F}{\sqrt{2}}[(1+\epsilon)\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu-2\epsilon\bar{e}(1-\gamma_5)\nu_e\bar{\nu}_\mu(1+\gamma_5)\mu], \tag{15}$$

where $\epsilon=g_H^2m_W^2/g^2m_{V_\pm}^2$, $g_H$ being the horizontal gauge coupling. The presence of the scalar term in Eq. (15) leads to deviation from the $V-A$ prediction in the $\mu$-decay asymmetry parameters. The limit[18] on the polarization asymmetry parameter $\xi$ ($\xi\geq 0.99677$) implies $\epsilon\leq 0.04$. (The Michel parameter $\rho$ is unaffected.) Another consequence of Eq. (15) is deviation from $e$-$\mu$-$\tau$ universality. Since $\tau$ is a singlet under $SU(2)_H$, its decay is not modified by the new gauge interactions. Therefore, $G_\tau=G_\mu/(1+\epsilon)$. This means that the $\tau$ lifetime could be longer than the universality prediction by as much as 8%. (However, see below.)

(b) The new contribution in Eq. (15) modifies the relation between $G_F$ measured in nuclear $\beta$ decay and $G_\mu$ from $\mu$ decay to $G_F|V_{ud}|^2=G_\mu/(1+\epsilon)$. Using the experimental value[19] of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ud}|$ and the unitarity of $V$, we obtain $\epsilon\leq 0.25\%$. This requires $m_{V_\pm}\gtrsim 1.6$ TeV (for $g_H=g$).

(c) A new low-mass horizontal gauge boson $V_\pm$ can have low-energy manifestations in rare decays through its mixing with the $Z$ boson. In our model, since we had $\langle\phi_1^0\rangle=0$, such mixing is forbidden at the tree level. If such mixing is introduced (by relaxing the $\sigma_2\to-\sigma_2$ symmetry, for example), it will give rise $\mu\to 3e$ and we must demand that $\langle\phi_1^0\rangle\langle\phi_2^0\rangle/m_{V_\pm}^2\leq 10^{-5}$.

(d) Since the vacuum expectation values $\langle\sigma_1\rangle$ and $\langle\sigma_2\rangle$ are both nonzero, there arises a mixing term $V_+V_+$ in the horizontal-gauge-boson mass matrix. This can lead to muonium-antimuonium oscillation with a strength $\beta\epsilon G_F$, where $\beta=v_2^2/2(v_1^2+v_2^2)$ is the mixing parameter.

(e) The low-mass neutral Higgs particle expected in this model modifies the $Z$ width and should be observable in $Z$ decay. Its contribution to the width is equivalent to half a neutrino (ignoring mixing effects) which should be measurable at the CERN $e^+e^-$ collider LEP.

Our work is particularly interesting in view of the recent observation that in the seesaw models, it is not possible to obtain a large transition magnetic moment, while keeping the neutrino masses small.[20] It should also be mentioned that the present model is much simpler in terms of its particle content than the model of Ref. 8. In particular, we do not need any new fermions beyond the standard model.

In conclusion, we have presented a simple horizontal-symmetry extension of the standard model, which leads to a pseudo Dirac neutrino having a large transition magnetic moment $\mu_{\nu_e \nu_\mu}$ required to solve the solar neutrino puzzle and a naturally small mass. The scale of horizontal-symmetry breaking is expected to be in the TeV range.

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