# Fractional Quantum Hall Effect in One Dimension

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For the first time, we have observed the fractional quantum Hall effect in a junction comprised of four one-dimensional ballistic constrictions, less than 100 nm wide, in a high-mobility two-dimensional electron gas. The four-terminal Hall resistance $R_{xy}$ at 280 mK is quantized near $(h/e^2)/i$, where $i$ denotes certain odd-denominator fractions, and surprisingly for $(h/e^2)/0.55 < R_{xy} < (h/e^2)/0.45$, as well.

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At low temperature, in a high magnetic field when only one Landau level is occupied the four-terminal Hall resistance of a high-mobility two-dimensional electron gas (2DEG) can be quantized at $(h/e^2)/i$, where $i$ denotes certain simple fractions.[1] The fractional quantum Hall effect (FQHE) is attributed to the Coulomb interaction between electrons and the formation of an incompressible quantum fluid, which is described by the Laughlin wave function.[2] Because the ground-state wave function is required to be *antisymmetric* under particle exchange, the resistance is expected to be quantized only if $i$ is an *odd-denominator* rational fraction and, so far, only odd-denominator fractions have been observed.[3] The quantization is exact only for a macroscopic sample at zero temperature, because it depends upon the long-range behavior of the Coulomb interaction.

Here, we report evidence of the FQHE in a ballistic, one-dimensional (1D) electron gas in a high magnetic field for temperatures $T > 100$ mK. We measured the resistance of a junction comprised of four variable-width constrictions in a high-mobility 2DEG at a GaAs/AlGaAs heterointerface. The four constrictions, configured in a cross geometry, are used as voltage and current terminals. The four-terminal Hall resistance, $R_{xy}$, of a junction comprised of constrictions 200 nm long and about 90 nm wide, is quantized at $(h/e^2)/i$, where $i$ is an odd-denominator rational fraction with an accuracy of about 96%. Our observations reveal that the length scale relevant to the fractional quantum Hall effect is less than 90 nm when the magnetic length $r_0 = (hc/eH)^{1/2} \approx 10$ nm. What is especially provocative is that the four-terminal Hall resistance is also quantized for $(h/e^2)/0.55 < R_{xy} < (h/e^2)/0.45$. This is surprising because the Hall resistance of a macroscopic 2DEG is usually featureless near $(h/e^2)/(\frac{1}{2})$.

The four constrictions were made using a split-gate geometry on a GaAs/AlGaAs heterostructure so that the width of each constriction can be independently controlled by changing the gate voltage. Using photolithography and a wet etch, a Hall-bar geometry comprised of wires about 100 $\mu$m wide is transferred onto the heterostructures. Four split-gate electrodes with a 300-nm gap between electrodes were then fabricated on top of the Hall bar using electron-beam lithography. The disposition of the gate electrodes is depicted schematically in the inset to Fig. 1(a). The electrodes were made of a gold-palladium alloy approximately 80 nm thick. Associated with each constriction in the device, numbered 1 through 4 in the inset, there are contacts to the 2DEG which are not shown. Devices were fabricated in two different heterostructures. The mobility, $\mu$, and carrier density, $n$, associated with the 2DEG at the lower AlGaAs/GaAs interface at 280 mK were $\mu = 110$ m$^2$/Vs and $n = 2.8\times10^{15}$ m$^{-2}$ for the first heterostructure, and $\mu = 107$ m$^2$/Vs and $n = 2.0\times10^{15}$ m$^{-2}$ for the second. The Hall effect in the 100-$\mu$m Hall bar was quantized to $(h/e^2)/i$, for integers $i \leq 5$, with at least 99.8% accuracy. The parameter $i$ is the Landau-level filling factor.

In the absence of a magnetic field, the two-terminal conductance of the constrictions comprising the junction is quantized in steps of $(1.9\pm0.1)e^2/h$ as a function of the gate voltage $V_g$, if the lithographic length of the constrictions is 200 nm, and if the other gate electrodes are grounded during the measurement.[4] (A detailed characterization of the junction is given elsewhere.[5]) By applying a voltage more negative than $-0.3$ V to the split gates, the 2DEG immediately beneath the gate electrodes is depleted and so the 2DEG is laterally constrained within the gap between the electrodes. As the gate voltage becomes more negative, the width of the constriction narrows, the carrier density within the constriction decreases, and plateaus are observed in the conductance. For large negative gate voltages, the 300-nm gap pinches off. Ideally, there is a direct correspondence between the number of occupied 1D subbands in a ballistic constriction and the two-terminal conductance given by the Landauer formula:[6-8] $G_{ij,ij} = (e^2/h)T_{ij} = (e^2/h) \times 2N$, where $T_{ij}$ is the transmission probability from lead $j$ to lead $i$, and $N$ is the number of occupied spin-degenerate subbands in the constriction. $G_{lm,jk}$ ($R_{lm,jk}$) denotes a conductance (resistance) measurement in which positive current flows into lead $l$ and out lead $m$ with a positive voltage measured between leads $j$ and $k$, respectively. The correspondence between the measured conductance and the Landauer formula implies that: (1) For a lithographic length of 200 nm, the constriction is ballistic; and (2) that the energy separation between subbands is much larger than 280 mK.

In a junction comprised of constrictions 200 nm long, the predominate scatterers are not impurities, but the constrictions. Scattering from the constrictions determines the resistance for magnetic fields $HW^2e/hc < 1$, where $W$ is the width of the constriction, and for fields between the integral Hall plateaus.[9] The effect of a lead on the resistance is especially evident in the four-terminal resistance. The four-terminal resistance expressed in terms of transmission probabilities is[10] $R_{mn,kl} = (h/2e^2)(T_{km}T_{ln} - T_{kn}T_{lm})/D$, where the denominator $D$ is always positive and depends on the lead geometry, but is independent of permutations in the indices $mn,kl$. If the gate electrodes of the junction are biased similarly, and if the junction is symmetric, then $R_{14,32} = (T_{31}T_{24} - T_{34}T_{21})/D < 0$ and $R_{12,43} = (T_{41}T_{32} - T_{42}T_{31})/D \approx 0$ because the probability for a carrier in a low-index-$N$ subband to propagate around a corner is small $(T_{31}, T_{24}, T_{41}, T_{23} \ll N)$, while $T_{21}, T_{34} \approx N$ for low $N$.[5] While this equation for the resistance applies for an arbitrary magnetic field,[11] but the transmission coefficients change for magnetic fields larger than $HW^2e/hc \approx 1$ because in this field range the conduction occurs via edge states of spatial extent $r_0$.[9] For $HW^2e/hc \gg 1$, the backscattering between opposite edges is reduced, and so the transmission probability between adjacent leads is exactly equal to the number of occupied edges states, while the transmission between leads which are not adjacent vanishes. Thus at high fields, $R_{14,32} \gtrsim 0$ and $R_{12,43} \approx (h/e^2)/i$, where $i$ denotes the number of edge states occupied in the 1D constriction, because $D = i^3$, $T_{13}, T_{24}, T_{41}, T_{32} \approx i$, while all other $T_{ij} \approx 0$.[10]

Figures 1(a)–1(c) show the magnetoresistances $R_{14,32}$ and $R_{12,43}$ observed at 100 mK when each of the constrictions comprising the junction are (a) not biased ($V_g = 0$ V), (b) biased so that only the two lowest subbands ($N = 2 \pm 1$) are occupied in each of the constrictions, and (c) biased so that only the lowest subband ($N = 1 \pm 1$) is occupied. $N$ is estimated using the following: (1) the quantization of the two-terminal resistances of each of the constrictions versus $V_g$ at $H = 0$ T; (2) the peaks found in $R_{14,32}$ vs $V_g$ at $H = 0$ T, associated with the change in the number of occupied subbands;[12] and (3) the quantization of $R_{12,43}$ vs $H$. Using a hard-wall confinement potential and the carrier density deduced from $R_{12,43}$ at $H = 8$ T and our estimate for $N$, we infer that $W = 105 \pm 25$ nm in Fig. 1(b) and $W \approx 70 \pm 25$ nm in Fig. 1(c).

A comparison of Fig. 1(a) with Figs. 1(b) and 1(c) shows that the four-terminal resistance of the junction is determined by the transmission through the constrictions and is not affected by the discontinuity in the density at the 2D contacts to the 1D constrictions or by the magnetoresistance in the 2D contact.[13] Notice that $R_{14,32} \gtrsim 0$ and $R_{12,43} = H/nec$ for $H < 100$ mT in the 2D junction of Fig. 1(a). In contrast, $R_{14,32}$ is negative and $R_{12,43}$ is

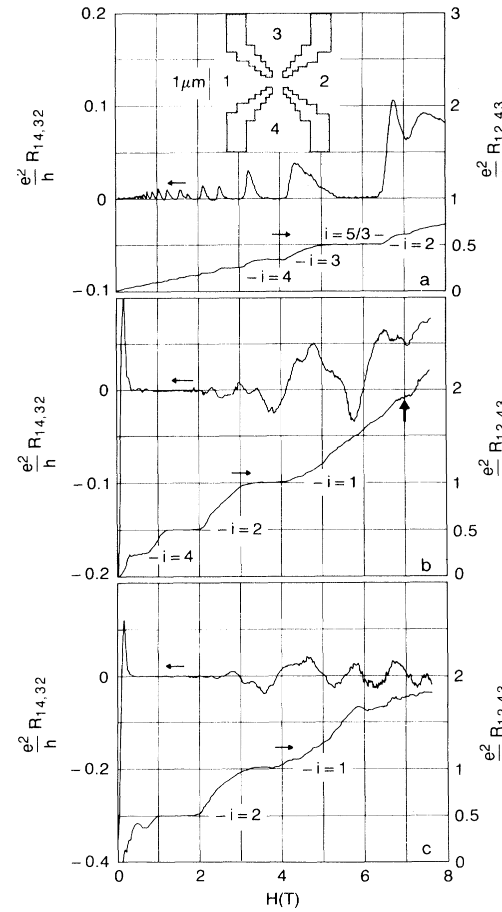


FIG. 1. The resistances $R_{12,43}$ and $R_{14,32}$ vs magnetic field found at 100 mK for a device made from the first heterostructure. In (a) the leads comprising the cross are 100 $\mu$m wide, while in (b) and (c) the leads are about 105 and 70 nm wide, respectively. The vertical arrow in (b) at $H = 7$ T indicates the position of an anomalous plateau in $R_{12,43}$ near $(h/e^2)/0.53$. Inset to (a): Schematic representation of the junction; the convention for labeling the leads or the constrictions between the electrodes is given here.

suppressed from the conventional 2D Hall resistance for $H < 100$ mT in Figs. 1(b) and 1(c),[14] in correspondence with our expectations for a narrow wire. For magnetic fields $HW^2e/hc > 1$, $R_{12,43}$ in Figs. 1(b) and 1(c) is quantized at approximately $(h/e^2)/i$ for integers $i \leq 2N$. In Fig. 1(b), $R_{12,43}$ reaches a plateau at approximately 300 mT, i.e., $R_{12,43} = (h/e^2)/(4.10 \pm 0.15)$, corresponding to the depopulation of the highest spin-polarized 1D subband, $i = 4$. The plateau associated with $i = 3$ is not well defined. However, with increasing magnetic field other plateaus are observed in the resistance centered at $(h/e^2)/(1.97 \pm 0.03)$ and $(h/e^2)/(0.99 \pm 0.03)$ near $H = 1.6$ and 4 T, respectively, corresponding to the $i = 2$ and 1 states. In Fig. 1(c), $R_{12,43}$ reaches plateaus at approximately 1 and 3.5 T, where $R_{12,43} = (h/e^2)/(2.02 \pm 0.03)$ and $(h/e^2)/(0.95 \pm 0.07)$, respectively, corresponding to $i=2$ and 1. For 300 mT $< H <$ 5 T, $R_{12,43}$ is not exactly quantized, however, and $R_{14,32}$ can be negative. We attribute these observations to the overlap between edge states due to the narrow width of the constrictions. For $H < 5$ T with $W \approx 70$ nm, the overlap between edge states is approximately $\exp[-(W/2r_0)^2] < 0.1$.[9] Because of the overlap, there is about (5–10)% backscattering between opposite edges, which gives rise to (5–10)% deviations in the transmission probability between adjacent leads, and ultimately to $0.1h/e^2$ deviations from exact quantization in $R_{12,43}$ and $-0.1h/e^2 < R_{14,32} < 0$.

While the value of the plateau resistance, $R_{12,43}$, is a measure of the number of occupied subbands, the magnetic field is not. Unlike a 2DEG where $i=nhc/eH$, the quantized resistance is not found at regular intervals in $H^{-1}$ in a constriction because the 1D subbands are defined by both magnetic and electrostatic confinement, and because the carrier density is determined by the Fermi energy in the 2D contacts which varies as a function of $H$.

Beyond $H=5$ T there are features in both $R_{14,32}$ and $R_{12,43}$ which we associate with the FQHE. In Fig. 1(b) there is a feature in $R_{12,43}$ centered near $(h/e^2)/0.53$ at $H=7$ T and, corresponding to the plateau, there is a minimum in $R_{14,32}$. There is also a minimum in $R_{14,32}$ near 5.7 T in Fig. 1(b), but there is only an inflection at $R_{12,43}=(h/e^2)/0.66$. Figure 1(c) does not show features which we can unambiguously associate with the FQHE; however, we do observe minima in $R_{14,32}$ near $H=5.25$, 6.2, and 7.2 T. Figure 2 shows the magnetoresistance observed at 280 mK in two devices in the two different heterostructures. In Fig. 2(a) each of the constrictions comprising the junction are biased so that $N=3\pm1$ and $W=140\pm40$ nm at $H=0$ T, while in Fig. 2(b), $N=2\pm1$ and $W=90\pm25$ nm. In addition to the integer quantization observed below 5 T, we again find plateaus in $R_{12,43}$, at fractional occupation of the lowest subband. In Fig. 2(a) there are plateaus centered near $(h/e^2)/0.33$ and $(h/e^2)/0.54$, at $H=10.8$ and 6.65 T, respectively, and, corresponding to the plateaus, there are minima in $R_{14,32}$. We associate these features with subband filling factors $i=\frac{1}{3}$ and $\frac{1}{2}$, respectively. In Fig. 2(b) there are plateaus centered near $(h/e^2)/0.340$, $(h/e^2)/0.395$, $(h/e^2)/0.47$, $(h/e^2)/0.685$, and $(h/e^2)/1.633$, at $H=11.5$, 9.8, 8.7, 6.9, and 2.88 T, respectively, and, corresponding to the plateaus, there are minima in $R_{14,32}$. We associate these features with subband filling factors $i=\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, or $\frac{6}{13}$, $\frac{2}{3}$, and $\frac{5}{3}$, respectively.

Figure 3 shows the dependence of the quantization of $R_{12,43}$ on $V_g$ exhibited by the device of Fig. 2(b). At $V_g=0$ V, the wires comprising the junction are 100 $\mu$m wide and $R_{12,43}=(h/e^2)/i$ precisely, where $i=1$, $\frac{5}{3}$, and 2. When $V_g=-0.68$ V, $W\approx170$ nm and $N=4\pm1$, and a plateau is evident at $R_{12,43}=(h/e^2)/0.667$ near $H=9.6$ T corresponding to $i=\frac{2}{3}$.[15] At $V_g=-0.87$ V, where $N=3\pm1$ and $W\approx130$ nm, a feature develops near $R_{12,43}=(h/e^2)/0.52$ at $H=9.3$ T. The arrows in Fig. 3 follow this feature through $R_{12,43}=(h/e^2)/0.50$ as a function of the gate voltage. At $V_g=-0.90$ V, the feature appears centered near $R_{12,43}=(h/e^2)/0.50$, while for $V_g=-0.94$ V, with $N=2\pm1$ and $W\approx90$ nm, the feature occurs near $(h/e^2)/0.47$. The definition of the plateaus found about $(h/e^2)/(\frac{1}{2})$ and the plateaus near $(h/e^2)/0.66$ and $(h/e^2)/0.40$ depends on $V_g$ generally.

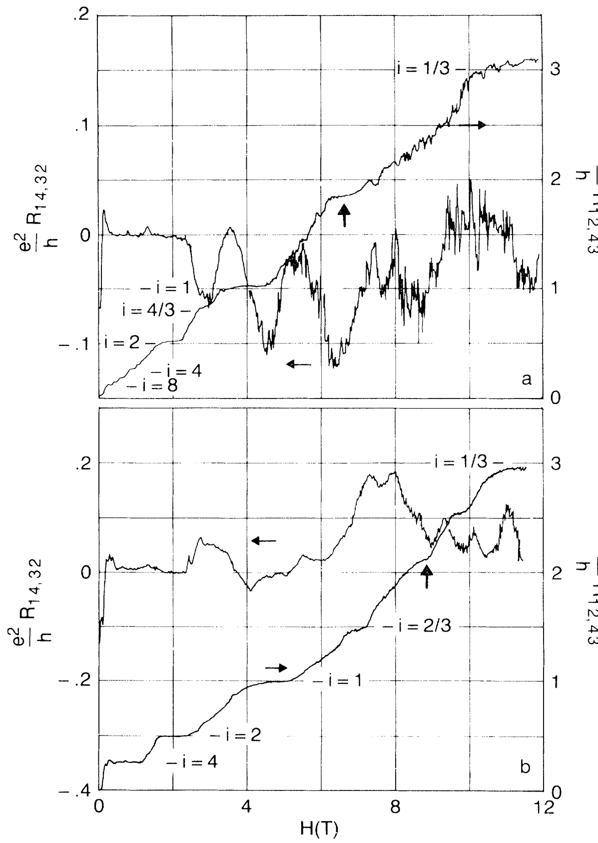


FIG. 2. The resistances $R_{14,32}$ and $R_{12,43}$ observed at 280 mK for two different devices made in the (a) second and (b) first heterostructure described in the text. In (a) the width of the constrictions comprising the junction is approximately 140 nm, while in (b) the width is approximately 90 nm. The arrows at $H=6.3$ T in (a) and at $H=8.7$ T in (b) indicate the positions of anomalous plateaus in the Hall resistance near $(h/e^2)/(\frac{1}{2})$.

Although there is not yet a model for transport in the fractional quantum Hall regime, the observation of the FQHE in a wire 90 nm wide where 7.5 nm $< r_0 <$ 10.5 nm is novel and it establishes an upper bound for the size of the current-carrying state at $i=\frac{2}{3}$ for $H=6.8$ T and at $i=\frac{1}{3}$ state for $H=11$ T. We have not observed the FQHE for constrictions narrower than $90\pm25$ nm, but we cannot set a lower bound because we have observed minima in $R_{14,32}$ vs $H$ in a junction in which $N=1\pm1$ and $W\approx70$ nm for fractional filling factors. If the

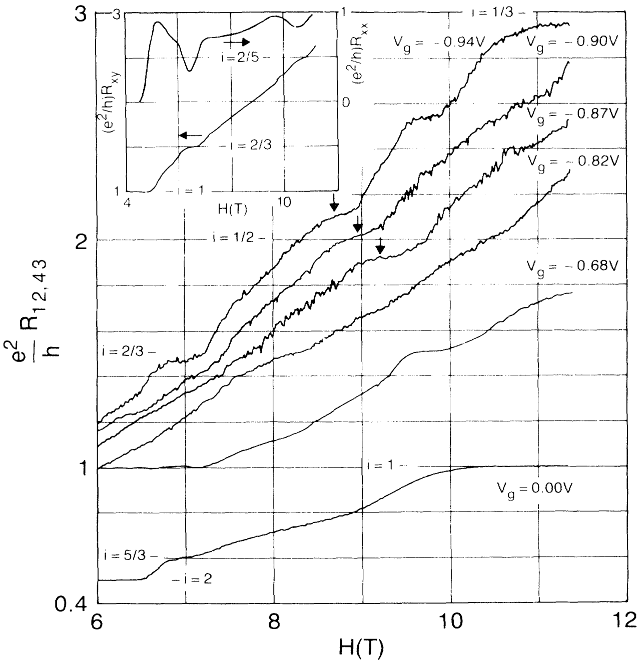


FIG. 3. The resistance $R_{12,43}$ vs magnetic field for $H > 6$ T found in the junction of Fig. 2(b) as a function of the gate voltage. Inset: Magnetoresistance found in a 2D Hall bar with an areal density and mobility comparable to that found at $V_g \simeq -0.9$ V in the device of Fig. 2(b).

FQHE is due to correlations which develop between electrons, then the range of the correlation can be less than 90 nm. The observation of a quantized Hall resistance near $(h/e^2)/(\frac{1}{2})$ is also novel. The inset to Fig. 3 shows the Hall resistance $R_{xy}$ and the longitudinal resistance $R_{xx}$ in a junction comprised of 100-$\mu$m-wide wires made from a heterostructure in which $n = 1.0 \times 10^{15}$ m$^{-2}$ and $\mu = 87$ m$^2$/V s. While $R_{xx}$ in a high-mobility 2DEG routinely exhibits a broad minimum for magnetic fields near $i = \frac{1}{2}$, no plateau has ever been observed in $R_{xy}$. The minimum found in $R_{xx}$ in a 2DEG is usually attributed to a sequence of unresolved, odd-denominator fractions. Because of the poor accuracy of the quantization of the FQHE in a narrow constriction, we cannot unambiguously distinguish between $\frac{1}{2}$ and odd-denominator fractions such as $\frac{5}{9}$.[3] However, the identification of the features found near $(h/e^2)/(\frac{1}{2})$ with odd-denominator fractions is unlikely because the size of the gap for $i = \frac{5}{9}$, $\frac{4}{9}$, etc., is supposed to be smaller than 280 mK.[1]

The observation of a plateau near $(h/e^2)/(\frac{1}{2})$ may be associated with the narrow width of the constrictions comprising the junction. Based on numerical calculations, Chui[16] has predicted an energy gap for $i = \frac{1}{2}$ as well as $i = \frac{1}{3}$ filling factors in a constricted 2DEG of spinless electrons where $W \approx (5-10)r_0$. According to Chui, the spatial dependence of exchange interactions and the boundaries of a 1D constriction can produce minima in the ground-state energy near filling factors of $i = \frac{1}{2}$ and $\frac{1}{3}$ for particular widths of the constriction. The size of the gap at $i = \frac{1}{2}$ can even be larger than that at $i = \frac{1}{3}$, which suggests that plateaus near $(h/e^2)/(\frac{1}{2})$ and $(h/e^2)/(\frac{1}{3})$ would be observed at the same temperature. Moreover, the dependence of the plateau found near $(h/e^2)/(\frac{1}{2})$ on $V_g$, and the poor precision of the quantization may be indicative of the dependence of the size of the calculated gap at $i = \frac{1}{2}$ on the width of the constriction. However, to establish the correspondence between our experimental results and Chui's calculations, the model should be reconciled with the four-terminal geometry in which the measurements are made. If the deviations observed from exact quantization in the FQHE are indicative of backscattering, then scattering from the leads for $W \approx (5-12)r_0$ may also influence the resistance.


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