Phenomenological Gravitational Constraints on Strings and Higher-Dimensional Theories

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We investigate measurable gravitational and cosmological effects in four dimensions that can arise from the compactification of higher-dimensional theories incorporating gravity. We identify the nature of effects due to massless scalar components of the compactified higher-dimensional metric and due to modifications of cosmological dynamics. Current experimental data impose constraints on the viability of many higher-dimensional theories, including Kaluza-Klein, supergravity, and string theories. The phenomenological problems can be avoided if the components of the metric in the higher dimensions acquire an effective mass. We survey some possible mechanisms for mass generation.

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During the past decade, the classical experimental constraints on gravitation theories have been significantly refined. The precise determination of the Eddington-Robertson parameters[1] β and γ is particularly noteworthy. These parameters arise in the expansion

$$ds^2 = -[1-2MG_N r^{-1} + 2\beta(MG_N r^{-1})^2 + \cdots]dt^2 + (1+2\gamma MG_N r^{-1} + \cdots)(dr^2 + r^2 d\Omega^2) \quad (1)$$

of the metric generated by a static, spherically symmetric body of mass M. Their current experimental values are $\beta = 1.003 \pm 0.005$ (Ref. 2) and $\gamma = 1.000 \pm 0.002$.[3] The values of cosmological parameters have also been improved, yielding the Hubble parameter[4] $H_0 = 55$–85 km s^{-1}Mpc^{-1}, the age of the Universe $t_0 = (1.5$–$1.9)\times 10^{10}$ yr, and an upper bound[5] on the variation with time of the Newton constant G_N, $|\dot{G}_N/G_N| < 6\times 10^{-12}\ \text{yr}^{-1}$.

In this paper, we consider the constraints placed on higher-dimensional theories by these experimental data. The notion that the Universe might have more than four dimensions has played an essential part in modern approaches to unification of the fundamental forces including gravity, such as Kaluza-Klein, supergravity, or string theory. Compactified higher-dimensional theories can induce measurable gravitational and cosmological effects in spacetime. Gravitational effects arise from the long-range propagation of massless modes of scalar components g_{jk}, $j,k \geq 4$, of the metric associated with the higher dimensions.[6] These modes are perturbatively generic in many compactification schemes[7] and represent massless particles that, like the graviton, cannot be directly detected at present. In this paper, we investigate the observational effects of such massless modes and show that their effects are subtle: They fail to modify Newtonian gravity but, nonetheless, induce measurable deviations from perturbative Einstein gravity. We discuss some possibilities for overcoming the phenomenological problems they generate. Observable cosmological effects are of two principal types: changes in cosmological parameters induced by the different dynamical evolution of the macroscopic scale parameter, and variations in the Newtonian gravitational constant induced by the evolution of the higher dimensions. We show that present measurements are sufficiently precise to exclude many models.

Consider a generic theory in D dimensions incorporating the Einstein-Hilbert action. The Lagrangian of the theory, assumed to have $O(D-1,1)$ invariance, has the form

$$\mathcal{L}_D = (16\pi G_d)^{-1}\sqrt{-g}R + \mathcal{L}_{R^k} + \mathcal{L}_{\text{matter}}, \quad (2)$$

where $\mathcal{L}_{R^k}$ contains higher powers of R. In a string theory, Eq. (2) might represent an effective Lagrangian; then, for example, the part of $\mathcal{L}_{R^k}$ involving quadratic terms in R forms[8] the Gauss-Bonnet combination while the terms in $\mathcal{L}_{\text{matter}}$ containing the massless vector A_μ form[9] the Born-Infeld action.

Since we observe only four macroscopic dimensions, the D-dimensional Lorentz symmetry must be broken. For the moment, assume that this occurs via some mechanism leaving the extra $n = D-4$ dimensions compactified and small, with volume V_n.

Given a localized matter distribution of total mass M, we wish to examine the behavior of the effective four-dimensional gravitational potential in the perturbative regime at large radial distance r. The leading large-r behavior is affected neither by the higher-R terms nor by the detailed nature of the mass distribution. We therefore approximate the matter distribution by a δ-function distribution at the origin.

It can be shown[10] that the resulting gravitational potential falls as r^{-1} for large r, independently of D. Whether the mass distribution is pointlike or extended in the extra dimensions, the long-range dependence on r of the gravitational potential is that of Newtonian gravity and is compatible with experiment.

However, Einsteinian gravity involves measurable corrections to Newtonian gravity. The perturbative solu-

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tion of Eq. (2) yields

$$h_{00}=\frac{4MG_D(n+1)}{V_n(n+2)r},\quad h_{jj}-\frac{4MG_D}{V_n(n+2)r}, \tag{3}$$

where $j=1,\ldots,D-1$ and $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$. The experimental definition of MG_N fixes the coefficient of the first r^{-1} term in Eq. (1) and gives $G_N=2(n+1)G_D/[(n+2)V_n]$. The resulting value for the parameter γ in Eq. (1) is $(n+1)^{-1}$. This is compatible with experiment only for $n=0$, i.e., $D=4$.

Thus, the assumptions made lead to the conclusion that higher dimensions are excluded by established experimental data. Note that the conclusion arises at the classical level. As such, it is independent of other difficulties with higher dimensions such as natural chirality generation[11] and renormalizability or finiteness.

Next, we investigate phenomenological constraints on the presence of higher dimensions arising from the observed cosmological evolution of the Universe. Consider cosmologies in a pure D-dimensional Einstein theory that involve two homogeneous spaces: a four-dimensional one governed by a scale parameter $a(t)$, and an n-dimensional one governed by an independent scale parameter $b(t)$. Approximate the cosmological matter distribution as a perfect fluid of D-dimensional density $\hat{\rho}$ and pressure $\hat{p}$. For the physical situation where $b \ll a$, the stress tensor is given to a good approximation[10] by the diagonal $D\times D$ matrix with entries $(\hat{\rho},\hat{p},\hat{p},\hat{p},0,\ldots,0)$. The Einstein equations then reduce to a set of three coupled second-order differential equations for a and b, with coefficients dependent on D.

A striking feature of the D-dimensional theory for $D>4$ is that the evolution of the Universe is governed by equations that are qualitatively different from the usual four-dimensional case. With $D=5$ and a vanishing cosmological constant, for example, a satisfies the equation

$$\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2=-\frac{k}{a^2}. \tag{4}$$

Contrary to the usual situation, this equation determines completely the evolution of the scale factor a, independently of the four-dimensional density ρ. Although this situation is somewhat special to the case $D=5$ in that b does not appear in Eq. (4), it remains true that for $D>4$ the cosmological equations are qualitatively different from the four-dimensional case.

Despite the different behavior one can still find experimentally acceptable solutions for a provided fine tuning of parameters is allowed.[10] This reflects in part our lack of experimental knowledge of the history of the Universe. It is possible to find models with b shrinking and a growing[12] and with cosmological properties matching current experimental constraints. An example is a model with $k=-1$ and $D=5$, for which $H_0t_0\approx 1$.

The resulting phenomenological constraints on higher dimensions are basically of two types. First, the age of the Universe is too low for certain models, such as the five-dimensional one with $k=+1$.

Second, the natural dynamical evolution of b means that G_N varies with time.[13] We find its value in the current epoch for the five-dimensional model is given by

$$\left.\frac{1}{H}\frac{\dot{G}_N}{G_N}\right|_0=1-\frac{3\Omega}{4}+\frac{k}{a_0^2H_0^2}, \tag{5}$$

where $\Omega=8\pi G_N\rho_0/3H_0^2$ and where k determines the geometry of the four-dimensional space. For the D-dimensional case we find instead

$$\left.\frac{1}{H}\frac{\dot{G}_N}{G_N}\right|_0=\frac{3n}{n-1}\pm\left(\frac{3n(n+2)}{(n-1)^2}+\frac{3n(n+2)\Omega}{(n-1)(n+1)}-\frac{6n}{(n-1)}\frac{k}{a_0^2H_0^2}-n^2\frac{k_n}{b_0^2H_0^2}\right)^{1/2}, \tag{6}$$

where $n>1$ and where k_n determines the geometry of the n-dimensional space. In some models, the variation of G_N with time exceeds the current experimental upper limit. For example, the six-dimensional model with $k=k_n=0$ and $\Omega=3$ is excluded.

It is also worth noting that the existence of higher dimensions seems to be a problem for the inflationary scenario.[14] Typically, if a inflates, so does b. For example, in the case $D=5$ with inflation driven by a cosmological constant $\Lambda>0$, both a and b are proportional to $\exp[(\Lambda/6)^{1/2}t]$. The extra dimensions would then be of cosmological size.

The phenomenological difficulties we have discussed can be avoided if fine tuning is permitted and if large-distance effects associated with massless scalar modes from the higher-dimensional components of $h_{\mu\nu}$ fail to arise. One reason for this could be the generation of an effective mass for these components. Their propagation would then be damped at large distances, so that, for example, the observed value of γ would be 1.

In particle gauge theories, a mass term at the Lagrangian level for any gauge field is forbidden by gauge invariance. Instead, gauge-field masses are generated via the Higgs mechanism. Now, it is possible to view gravity as analogous to a non-Abelian gauge theory with the connection and the Lorentz group playing the roles of the gauge field and the gauge group, respectively.[15] It is therefore natural to investigate whether a gravitational version of the Higgs effect could generate masses for certain components of $h_{\mu\nu}$.

This suggestion is particularly pertinent as spontaneous Lorentz-symmetry breaking may occur naturally in string theory.[16] String field theory[17] contains cubic interaction terms of the form ST^MT_M, where S is a generic Lorentz-scalar field and T^M is a generic Lorentz-tensor field with M representing one or more Lorentz vector indices. Cubic couplings of this type are absent in renormalizable particle theories. If any scalars S acquire finite expectation values of the appropriate sign and magnitude, some Lorentz tensors T^M acquire mass-squared terms of the wrong sign. This results in spontaneous breaking of the Lorentz symmetry. In the bosonic string, for example, a candidate scalar that may acquire an appropriate expectation value is the tachyon field. Its presence signals an instability in the naive perturbation vacuum and therefore may even be desirable, given the phenomenological need to break the $O(3+n,1)$ Lorentz symmetry and the large gauge groups often present in strings.

Let us explore the possibility of generating an effective mass for $h_{\mu\nu}$ via spontaneous Lorentz-symmetry breaking. We treat a model that mimics the string case. For simplicity, suppose that the only Lorentz tensor that gets a wrong-sign squared mass is a vector A_μ. An effective action can be constructed by integrating over all fields except A_μ and $g_{\mu\nu}$. Neglecting higher-derivative effects and setting the cosmological constant to zero, this action is described by the Einstein-Maxwell action in D dimensions together with a potential $V=V(A_\mu A^\mu - a^2)$ for A_μ causing the spontaneous breaking of Lorentz symmetry:

$$\mathcal{L}=\sqrt{-g}\,[(16\pi G_D)^{-1}R-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}V]\,. \qquad (7)$$

Here, $V(x)$ is taken positive except at $x=0$, where it vanishes.

The potential V is minimized when A_μ has a constant expectation value satisfying $A_\mu g^{\mu\nu} A_\nu = a^2$. After shifting A_μ by $a\delta_{\mu D-1}$, an analysis reveals that no component of $h_{\mu\nu}$ acquires a mass. There is no version of the Higgs effect in gravity because the graviton couples to tensor fields via derivative interactions.

The value of γ is affected despite the absence of a Higgs mechanism. With $A_\mu = a\delta_{\mu D-1}$, we find the solution to the Einstein equations is

$$h_{00}=\frac{4MG_D(l-1)}{lr}\,,\quad h_{jj}=\frac{4MG_D}{lr}\,,\qquad h_{D-1\,D-1}=\frac{4MG_D}{l(k+1)r}\,, \qquad (8)$$

where

$$k=8\pi G_D a^2,\quad l=(1+k)^{-1}[(n+2)+(n+1)k]\,. \qquad (9)$$

This gives

$$\gamma=\frac{1+k}{1+n+nk}\,, \qquad (10)$$

where $n\geq 1$. There is still a phenomenological problem unless $D=5$ and $k\to\infty$. The latter is an unnatural choice, since a value of a of order of the Planck scale and hence a value of k of order 1 is to be expected in the context of a quantum theory of gravity such as string theory.

Is there some other mechanism for preventing the long-range propagation of $h_{\mu\nu}$ modes? The analogy with non-Abelian gauge theories again suggests a possibility. At large distances, these theories are believed to be strongly coupled and confining, which leads to bound states. For example, quantum chromodynamics is a non-Abelian gauge theory in which perturbatively there are massless gauge vectors, the gluons, yet long-range forces are absent. Because of confinement a single gluon cannot be isolated at macroscopic scales. Effectively, the mass of a gluon increases as it moves away from a bound state, which prohibits its long-range propagation.

In a higher-dimensional gravity theory, a mechanism analogous to confinement involving the extra components of $g_{\mu\nu}$ might avoid the phenomenological difficulties discussed above. These components must then be strongly coupled at large distances. Strong coupling is a possibility here because gravity is a highly nonlinear theory. Note that string theory, which has higher-R interactions and an infinite number of particle fields, is even more nonlinear. Note also in this context that arguments based on the effective dilaton potential[18] indicate a need for strong coupling in string theories.

Given this idea, it is then necessary to understand why higher-dimensional gravity is strongly coupled while four-dimensional gravity is not. As the gravitational force is nondirectional, it is also necessary to understand why the extra components remain strongly coupled while the usual metric is perturbative. These asymmetric requirements are difficult to implement in most higher-dimensional theories.

Theories in which Lorentz-symmetry breaking may occur naturally could avoid this problem. String theories fall into this class. The idea is that, even without directly generating masses, Lorentz-symmetry breaking may be responsible for the needed asymmetry. The situation might be compared to that of a non-Abelian gauge group G of a grand unified theory, for example, being broken to one or more U(1) groups together with a non-Abelian subgroup H. The U(1) gauge bosons would then be the analogs of the four-dimensional metric components, whereas the gauge bosons of H would be the analogs of the extra degrees of freedom. The former are massless and result in long-range interactions; the latter are confining and macroscopically nonpropagating. The explicit implementation of this analogy is an interesting and open problem.

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