Collapse of a "Global Monopole"

Barriola and Vilenkin[1] (BV) have discussed the intriguing behavior of the gravitational field surrounding the center of a "global monopole," an object which exhibits a spherically symmetry energy density falling as $1/r^2$, where r is the distance from the center. They find that there is no appreciable force on a nonrelativistic particle in this field, yet passing light beams are focused so that the image of a point on one side of the monopole is seen by an observer on the other side as a ring. The purpose of the present Comment is to study the dynamics of a cylindrically symmetric distortion mode which can reduce the static energy monotonically to zero as it "unties the knot" which defines the monopole.

A simple change of variable $y = \ln\tan(\bar{\theta}/2)$ transforms the energy density of the BV spherically symmetry *Ansatz* on the surface of a sphere of radius r to

$$\rho = [\sin^2\bar{\theta} + (\partial\bar{\theta}/\partial y)^2]\phi^2/2 + (\partial\phi/\partial y)^2/2 + [\lambda(\phi^2 - \eta^2)^2/2 + r^2(\partial\bar{\theta}/\partial t)^2]/2\cosh^2 y\,,$$

where ϕ is the magnitude of the isovector field $\boldsymbol{\phi}$, η is the value of ϕ which minimizes the familiar quartic potential, and $\bar{\theta}$ is the polar angle giving the orientation of $\boldsymbol{\phi}$. Note that if ϕ is held fixed and equal to η, then the expression is just that which makes the energy minimizing y dependence of $\bar{\theta}$ identical to that of a sine-Gordon soliton (which for BV is centered on $y = 0$), with perfect translation invariance in y. While the sine-Gordon character is quite pretty, only the translation invariance is important in what follows.

The above energy expression exhibits an angular instability, since translating the soliton to large $-y$ and sending ϕ slowly to zero in that direction can make the static energy arbitrarily small. Thus, one could construct an isolated monopole of finite radius R by letting the $\boldsymbol{\phi}$ field rotate away from spherical symmetry and disappear into the North Pole as r increases from 0 to R, yielding a north-pointing teardrop shape for the monopole mass density. This object has a scale instability: It can shrink to smaller radius, as not only the static mass but also the kinetic energy associated with the time derivative of the scale is proportional to R, so that once such an implosion has begun it will accelerate until radiative dissipation sets in. This example illustrates vividly that "topological stability" depends critically on the precise choice of energy functional, and not on continuity alone.[2]

One is led to conclude that something other than a global monopole should be sought as a matter source for the BV gravitational field. A further question which one would like to understand is the difference between general relativity, where there is negligible force on any particle moving slowly through the monopole,[1] and Newtonian gravity, which implies a central attractive force falling inversely with radius.

I first learned about the "soliton-in-the-soliton" some time ago through discussions with George Chapline about SU(5) monopoles in spontaneously broken QCD. Andrew Jackson independently came across this phenomenon while considering special Skyrme field configurations. This work, supported in part by the Department of Energy and in part by the National Science Foundation, was carried out with the hospitality of the Theoretical Division of Los Alamos National Laboratory.

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