Boundary Scattering in Quantum Wires

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(Received 25 August 1989)

Narrow quantum wires of sufficiently high mobility exhibit an anomalous, low-field magnetoresistance peak at a position scaling as the ratio of the Fermi wave vector to the wire width. We investigate this effect varying temperature, electron density, sample geometry, and method of lateral confinement. The data we obtain are consistent with a classical explanation based upon a diffuse component of electron scattering at the edges of the wire.

PACS numbers: 73.50.Jt, 73.40.Kp, 73.50.Bk

Imposing additional confinement upon a two-dimensional electron-gas (2DEG) creates highly conducting one- (1D) and zero-dimensional (0D) devices commonly called "quantum wires"[1-4] and "quantum dots."[5] If the potentials are not smooth, however, the resulting electron eigenstates will have short lifetimes and confinement effects may be obscured. In a 2DEG, for example, interface scattering[6,7] can seriously limit the in-plane electron mobility. In this Letter, we consider the analogous problem of electron scattering from the lateral boundaries of quasi-1D wires.[2]

Recent experiments have shown that boundary scattering in microfabricated structures is predominantly specular;[2,8] i.e., the probability of specular scattering, p,[9] is close to unity. This implies that an electron's longitudinal momentum is conserved even after many collisions with the edges. In narrow 2DEG conductors, however, the zero-field sheet resistance is generally seen to markedly increase as the conducting path width decreases and a characteristic feature of transport in such devices is a *negative* magnetoresistance.[10-13] Recently Roukes *et al.*[4] reported a *positive* zero-field magnetoresistance in narrow high-mobility wires increasing to an anomalous low-field ($B < 0.5$ T) maximum at a field position, B_{max}, scaling approximately inversely with the wire width, W. Similar features are evident in some of the data of Timp *et al.*[3] and Ford *et al.*[14] Here, we describe a possible origin of this unusual anomaly which demonstrates that a small, but significant, amount of the boundary scattering is diffuse (i.e., $p < 1$).

The wires used in this work were defined either by low-energy ion exposure[15] or confinement between split gates.[1] In the former case, areas of a high-mobility 2DEG are protected from the deleterious effects of an ion beam by a narrow mask deposited on the surface. The mask can either be an insulating material[15] or a metal which acts as a self-aligned Schottky-barrier gate so that the carrier density, n, can be varied while keeping a constant conducting width.[16] In either case the ion dose can be optimized so that the electrical width of the wire closely resembles the width of the mask.[17] For split-gate confinement a narrow conducting channel is established in the gap between a pair of reverse-biased gate electrodes deposited on the surface of the heterojunction and in this case increasing the reverse bias to the gate reduces the wire width.

In Fig. 1 we show the high-field magnetoresistance of ion exposed wires. The positive zero-field magnetoresistance reaches a maximum with an amplitude and field position that increase as the width of the wire is reduced. A similar feature also appears in split-gate wires as the width of the wire is reduced (inset Fig. 1). In this case the resistance maximum at ~0.2 T is only 5%–10% of the background and is simultaneously present with a small peak at $B = 0$. We argue below that the anomalous resistance maximum at B_{max} is due to electrons which scatter diffusively from the wire edges before they can make an elastic collision within the bulk of the wire.

Size effects[18] in thin metal films have been observed when a magnetic field is directed parallel to the plane of the film and perpendicular to the current flow.[19] These systems resemble narrow wires in a perpendicular field in that both have elastic lengths larger than the smallest dimensions so that the electrons interact with the walls before suffering internal collisions. A model based on clas-

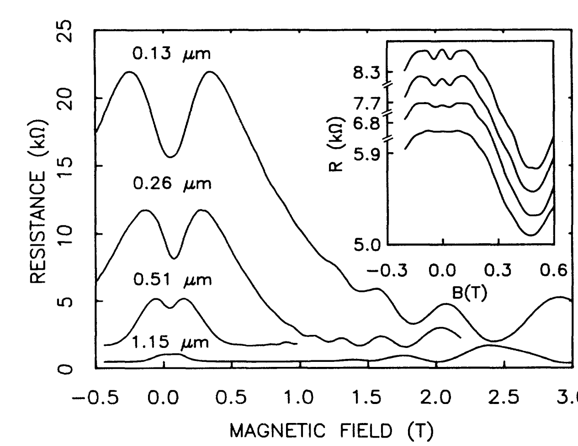

FIG. 1. The magnetoresistance of 12-μm-long ion-exposed wires of differing widths at 4.2 K. Inset: The low-field data from a 12-μm split-gate wire for gate voltages $V_g = -0.78$, -0.83, -0.86, and -0.89 V.

sical electron trajectories[20] was able to reproduce the positive magnetoresistance observed in aluminum[19] assuming completely diffuse scattering ($p=0$) at the surfaces of the film. This work identified two field regimes in which the boundary scattering did not contribute significantly to the resistivity. At low fields the trajectories are little different from those at $B=0$ and the conductivity is dominated by the electrons with large longitudinal momentum which scatter internally before colliding with the edges. At higher fields when the cyclotron orbit is smaller than the wire width an electron leaving one edge can only reach the opposite edge by making an internal collision and once again the resistivity is dominated by scattering within the wire.

For our purposes the interesting case occurs at intermediate fields when a large number of trajectories acquire sufficient curvature that they are forced to interact with the walls before making an internal collision. For an arbitrarily small amount of diffuse scattering the resistance increases until it reaches a maximum at a field B_{max} which scales with the ratio of the cyclotron length, L_c, to the wire width as $W/L_c = WeB_{max}/\hbar k_F = 0.55$,[20,21] where $k_F = k_F(2D) = (2\pi n)^{1/2}$. The resistance maxima in our data scales in agreement with this expression as shown in Fig. 2. In most cases the data have been obtained from wires patterned by ion exposure using the optimum dose so that the actual conducting width of the wire is the nominal mask width.[17] For the narrow wires the electrical width could be estimated by considering the magnetic depopulation of 1D subbands.[22] The best fit to the data suggests that $W/L_c = 0.55 \pm 0.05$ in agreement with Ref. 20 to a degree which at first seems surprising for such a simple classical model.[23] However, recent calculations using classical electron trajectories have been able to reproduce many of the magnetoresistance effects observed in ballistic, multiprobe conductors.[8,24] This gives us some confidence here in the use of this approach to describe magnetosize effects in wires where several 1D subbands are occupied. The explicit $\sqrt{n}$ dependence is shown in the inset to Fig. 2 for gated wires of different widths. For wires with a large resistance maximum (i.e., greater than 30% of the $B=0$ resistance) the field position B_{max} scales nicely with $\sqrt{n}$, the solid lines being the expected fit to $B_{max} = 0.55\hbar k_F/We$.

At low carrier concentrations when the peak at B_{max} is not so well developed the behavior is qualitatively different. As n decreases the mobility drops rapidly[16,25] and there is a large increase in the zero-field resistance. In this regime the transport mean free path is decreasing so electrons make fewer collisions with the walls before scattering internally. Ultimately the boundary scattering contributes only a small part to the total resistivity and the relative amplitude of the peak at B_{max} decreases until it is overwhelmed by the zero-field resistance. In Fig. 3 we have scaled the resistivity to its value at $B=0$ to show explicitly how the normalized resistance of the peak at B_{max}, $\mathcal{R} = R(B_{max})/R(0)$, decays with decreasing n for a

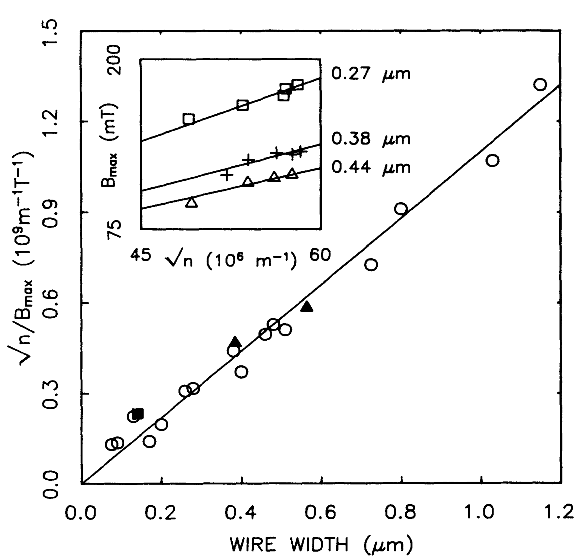

FIG. 2. The value of $\sqrt{n}/B_{max}$ plotted against wire width. The solid symbols are taken from Refs. 3 (square) and 14 (circles). The solid line is derived from the condition for a maximum in the resistance $W/L_c = 0.55$. Inset: The explicit $\sqrt{n}$ dependence for three wires with self-aligned gates.

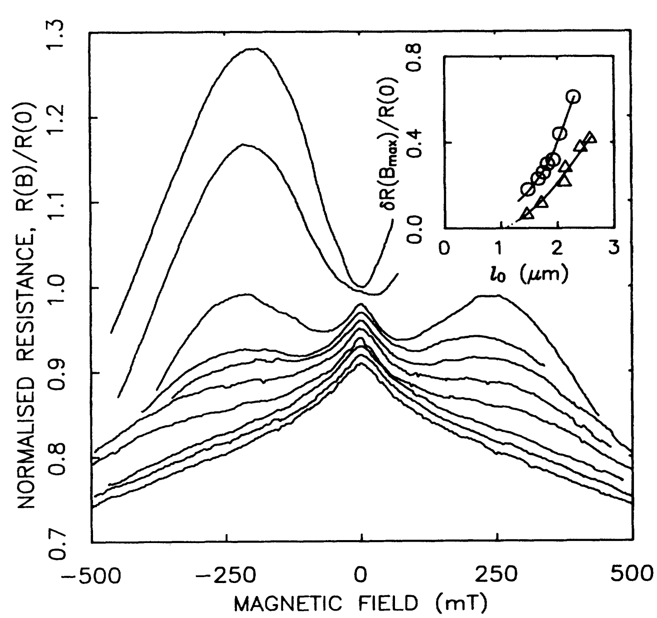

FIG. 3. The normalized resistivity, $R(B)/R(0)$, for a 0.27-μm-wide wire showing the decay of the anomaly with decreasing carrier concentration. n varies from 4.5×10^{15} (top curve) to 2.2×10^{15} m^{-2} (bottom curve). Each curve has been offset downwards from the one above by 1%. Inset: The amplitude of the maximum is plotted against the transport mean free path, l_0, for 0.27- (triangles) and 0.1-μm (circles) wires.

wire of width 0.27 μm. At some intermediate values of n the minimum at $B=0$ is replaced by a small maximum reminiscent of the data from split-gate devices (see inset of Fig. 1). For still lower carrier concentrations the peak at $B_{\max}$ all but disappears and for a wide range of gate voltage the normalized resistance lies on an approximately universal curve independent of n. We subtract the normalized background resistance from the data and plot the excess amplitude at $B_{\max}$ against the transport mean free path, $l_0 = hL/e^2 k_F RW$, where R is the resistance at $B=0$. The peak at $B_{\max}$ is seen to vanish at an extrapolated value of ~ 1.1 μm (inset, Fig. 3) which we interpret as the distance an electron travels down the wire before the total probability of it scattering diffusively from the edges is close to unity. This length we designate as l_b. Although significantly smaller than the 8-μm transport mean free path deduced from the mobility of a wide sample of the same material, l_b is not necessarily the dominant scattering length at $B=0$. At zero field the distribution of electrons is weighted in favor of those with large longitudinal momentum[24] and for these states the transport mean free path deduced from the wide sample is probably the more relevant length scale.

The value of l_b will be larger for wider wires where there is less interaction with the edges. However, the probability for a diffuse scattering event at each collision with the boundaries, $1-p$, is a property of the boundaries themselves and is expected to be the same for wires defined under the same conditions. For a wire of width W an electron will, on average, make approximately l_b/W collisions with the walls before traveling a longitudinal distance l_b. If it then scatters diffusely at the boundary, the coefficient of specular scattering is given by $1-p=W/l_b$. For the wire considered above ($W=0.27$ μm) this expression gives $p=0.75$. We expect l_b to scale with the wire width and a similar analysis for a 0.1-μm wire (see inset of Fig. 3) gives $l_b \sim 0.5$ μm corresponding to $p=0.8$.

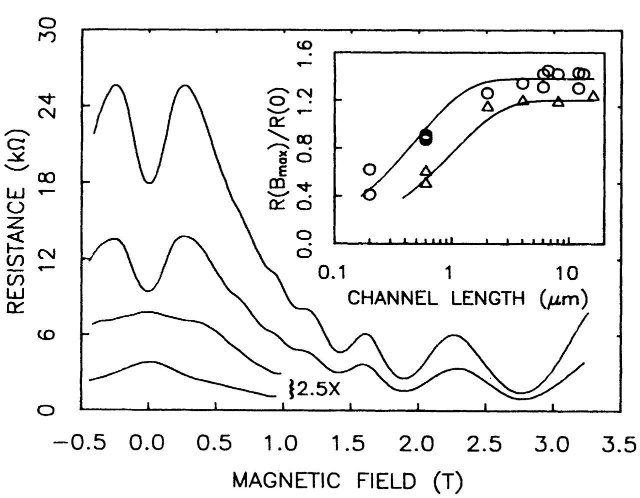

FIG. 4. The magnetoresistance of 0.13-μm wires with channel lengths of 0.2, 0.6, 8, and 16 μm. Inset: The decay of the normalized amplitude is plotted for two sets of wires of nominal width 0.13 (circles) and 0.30 μm (triangles).

The normalized peak resistance $\mathcal{R}$ is expected to be strongly length dependent for wires shorter than l_b. In Fig. 4 the magnetoresistance of nominally identical wires is shown for a number of different lengths. Each of the wires had values of W and n in the range 0.13 ± 0.03 μm and $(2.5 \pm 0.3) \times 10^{15}$ m^{-2}, respectively, and were defined by exposure to the same optimal dose of Ne ions so that the sidewall roughness is expected to be similar in each case. For long wires the value of $\mathcal{R}$ is approximately independent of length but drops rapidly for $L < 2$ μm. The length dependence of $\mathcal{R}$ is plotted in the inset and fitted to a function of the form $\mathcal{R} = A[1-\exp(-L/l_b)]$. The deduced value of l_b is ~ 0.5 μm so that $p=0.74$ similar to the estimates obtained above. To illustrate the trend as the width of the wire is increased the inset of Fig. 4 shows similar behavior for wires of width $W=0.3 \pm 0.02$ μm. Here the larger value of $l_b \sim 1$ μm ($p=0.73$) is consistent with the increased width.

The value of $\mathcal{R}$ decreases with increasing temperature (because of the reduction in l_0) and for wires of width 0.13 and 0.51 μm reaches a value of unity at temperature of 80 and 35 K, respectively. For a wide sample of the same material we have measured the elastic length to be approximately 1.2 and 5 μm at these temperatures corresponding to values of p of 0.89 and 0.9, respectively, somewhat larger than the previous estimates.

For nominally identical split-gate and ion-exposed wire the very much smaller value of $R(B_{\max})/R(0)$ for the split-gate devices (see inset of Fig. 1) suggests a qualitative difference in the nature of the scattering at electrostatic and ion-exposed boundaries and it is therefore of interest to compare the value of p for the two cases. Squeezing the split-gate wires leads to a small drop in n and the corresponding reduction in mobility, along with the reduction in width, produces a large increase in the zero-field resistance (inset, Fig. 1). The peak at $B_{\max}$ therefore eventually decays with decreasing l_0 in a similar fashion to ion-exposed wires (Fig. 3). Again we take the transport mean free path at the point at which the peak at $B_{\max}$ is vanishingly small as an upper bound to l_b. For the device considered in the inset to Fig. 1 this occurs when the wire width is ~ 0.1 μm and $l_0 \sim 1.85$ μm giving $p=0.95$. This suggests that the probability for diffuse scattering from an electrostatically defined boundary is smaller than from one defined by ion exposure but we stress that our deduced values of p are only rough estimates.

The important role of boundary scattering in high-mobility wires has been indirectly inferred in other experiments. van Houten *et al.*[2] modeled the low-field negative magnetoresistance observed in etched wires using localization theory[10] modified to include scattering at the edges. Their data were best fitted assuming predominantly *specular* boundary scattering. Molenkamp *et al.*[26] have recently conjectured that a *diffuse* component of scattering at the edges might act to suppress the negative resistance phenomenon first observed by Takagaki *et al.*[27] at a narrow wire junction, especially in samples where current and voltage probes are spatially separated. Scherer and Van der Gaag[28] have directly measured the spatial decay of this "transfer" resistance and find that it is measurable at distances $>6\ \mu m$ with an exponential decay length exceeding $1\ \mu m$ in close agreement with the boundary scattering lengths deduced here.

Recent classical trajectory calculations[24] and quantum-mechanical simulations[29] of electron scattering at junctions of narrow wires have shown distinct peaks in the low-B magnetoresistance. We have ascertained that the phenomenon reported here, however, is an *intrinsic* property of our wires since it scales linearly with the length of the wire segment between voltage probes and can be seen in junctionless wires formed by a long-wire segment connecting broad 2DEG regions.

In summary, we have investigated the anomalous low-field magnetoresistance peak seen in high-mobility quantum wires. We explain the data using a classical model of partially diffuse scattering at the wire edges for two important types of microfabricated boundaries we extract a probability of specular boundary scattering p close to unity.

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