# Quantum Magnetotransport of a Periodically Modulated Two-Dimensional Electron Gas

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A quantum-mechanical theory is developed for the recently discovered magnetoresistance oscillations in a periodically and weakly modulated two-dimensional electron gas. The bandwidth of the modulation-broadened Landau levels at the Fermi energy oscillates with magnetic field and gives rise to magnetoresistance oscillations parallel ($\rho_{xx}$) and perpendicular ($\rho_{yy}$) to the modulation. *Diffusion*-current contributions, proportional to the square of the bandwidth, dominate $\rho_{xx}$; *collisional* ones, which are large for small bandwidths, dominate $\rho_{yy}$. $\rho_{yy}$ and $\rho_{xx}$ oscillate *out of phase* as observed. New oscillations in the Hall resistance, the cyclotron resonance position, and the linewidth are predicted.

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Over the last two decades[1] the transport properties of a two-dimensional electron gas (2DEG) have been studied intensively by many research groups. Because of the singular nature of the density of states (DOS) of a 2DEG in a magnetic field well defined oscillations are found in the thermodynamic quantities, such as specific heat, magnetization, magnetocapacitance, etc., and in the resistance, such as Shubnikov–de Haas (SdH) oscillations, magnetophonon oscillations, etc., but also novel phenomena such as the quantum Hall effect (QHE) and the fractional quantum Hall effect (FQHE) have been observed. Recently[2] a weak 1D modulation (taken along the $x$ direction) of a high-mobility 2DEG has been realized which leads to novel oscillations in the magnetoresistance. These oscillations are connected to the commensurability between the modulation period $a$ and the diameter of the cyclotron orbit $2R_c=2(2\pi n_e)^{1/2}l^2$ at the Fermi energy, with $l=(\hbar/eB)^{1/2}$ the magnetic length and $n_e$ the electron density. These oscillations[2-4] have the following features: (1) They are periodic in $1/B$ like the SdH oscillations. (2) The periodicity depends on the electron density like $\sqrt{n_e}$ while the SdH have a $n_e$ dependence. (3) The amplitude of these oscillations has almost no temperature dependence in contrast with that of the SdH oscillations. (4) They show up most clearly at small magnetic fields, because at higher fields they are obscured by the SdH oscillations. (5) Weiss *et al.*[2] also found oscillations in $\rho_{yy}$ which are much weaker in amplitude and are out of phase with the oscillations in $\rho_{xx}$. (6) Also, modulations in the magnetocapacitance oscillations were observed.[5]

Different theoretical models have been given which are able to explain the oscillations in $\rho_{xx}$. Gerhardts, Weiss, and von Klitzing[3] presented a quantum-mechanical calculation based on a Kubo-type formula. Theoretically no noticeable oscillations in $\rho_{yy}$ and $R_H$ were obtained. Winkler, Kotthaus, and Ploog[4] calculated the diffusive contribution to $\rho_{xx}$ in the high-temperature and classical (large Landau-level index) limit. This approach leads to a simple expression for the oscillations which agrees very well with the experimental results in the very small magnetic field limit, but for higher magnetic fields the theoretical result did not recover the SdH oscillations. Beenakker[6] presented an alternative explanation for the oscillations in $\rho_{xx}$ on the basis of a classical picture, in which a resonance between the periodic cyclotron orbit motion and the induced (by the periodic potential) oscillatory motion of the center of the orbit leads to oscillations in $\rho_{xx}$. Because the theory is classical the transition to SdH oscillations in $\rho_{xx}$ for larger magnetic field is not obtained. No oscillations are found in the other components of the resistivity tensor indicating that the weak oscillations in $\rho_{yy}$ have a purely quantum-mechanical origin. At present no explanation is available for the antiphase oscillations in $\rho_{yy}$.

In the Letter we demonstrate that a *quantum* Boltzmann equation,[7] derived in the framework of Kubo's linear-response formalism, accounts well for all the observations mentioned above. The antiphase oscillations in $\rho_{yy}$ are explained and *new* oscillations in the Hal resistance and the cyclotron resonance position and linewidth are predicted.

We consider a two-dimensional electron gas, in the $(x,y)$ plane, in the presence of a magnetic field $\mathbf{B}$ along the $z$ axis, and periodically modulated in the $x$ direction by the potential $U(x)=V_0\cos(Kx)$, with $K=2\pi/a$, $a$ being the modulation period. To evaluate the resistivity tensor $\rho_{\mu\nu}$ $(\mu,\nu=x,y)$ we will use the components $\sigma_{\mu\nu}$ of the conductivity tensor in the standard expression: $\rho_{xx}=\sigma_{yy}/S$, $\rho_{yy}=\sigma_{xx}/S$, and $\rho_{yx}=-\sigma_{yx}/S$, where $S=\sigma_{xx}\sigma_{yy}-\sigma_{xy}\sigma_{yx}$ with $S\approx\sigma_{xy}^2=n_e^2e^2/B^2$ in the experiments under consideration.

We consider a many-body system described by the Hamiltonian $H=H_0+H_I-\mathbf{R}\cdot\mathbf{F}(t)$, where $H_0$ is the unperturbed part, $H_I$ is a binary-type interaction (e.g., between electrons and impurities or phonons), and

 

$-\mathbf{R}\cdot\mathbf{F}(t)$ is the interaction of the system with the external field $\mathbf{F}(t)$. For conductivity problems the external field is given by $\mathbf{F}(t)=-e\mathbf{E}(t)$, where $\mathbf{E}(t)$ is the electric field, $\mathbf{R}=\sum_i \mathbf{r}_i$, $-e$ is the electron charge, and $r_i$ is the position operator of electron $i$. In the representation in which $H_0$ is diagonal the density operator $\rho=\rho^{\mathrm{d}}+\rho^{\mathrm{nd}}$ has a diagonal part $\rho^{\mathrm{d}}$ and a nondiagonal part $\rho^{\mathrm{nd}}$. For weak electric fields (i.e., for linear responses) and weak scattering potentials the diagonal part of the current density reads[7]

$$\langle J_\mu^d\rangle=\mathrm{Tr}\{\rho^{\mathrm{d}}J_\mu\}=\frac{|e|}{\Omega}\sum_\zeta[-\mathcal{B}_\zeta\langle n_\zeta\rangle_t\alpha_\mu^\zeta+\langle n_\zeta\rangle_t v_\mu^\zeta]\,, \quad (1)$$

where $\Omega$ is the volume of the system, $\alpha_\mu^\zeta=\langle\zeta|r_\mu|\zeta\rangle$, $v_\mu^\zeta=\dot{\alpha}_\mu^\zeta=\langle\zeta|\dot{r}_\mu|\zeta\rangle$, with $|\zeta\rangle$ being the one-particle eigenstate of $H_0$ with eigenvalue $E_\zeta$, and $\langle n_\zeta\rangle_t$ is the average occupancy of the state $|\zeta\rangle$. $\mathcal{B}_\zeta\langle n_\zeta\rangle_t$ is the collision integral of the *quantum* Boltzmann equation for scattering between different[7] or like particles.[8] The second term of Eq. (1) is the usual *diffusive* current; the first term of Eq. (1), absent in semiclassical treatments,[6] represents *collisional* contributions to the current and is called "collisional" current. The latter is only current for transport through localized states,[7-9] e.g., Landau states, or for hopping conduction in disordered materials. In such a situation the diffusion contribution vanishes identically.

If only diffusive current is present, the dc conductivity is given by[7(a)]

$$\sigma_{\mu\nu}^{\mathrm{d}}=\frac{\beta e^2}{\Omega}\sum_\zeta f_\zeta(1-f_\zeta)\tau(E_\zeta)v_\mu^\zeta v_\nu^\zeta\,, \quad (2)$$

provided that the scattering (between different particles) is elastic or quasielastic. Here, $f_\zeta=\langle n_\zeta\rangle_{\mathrm{eq}}$ is the Fermi-Dirac function, $\beta=1/k_BT$, and $\tau(E_\zeta)$ is the relaxation time. If the problem is such that there is only collisional current, the dc conductivity takes the simple form (for $\mu=\nu$)[7(a)]

$$\sigma_{\mu\mu}=\frac{\beta e^2}{2\Omega}\sum_{\zeta\zeta'}f_\zeta(1-f_{\zeta'})W_{\zeta\zeta'}(\alpha_\mu^\zeta-\alpha_\mu^{\zeta'})^2\,, \quad (3)$$

where $W_{\zeta\zeta'}$ is the transition rate between the states $|\zeta\rangle$ and $|\zeta'\rangle$. This is the well-known hopping-type formula for transport in the presence of a magnetic field.[9] Conduction occurs by transitions through spatially separated states from $\alpha_\mu^\zeta=\langle\zeta|r_\mu|\zeta\rangle$ to $\alpha_\mu^{\zeta'}=\langle\zeta'|r_\mu|\zeta'\rangle$.

The nondiagonal part of the current density leads[7] to the dc conductivity

$$\sigma_{\mu\nu}^{\mathrm{nd}}=2\frac{i\hbar}{\Omega}\sum_{\zeta\neq\zeta'}f_\zeta(1-f_{\zeta'})\langle\zeta|v_\mu|\zeta'\rangle \times\langle\zeta'|v_\nu|\zeta\rangle\frac{1-e^{\beta(E_{\zeta'}-E_\zeta)}}{(E_\zeta-E_{\zeta'})^2}\,. \quad (4)$$

The total conductivity is then given by the sum $\sigma_{\mu\nu}=\sigma_{\mu\nu}^{\mathrm{d}}+\sigma_{\mu\nu}^{\mathrm{nd}}$. The above formulas have been successfully applied to many situations:[7,8] hopping conduction and magnetophonon resonances, quantum Hall effect, Aharonov-Bohm effect, etc.

To apply Eqs. (2)-(4) to the present problem we need the eigenfunctions and eigenvalues of the one-electron Hamiltonian

$$H_0=(2m^*)^{-1}(\mathbf{p}-e\mathbf{A})^2+U(x)\,, \quad (5)$$

where $\mathbf{p}$ is the momentum operator, $\mathbf{A}$ is the vector potential, and $U(x)$ is the spatial 1D modulation. In the homogeneous case [i.e., $U(x)=0$] the eigenfunctions in the Landau gauge $\mathbf{A}=(0,Bx,0)$ are given by $\phi_n(x-x_0)\times e^{ik_yy}/\sqrt{L_y}$, where $\phi_n(x-x_0)$ are the well-known harmonic-oscillator functions centered at $x_0=l^2k_y$, $n$ is the Landau-level index, and $L_y$ is the length of the system in the $y$ direction. The energy of the $n$th level $E_n=(n+\frac{1}{2})\hbar\omega_c$ is degenerate with respect to the wave vector $k_y$.

In the presence of the 1D modulation the exact eigenstates are difficult to obtain. In the experimental systems under study the amplitude of the modulation is small and we may evaluate the correction to the energy levels by first-order perturbation theory[3] using the unperturbed wave functions given above. This gives

$$E_{n,k_y}=(n+\tfrac{1}{2})\hbar\omega_c+V_0\cos(Kx_0)e^{-u/2}L_n(u)\,, \quad (6)$$

where $u=K^2l^2/2$ and $L_n(u)$ is Laguerre polynomial.[10] We see that the modulation lifts the $k_y$ degeneracy of

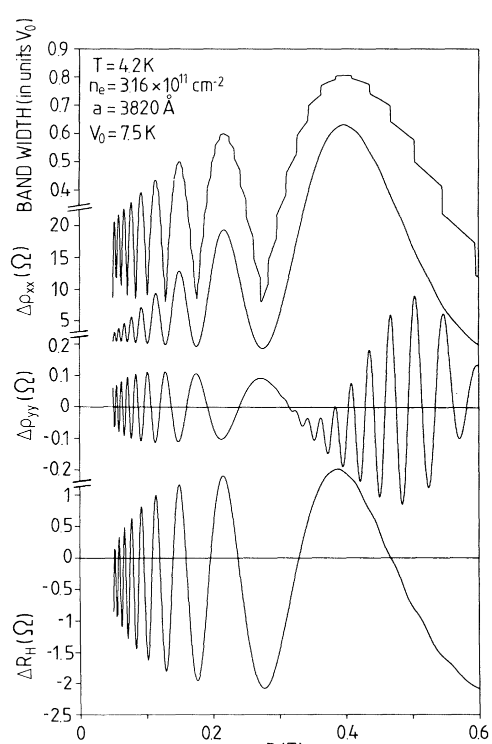


FIG. 1. The bandwidth at the Fermi energy, and the corrections to the magnetoresistances $\rho_{xx}$ and $\rho_{yy}$ and the Hall resistance, as functions of the magnetic field. The 1D modulation of the 2DEG is along the $x$ direction.

the unperturbed Landau levels which are broadened into bands with a bandwidth that oscillates with band index $n$ and magnetic field. What will be important is the bandwidth *at* the Fermi energy which is illustrated in Fig. 1. Its steplike character is due to the fact that the electron is taken to be in a definite Landau level $n=E_F/\hbar\omega_c - 1/2$.

When the modulation is absent the diffusive contribution to the current vanishes identically because $v_x^\zeta$ and $v_y^\zeta$ are zero. The only current contribution left for transport along the electric field $(\mu=\nu)$ is the collisional one, as given by Eq. (3). However, in the presence of the modulation the carriers acquire a mean velocity in the $y$ direction

$$v_y=-\frac{\partial E_{n,k_y}}{\hbar\,\partial k_y}=-\frac{2V_0}{\hbar K}\sin\left(\frac{2k_y}{K}u\right)ue^{-u/2}L_n(u)\,, \tag{7}$$

whereas $v_x^\zeta$ is again zero. Thus $\sigma_{xx}$ have only a collisional contribution while $\sigma_{yy}$ will have two contributions, a collisional and a diffusive. This already implies that the resistivity tensor is asymmetric.

For the evaluation of formulas (2)-(4) we assume that the electrons are scattered elastically by randomly distributed impurities. This is a very good approximation for the experimental temperatures $T<10$ K. To evaluate $\Delta\rho_{xx}$ we have to calculate the correction to the conductivity $\Delta\sigma_{yy}$ due to the modulation which is dominated by a diffusive component. To leading order in $V_0$ we found

$$\Delta\sigma_{yy}=\frac{e^2}{h}\frac{2\pi^2}{\hbar}\tau V_0^2\frac{l^2}{a^2}e^{-u}\times\sum_{n=0}^{\infty}[L_n(u)]^2\left(-\frac{\partial f(E)}{\partial E}\right)_{E=E_n}, \tag{8}$$

with $\tau$ an energy-independent relaxation time which we have approximated by $\tau=\mu m^*/e$, where $\mu$ is the mobility of the 2DEG. It is evident from Eq. (8) that $\Delta\sigma_{yy}$ and thus $\Delta\rho_{xx}$ are proportional to the square of the bandwidth at the Fermi energy. This is a generalization of the result of Ref. 4 to arbitrary temperature which is the reason why Eq. (8) also contains the SdH oscillations. The result of Eq. (8) leads to $\Delta\rho_{xx}$ which is shown in Fig. 1; we took $\mu=1.3\times10^6$ cm$^2$/Vs, $n_e=3.16\times10^{11}$ cm$^{-2}$, and a 1D modulation with period $a=3820$ Å and amplitude $V_0=7.5$ K$=0.65$ meV which corresponds to the experimental condition of Ref. 2. For $T=4.2$ K and $B<0.6$ T the SdH oscillations are not yet visible in $\Delta\rho_{xx}$. For $T=4.2$ K the SdH oscillations appear for $B>0.6$ T. When $T<4.2$ K they are also present for $B<0.6$ T. No attempt has been made to fit the theoretical result to the experimental one because this has been done already in Refs. 3-6 and our result is in essence the same as that of these references. The position of the minima are accurately described[4] by the condition $2R_c/a=n+3/4$.

The magnetoresistance along the modulation $\rho_{yy}\approx\sigma_{xx}/\sigma_{xy}^2$ is proportional to the conductivity $\sigma_{xx}$ which has only collisional contributions and can be evaluated from Eq. (4),

$$\sigma_{xx}=\frac{e^2}{h}\frac{N_IU_0^2}{\pi\Gamma a}\sum_n(2n+1)\int_0^{a/l^2}dk_y\,\beta f_{nk_y}(1-f_{nk_y})\,, \tag{9}$$

where $N_I$ is the impurity density with $U_0=2\pi e^2/\epsilon k_s$ the impurity potential in Fourier space in the limit $k_s\gg q$; $k_s$ is the screening wave vector and $\epsilon$ is the dielectric constant. In the absence of modulation Eq. (9) gives the standard two-dimensional result. In the following we will calculate the correction due to the modulation: $\Delta\rho_{yy}=\rho_{yy}(V_0)-\rho_{yy}(V_0=0)$, which is shown as the third curve in Fig. 1. The conduction along the modulation occurs through hopping between the Landau states. This type of conduction is smallest (and thus also $\rho_{yy}$) when the density of states at the Fermi level is smallest, which is the case when the bandwidth at the Fermi level is largest. This explains why the modulations in $\rho_{yy}$ are out of phase with those of $\rho_{xx}$. The fact that the oscillations in $\rho_{yy}$ are much weaker is also evident because they are only a consequence of small perturbations on the collisional current which is also present without modulation. This is different from $\rho_{xx}$ where the modulation opens up an *extra* conduction mechanism. For $B>0.3$ T the rapid oscillations in Fig. 1 are SdH oscillations whose amplitude is now modulated by these new oscillations. Increasing the temperature (but such that $T<10$ K) will not influence the amplitude of the oscillations for $B<0.3$ T but will wash out the rapid SdH oscillations visible for $B>0.3$ T.

The Hall conductivity is evaluated from Eq. (4) along the lines of Ref. 7(b). The result is

$$\sigma_{yx}=\frac{e^2}{m^*\omega_c\pi a}\sum_{n=0}^{\infty}(n+1)\int_0^{a/l^2}dk_y\frac{f_{n,k_y}-f_{n+1,k_y}}{[1+\lambda_n\cos(2uk_y/K)]^2}\,, \tag{10}$$

where $\lambda_n=V_0/\hbar\omega_c e^{-u/2}L_{n+1}^{-1}(u)$. In the absence of the modulation $\lambda_n=0$, $f_{n,k_y}\equiv f_n$, and $a\equiv L_x$; then for strong magnetic fields Eq. (10) leads to the integral quantum Hall effect[7(b)] with the assumption that the Fermi level lies in a region of localized states between two successive Landau levels. In the experiments with modulation the magnetic field is very weak, $\lambda_n\neq0$, and the term $[1+\lambda_n\cos(2uk_y/K)]\hbar\omega_c$, expressing the energy difference $E_\zeta-E_{\zeta'}$ between successive Landau levels, oscillates with magnetic field and leads to oscillations in $\sigma_{yx}$. $\Delta R_H=R_H(V_0)-R_H(V_0=0)$ versus the magnetic field is shown by the last curve of Fig. 1. The oscillations are in phase with those of $\rho_{xx}$ and are small since they result from the term $\lambda_n\cos(2uk_y/K)$ which is the difference of the bandwidths of two neighboring Landau levels.

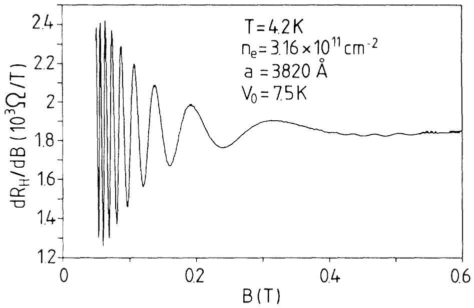


FIG. 2. The derivative of the Hall resistance with respect to the magnetic field for the same physical system as in Fig. 1.

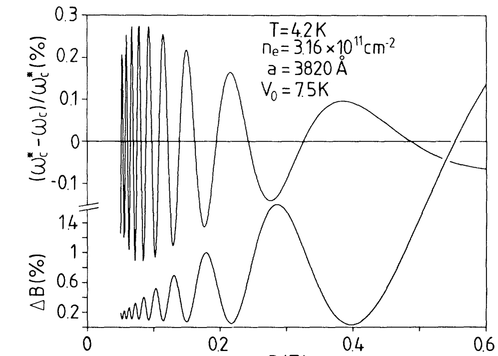


FIG. 3. The percentage shift in the cyclotron frequency and the percentage increase of the cyclotron linewidth as functions of the magnetic field.

If we take the derivative of the Hall resistance with respect to the magnetic field as illustrated in Fig. 2, the oscillations should be much more visible.

Previously Chaplik[11] predicted that the cyclotron resonance of electrons in a lateral superlattice in a strong magnetic field perpendicular to the growth axis exhibits a two-peak structure due to the singular nature of the density of states at the band edges. Up to now the experiments[12] have shown only a broadening of the linewidth. The present system under study is the weak modulation limit of the system studied in Ref. 11. We found the following expression for the cyclotron resonance power spectrum:

$$P(\omega)=\frac{e^2}{h}\frac{E^2}{2}(\hbar\omega_c)^2\frac{l^2}{a}\sum_{n=0}^{\infty}(n+1)\int_0^{a/l^2}dk_y\frac{f_{n,k_y}-f_{n+1,k_y}}{\Delta_{n,k_y}}\frac{\Gamma}{(\Delta_{n,k_y}-\hbar\omega)^2+\Gamma^2}\,, \tag{11}$$

with

$$\Delta_{n,k_y}=\hbar\omega_c[1+\lambda_n\cos(2\pi k_y l^2/a)]\,,$$

and $E$ the strength of the oscillating electric field with frequency $\omega$. For the broadening a typical value of $\Gamma=2$ K was used, but we have checked that the numerical conclusions do not depend on the value of $\Gamma$. The numerical results for the percentage change in the position of the cyclotron resonance frequency and the linewidth are shown in Fig. 3. The position of the cyclotron resonance frequency oscillates around the unperturbed value; it reaches a maximum at maximum bandwidth and its minimum at zero bandwidth. The width of the cyclotron resonance peak oscillates in phase with the oscillations of $\rho_{yy}$.

In conclusion, we have presented a full quantum-mechanical calculation of the resistivity tensor for a 2DEG in a weak 1D periodic potential. All available experimental data can be explained by our model. An interpretation of the antiphase oscillations in $\rho_{yy}$ is given. New oscillations in the Hall resistance, the cyclotron resonance frequency, and the cyclotron resonance linewidth are predicted. We find numerically that the amplitude of the oscillations increases quadratically with the amplitude of the modulation potential $V_0$. Furthermore, lowering the electron density also increases the amplitude of the oscillations. The reason is that the Fermi energy is lowered and the electrons are thus located much closer to the bottom of the modulation potential.

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