Four-Body Calculation of $dd \rightarrow dd$ and $dd \rightarrow p\,^3\mathrm{H}$ Tensor Analyzing Powers

A. C. Fonseca
Centro de Física Nuclear, Avenue Gama Pinto 2, 1699 Lisbon, Portugal[a]
and University of Surrey, Guildford GU25XH, United Kingdom
(Received 5 June 1989)

Four-body integral equations are used to calculate $dd \rightarrow dd$ and $dd \rightarrow p\,^3\mathrm{H}$ amplitudes using a single-term separable nucleon-nucleon potential in channels $^1S_0$ and $^3S_1$-$^3D_1$. First-order perturbation theory is used to include in the four-body kernel the contribution of d-wave two- and three-nucleon channel components that result from the tensor force. Cross sections and analyzing powers are calculated using all positive- and negative-parity four-body amplitudes with total angular momentum $J \leq 4$. Comparison with data is presented.

PACS numbers: 25.10.+s, 24.70.+s, 25.45.De, 25.45.Gh

In a recent work,[1] thereafter named I, a four-body calculation of $n\,^3\mathrm{H} \rightarrow n\,^3\mathrm{H}$, $p\,^3\mathrm{He} \rightarrow p\,^3\mathrm{He}$, $dd \rightarrow p\,^3\mathrm{H}$, and $dd \rightarrow dd$ amplitudes was performed using Alt, Grassberger, and Sandhas[2] (AGS) equations in the form that the (2)+(2) subamplitudes are treated exactly by convolution.[3] This work clearly confirmed the earlier indications by Tjon[4] that p-wave three-nucleon subamplitudes cannot be left out from the four-nucleon kernel, if agreement with data is to be achieved. The effect is particularly strong in $p\,^3\mathrm{He}$ elastic cross sections, but it is also noticeable in $dd \rightarrow p\,^3\mathrm{H}$ and $dd \rightarrow dd$. In I a single-term separable potential in channels $^1S_0$ and $^3S_1$-$^3D_1$ is used between pairs, but the triplet d-wave channel component of the NN t matrix is neglected, leading to the "t_{00}" approximation. Therefore three-nucleon channel spin s is conserved, together with particle-pair relative orbital angular momentum l. The four-nucleon equations one solve take into account contributions from $s = \frac{1}{2}$ and $\frac{3}{2}$ as well as $l = 0$ and 1. One additional approximation involves the neglect of terms in the four-body kernel where two $l = 1$ three-nucleon states are coupled, leading again to the conservation of four-nucleon channel spin S and relative orbital angular momentum L between two-body clusters of (3)+1 or (2)+(2) type. The results that were obtained for $p\,^3\mathrm{He} \rightarrow p\,^3\mathrm{He}$ cross sections showed an excellent agreement with data, but for $dd \rightarrow p\,^3\mathrm{He}$ a persistent discrepancy around 90° in the center of mass remained unexplained.

To solve this problem we attempt for the first time to include the tensor components of the NN force in a scattering calculation. This has already been done successfully for the ground state of ^{4}He and excited 0^+ energies,[5] using different one-term separable Yamaguchi potentials in channels $^1S_0$ and $^3S_1$-$^3D_1$. There we included only the $\frac{1}{2}^+$ three-nucleon subamplitude that is shown to be responsible, together with all (2)+(2) subamplitudes, for 99.99% of the ground-state energy. Nevertheless, as mentioned in I, in a scattering calculation one needs at least all three-nucleon subamplitudes with channel spin $s = \frac{1}{2}$ or $\frac{3}{2}$ and $l = 0$ or 1. In the presence of the three two-nucleon channels that are consistent with the NN interaction chosen above, the three-nucleon subamplitudes of interest have spin and parity $j^p = \frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$, and $\frac{3}{2}^+$, and isospin i; the corresponding three-nucleon channels are shown in Table I for $i = \frac{1}{2}$. The resulting number of four-nucleon channels depends on the total angular momentum J, parity P, and isospin $I = 0$ and rises from seven ($J^P = 0^+$) to seventeen ($J^P = 2^+$).

Given the large number of four-nucleon channels, the underlying subcluster structure, and the need to represent accurately all (3)+1 subamplitudes in a separable form[5] to obtain converged four-nucleon amplitudes, one quickly exceeds the capabilities of present-day computers such as the VAX 8550 or the Cray XMP 48 in terms of memory. Therefore, as a first step, we calculate the contribution of the tensor components in the four-nucleon kernel in first-order perturbation theory.

As shown in Ref. 5 the exact four-nucleon kernel involves the calculation of the driving term $\mathcal{B}$ shown diagrammatically in Fig. 1, which may be written as

$$\mathcal{B} = B + X + Y. \tag{1}$$

The first term B is the Born term which involves the ex-

TABLE I. Three-nucleon channels for different j^p states and $i = \frac{1}{2}$. A singlet (triplet) pair is denoted by $v = 1$ ($v = 2$). A singlet pair carries orbital angular momentum $\tilde{l} = 0$ while a triplet pair carries $\tilde{l} = 0$ or 2.

v	s	l	j^p
1	$\frac{1}{2}$	0	$\frac{1}{2}^+$
2	$\frac{1}{2}$	0	$\frac{1}{2}^+$
2	$\frac{3}{2}$	2	$\frac{1}{2}^+$
1	$\frac{1}{2}$	1	$\frac{1}{2}^-$
2	$\frac{1}{2}$	1	$\frac{1}{2}^-$
2	$\frac{3}{2}$	1	$\frac{1}{2}^-$
1	$\frac{1}{2}$	2	$\frac{3}{2}^+$
2	$\frac{1}{2}$	2	$\frac{3}{2}^+$
2	$\frac{3}{2}$	0	$\frac{3}{2}^+$
2	$\frac{3}{2}$	2	$\frac{3}{2}^+$
1	$\frac{1}{2}$	1	$\frac{3}{2}^-$
2	$\frac{1}{2}$	1	$\frac{3}{2}^-$
2	$\frac{3}{2}$	1	$\frac{3}{2}^-$
2	$\frac{3}{2}$	3	$\frac{3}{2}^-$

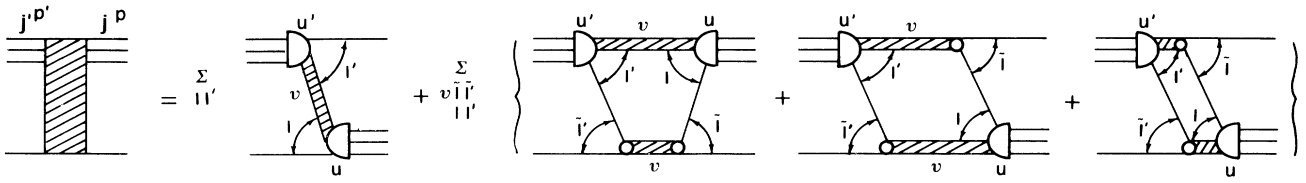

FIG. 1. Driving term for $3+1 \rightarrow 3+1$ amplitudes.

change of an interacting pair of nucleons between $3+1$ states, while X corresponds to the first box amplitude, and Y to the sum of the last two. Explicit expressions for all three terms are given in Ref. 5. Because we are mainly interested in $I=0$ reactions, the three-nucleon subamplitudes carry isospin $i=\frac{1}{2}$ and the $(2)+(2)$ subamplitudes only involve identical pairs. The four-nucleon amplitudes are characterized by $\langle k'S'L'j'^{p'}| \times \mathcal{T}_u^{JP}(E)|kSLj^p\rangle$, where k is the relative momentum between two-body clusters, u is a given separable term of the three-nucleon subamplitude j^p, and E is the four-body center-of-mass energy. Denoting $\mathcal{T}_1$ as the $3+1 \rightarrow 3+1$ elastic amplitude, the corresponding integral equation in operator form reads[6]

$$\mathcal{T}_1 = \mathcal{B} + \mathcal{B}D\mathcal{T}_1 , \tag{2}$$

where D is the three-nucleon propagator as defined by the energy-dependent pole expansion[5] of the $(3)+1$ subamplitude. Likewise, defining $\mathcal{T}_2$ as the $2+2 \rightarrow 3+1$ amplitude, one may write a similar integral equation

$$\mathcal{T}_2 = \mathcal{A} + \mathcal{B}D\mathcal{T}_2 , \tag{3}$$

where $\mathcal{A}$ is the transfer Born term which, using Eqs. (13) and (14) in Ref. 5, reads

$$\langle k'S'L'j'^{p'}|\mathcal{A}_u^{JP}(E)|q\bar{S}\bar{L}vv'\rangle = [1+(-1)^{\bar{S}+\bar{L}+I}]A_{u'}(E;k'q;S'L'j'^{p'};\bar{S}\bar{L}vv')\delta_{vv'} . \tag{4}$$

Finally, defining $\mathcal{T}_3$ as the $2+2 \rightarrow 2+2$ amplitude, one gets $\mathcal{T}_3$ from $\mathcal{T}_2$ through an integral relation

$$\mathcal{T}_3 = \mathcal{A}D\mathcal{T}_2 . \tag{5}$$

If in these formally exact equations one drops the triplet NN channel that carries angular momentum $\tilde{l}=2$, one falls into the t_{00} approximation; that is, all $s \neq j$ three-nucleon channels in Table I disappear, leading to s and l conservation. If, in addition, one neglects the coupling between two negative-parity $3+1$ states, one obtains the equations shown in I. Apart from very small numerical differences (of the order of one or two percent in the phase shifts) the above-mentioned procedure is identical to dropping all terms in $\mathcal{B}$ or $\mathcal{A}$ where $\tilde{l}=2$, $s \neq j$ three-nucleon channels are involved, $l \neq 0$ or 1, and two negative-parity $3+1$ states are coupled. Therefore, for simplicity, we take our amplitudes from I as our zeroth-order amplitudes, and proceed to calculate first-order corrections to the $2+2 \rightarrow 3+1$ and $2+2 \rightarrow 2+2$ amplitudes that result from the tensor components in the kernel. Following the above-mentioned prescription we can separate both $\mathcal{A}$ and $\mathcal{B}$ into

$$\mathcal{A} = \mathcal{A}^{(0)} + \mathcal{A}^{(1)} , \tag{6}$$

$$\mathcal{B} = \mathcal{B}^{(0)} + \mathcal{B}^{(1)} , \tag{7}$$

where $\mathcal{A}^{(0)}$ and $\mathcal{B}^{(0)}$ have $\tilde{l}=0$, $s=j$ three-nucleon channel components with $l=0$ or 1, and no terms that couple two negative-parity $3+1$ states. Using $\mathcal{A}^{(0)}$ and $\mathcal{B}^{(0)}$ in Eqs. (2)–(4) we get $\mathcal{T}_i^{(0)}$ which are, within a few percent, numerically identical to those obtained in I. Next, using standard[7] identities in multichannel scattering theory, we write

$$\mathcal{T}_2^{(1)} = \mathcal{T}_2^{(0)} + \mathcal{A}^{(1)} + \mathcal{T}_1^{(0)}D\mathcal{A}^{(1)} + \mathcal{B}^{(1)}D\mathcal{T}_2^{(0)} + \mathcal{T}_1^{(0)}D\mathcal{B}^{(1)}D\mathcal{T}_2^{(0)} , \tag{8}$$

and

$$\mathcal{T}_3^{(1)} = \mathcal{T}_3^{(0)} + \mathcal{A}^{(1)}D\mathcal{T}_2^{(0)} + \mathcal{T}_2^{(0)}D\mathcal{A}^{(1)} + \mathcal{T}_2^{(0)}D\mathcal{B}^{(1)}D\mathcal{T}_2^{(0)} . \tag{9}$$

To simplify the calculation even further we consider terms in $\mathcal{A}^{(1)}$ and $\mathcal{B}^{(1)}$ that involve the $j^p=\frac{1}{2}^+$ and $\frac{3}{2}^+$ three-nucleon states alone. Therefore $\mathcal{A}^{(1)}$ involves terms with $\tilde{l}+l \geq 2$ and $\mathcal{B}^{(1)}$ requires $l+l' \geq 2$ or $\tilde{l}+\tilde{l}'+l+l' \geq 2$; $\mathcal{A}^{(1)}$ and $\mathcal{B}^{(1)}$ carry at least a vertex with a nonzero angular momentum corresponding to an $\tilde{l}=2$ NN form factor or a $l=2$ N-$(2N)$ generalized three-nucleon form factor with quantum numbers $j^p=\frac{1}{2}^+$ or $\frac{3}{2}^+$ and channel spin s. Since d-wave form factors are in general weaker than s-wave ones, we take 4 as the upper limit on the sum of all $\tilde{l}$'s. In Ref. 5, we showed that the binding energy is 99.9% converged when a similar prescription is used on $\mathcal{B}$. All half-shell and off-shell matrix elements $\mathcal{A}^{(1)}$ and $\mathcal{B}^{(1)}$ are calculated for complex momentum k using the same contour-rotated integration mesh as in I. This procedure is valid as long as one stays below the four-body breakup threshold.

On a VAX 8550 computer, the calculation of all terms needed in (8) and (9) to get $\mathcal{T}_2^{(1)}$ and $\mathcal{T}_3^{(1)}$, with three-nucleon tensor coupling effects resulting from the j^p

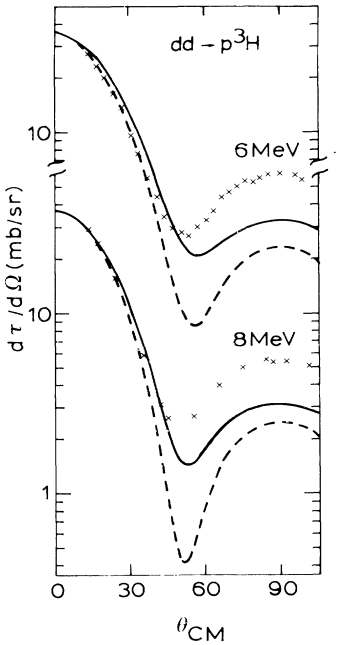

FIG. 2. Cross section for $dd \rightarrow p\,^3\mathrm{H}$ at 6- and 8-MeV deuteron laboratory energy. The data are from Ref. 6.

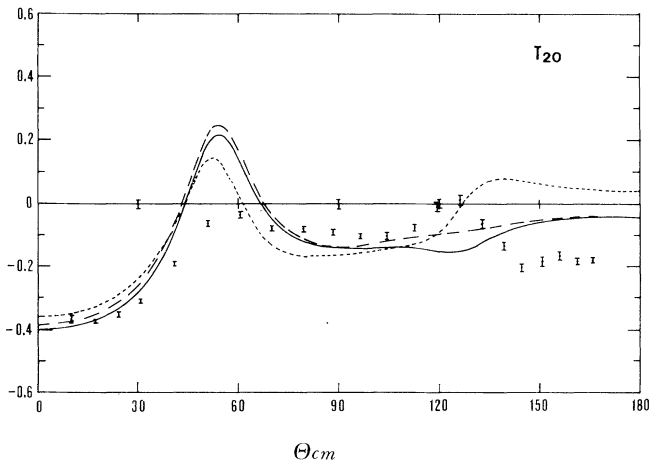

FIG. 3. Tensor analyzing power T_{20} for $dd \rightarrow p\,^3\mathrm{H}$ at 6-MeV deuteron laboratory energy. The data are from Ref. 8. The solid line corresponds to including all five terms in (8) while the dashed line only involves four terms and the dotted line two. Only $j^p = \frac{1}{2}^+$ tensor components were included in $\mathcal{B}^{(1)}$ and $\mathcal{A}^{(1)}$.

$= \frac{1}{2}^+$ subamplitude alone, takes 51 h of CPU time. Adding the $j^p = \frac{3}{2}^+$ subamplitude requires 350 h. For this reason we neglected the last term in (8) and (9) when the coupling to the $\frac{3}{2}^+$ state is included in $\mathcal{A}^{(1)}$ and $\mathcal{B}^{(1)}$. In all calculations we take the negative- and positive-parity four-nucleon amplitudes with $J \leq 4$. The results are shown in Figs. 2–4 for $dd \rightarrow p\,^3\mathrm{H}$ and Fig. 5 for $dd \rightarrow dd$. The point Coulomb phases were used to multiply the nuclear amplitudes and the point Coulomb amplitude added in the case of $dd \rightarrow dd$.

In Fig. 2 we show the cross section for $dd \rightarrow p\,^3\mathrm{H}$ at both 6- and 8-MeV deuteron laboratory energy. The solid line is the new result using Eq. (8) and three-nucleon tensor contributions from $j^p = \frac{1}{2}^+$ alone. The dashed line corresponds to the results of I. The tensor analyzing powers (TAP's) T_{20} and T_{21} are shown in Figs. 3 and 4 for $E_d = 6$ MeV. Although the agreement with the data is not perfect, the calculated TAP's show the correct qualitative behavior; also, a considerable improvement in the cross section takes place over the results of Ref. 1. Nevertheless, a note of care should be added concerning this calculation. Since the $\mathcal{T}^{(0)}$ amplitudes are diagonal in S and L, the coupling in first order through the $\frac{1}{2}^+$ three-nucleon tensor components alone leaves many positive-parity four-nucleon amplitudes unchanged or changed by a few percent. When the $\frac{3}{2}^+$ tensor components are added in Eq. (8) with four terms alone, a stronger coupling emerges that changes the TAP's, while keeping the cross section unchanged. Nevertheless, such results are inconclusive for two reasons: (i) The fifth term in (8) may equally well introduce important changes; (ii) in the presence of strong coupling, perturbation theory may no longer be justified, and one may be required to perform an exact calculation, particularly for lower values of J (see I for the discussion of a similar problem). Work is under way to investigate the effect of the $\frac{3}{2}^+$ tensor coupling.

Finally, in Fig. 5 we show the $dd \rightarrow dd$ TAP's at $E_d = 6$ MeV. Again we find a very remarkable qualitative agreement, particularly for T_{21}, which, in this case, improves by adding tensor coupling effects through the $\frac{3}{2}^+$ three-nucleon state. For the reasons mentioned above, one should not take these changes very seriously, but only as a possible indication of the trends one may expect. Nevertheless, it is remarkable that one is able to simultaneously describe TAP's for $dd \rightarrow p\,^3\mathrm{H}$ and dd

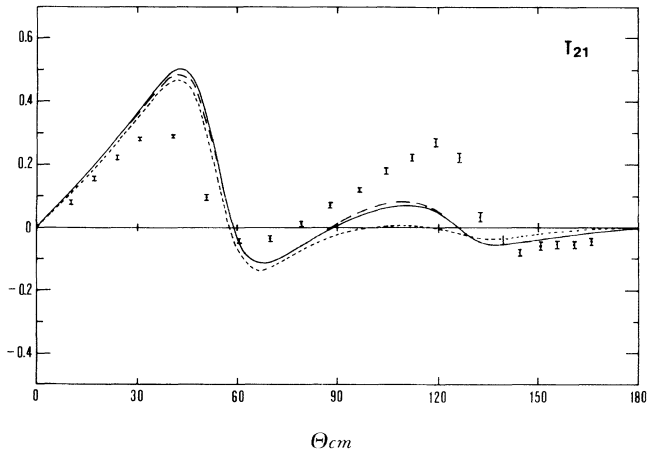

FIG. 4. Same as Fig. 3 for T_{21}.

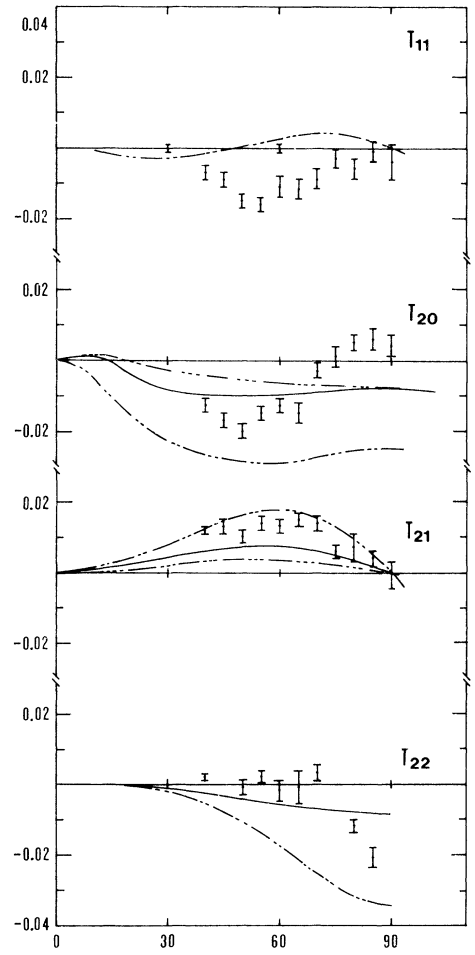

FIG. 5. Analyzing powers for $dd \rightarrow dd$ at 6-MeV deuteron laboratory energy. The data are from Ref. 9. The solid curves correspond to all four terms in (9) while the triple-dot-dashed curves include the first three. Both curves only take $j^p = \frac{1}{2}^+$ tensor components. The double-dot-dashed curves take $j^p = \frac{1}{2}^+$ and $\frac{3}{2}^+$ tensor components but include only the first three terms in (9).

$\rightarrow dd$ that are 1 order of magnitude different without using any free parameter. Instead one uses the correct four-body dynamics together with a simple NN force that fits the low-energy nucleon-nucleon observables, leads to a reasonable triton binding energy ($\epsilon_t = -8.57$ MeV), and generates low-energy p-d scattering results that are quantitatively acceptable, though not perfect. Compared with the most recent resonating-group-method (RGM) calculation[10] for $dd \rightarrow p\,^3\mathrm{H}$, our work seems to provide a better description of the differential-cross-section data particularly for $\Theta_{\mathrm{c.m.}} \leq 30°$. No TAP's are displayed in Ref. 10 for the same reaction.

We also find that the results we now obtain for the $dd \rightarrow dd$ cross section hardly differ from those obtained in I. Unless some major change takes place when one adds the last term in Eq. (9) together with the $\frac{3}{2}^+$ three-nucleon state, or eventually one performs an exact calculation, this persistent discrepancy may have to be related to the nature of the chosen NN potential or to the incorrect treatment of the Coulomb force. The RGM results for the $dd \rightarrow dd$ differential cross section are, relative to the data, substantially better than ours, but unlike ours, they include the correct treatment of the Coulomb distortion.

We would like to thank R. C. Johnson for many useful discussions as well as for his notes on how to calculate tensor analyzing powers given the amplitudes. We would also like to acknowledge the hospitality of the University of Surrey where this work started.

[a]Permanent address.

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