Effects of Superlattice Structure on Weak Localization in Parallel Transport

W. Szott, C. Jedrzejek, ^(a) and W. P. Kirk Department of Physics, Texas A&M University, College Station, Texas 77843-4242 (Received 15 May 1989)

New results of parallel transport effects in a GaAs/AlGaAs superlattice are reported. The observations show a positive magnetoconductivity and a linear variation of the conductivity with temperature. The results are best explained by a recent weak-localization theory which explicitly accounts for the superlattice periodic structure. In contrast, neither the standard three-dimensional anisotropic weaklocalization theory nor the two-dimensional one is found to be adequate. This analysis demonstrates the influence of the superlattice structure on parallel charge conduction.

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Semiconductor superlattices¹ have recently attracted a great deal of attention because of both their unique physical properties and potential device applications. Because of the unprecedented opportunity to control their energy-band structures, superlattices are particularly well suited for studying quantum aspects of electron transport. In this paper we discuss the effects of superlattice structure on weak-localization contributions to parallel transport.

In particular, we show from systematic magnetoresistance measurements of a GaAs/AlGaAs superlattice in low magnetic fields that weak-localization effects in a semiconductor superlattice cannot be simply interpreted as a *n*-layered stack of parallel-conducting, two-dimensional, weakly localized electrons. This result is contrary to the assumptions used in previous analyses of superlattice effects.^{2,3} In addition, we provide an analysis of the results which shows for the first time that (i) lowtemperature parallel transport of charge in a superlattice obeys anisotropic three-dimensional behavior and (ii) this behavior can be related directly to the structural properties of a superlattice. Finally, we find that the an-



FIG. 1. Zero-field conductance of GaAs/(AlGa)As superlattice as a function of temperature. The dashed line is a linear fit to the data from 1 to 7 K. Inset: Planar size of the sample.

isotropic three-dimensional weak-localization theory based on the Kawabata⁴ anisotropic mass-tensor concept cannot account for the measurements. Instead, our analysis shows that the data are well explained by a recent weak-localization theory⁵ which accounts explicitly for the superlattice structure.

A GaAs/AlGaAs superlattice sample was grown by molecular-beam epitaxy on a semi-insulating, Cr-doped GaAs substrate with a 120-nm undoped GaAs buffer. It consisted of thirty periods of four layers each: 18.8-nm undoped GaAs, 1.0-nm undoped Al_{0.3}Ga_{0.7}As spacer, 1.8-nm Al_{0.3}Ga_{0.7}As doped with Si $(1.0 \times 10^{24} \text{ m}^{-3})$, and 1.0-nm undoped Al_{0.3}Ga_{0.7}As spacer. A 100-nm undoped GaAs cap layer was deposited on the top of superlattice structure.⁶ The sample was patterned into a Hall bar consisting of three pairs of voltage probes with sizes shown in the inset of Fig. 1. The experiments were carried out in a ³He-⁴He dilution refrigerator from 0.025 to 7.0 K with excitation currents applied to the sample as square dc pulses of $5-\mu A$ amplitude for 500-ms durations, using negative and positive polarities to eliminate thermal emf's. We estimated the electric field between the potential probes to be less than 0.4 V/m. The measurements were made in low magnetic fields between -0.1 and 0.1 T with the field applied parallel to the superlattice growth direction, and perpendicular to the current density. Measurements of the Hall carrier density and Hall mobility at 0.1 K yield $n_H = 2.03 \times 10^{23}$ m⁻³, and $\mu_H = 0.59 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$, assuming a thickness of $30 \times (18.8 \text{ nm} + 3.8 \text{ nm}).$

The temperature dependence of the conductance in the absence of magnetic field is shown in Fig. 1. A linear temperature T dependence is observed above ≈ 0.5 K, while the conductance approaches a constant value as $T \rightarrow 0$. In Fig. 2 the conductivity data are shown as a function of magnetic field at various temperatures. The magnetoconductivity is clearly seen to increase with magnetic field and temperature. These experimental results are interpreted in light of a weak-localization theory. In order to apply the existing weak-localization theory of positive magnetoconductivity to an extremely anisotropic, periodic, three-dimensional system, we de-



FIG. 2. Conductivity as a function of magnetic field B in a GaAs/(AlGa)As superlattice for several temperatures. Points represent experimental data. Solid lines are theoretical fits calculated using Eq. (6).

rived a theoretical model of weak localization in superlattices,⁵ which we refer to as the superlattice weaklocalization theory (SLWL). This theory assumes the following: (1) a small width 2w of the superlattice subbands, i.e., $2w \ll E_F$; and (2) a short-range isotropic scattering potential.

The miniband structure of our superlattice sample was calculated using a Krönig-Penney model. We used the following parameters: effective carrier mass, $m^* - 0.0667m_e$; barrier height, $V_b = 0.246$ eV; barrier width, 3.8 nm; and well width, 18.8 nm. From the resulting dispersion relation and the Hall carrier density, we found the Fermi energy, $E_F = 17.1$ meV. We also estimated the width of the ground miniband 2w = 1.12 meV, and found the bottom of the first excited miniband at 49.0 meV. Consequently, only the lowest miniband is occupied and the condition $2w \ll E_F$ is satisfied.

According to the SLWL model, the contribution of weak localization to the conductivity at zero magnetic field is given by

$$\sigma_{\rm WL}(0) = \frac{e^2}{\pi^3 \hbar} (D_z \tau_{\rm in})^{-1/2} + c , \qquad (1)$$

where τ_{in} is the inelastic scattering time, c is a temperature-independent term that does not enter into further analysis, and D_z is the effective diffusion constant in the superlattice growth direction,

$$D_z = \left(\frac{wa}{\hbar}\right)^2 \frac{\tau}{2} \,. \tag{2}$$



FIG. 3. Anisotropy factor α vs temperature for the superlattice obtained from the fit to magnetoconductivity data. The dotted line is a guide to the eye through the experimental points. Dashed lines are theoretical results for α obtained according to our model [Eq. (5)], and α_{κ} [Eq. (4)] for Kawabata (Ref. 4).

Here 2w is the width of the miniband, *a* is the superlattice period, and τ is the elastic scattering time. The magnetoconductivity at low fields takes a form analogous with one given by the Kawabata theory, ⁴ viz.,

$$\Delta \sigma_{\rm WL}(B) = \sigma_{\rm WL}(B) - \sigma_{\rm WL}(0) = \frac{\alpha e^2 F(\delta)}{2\pi^2 \hbar l}.$$
 (3)

Here $l = (\hbar/eB)^{1/2}$ is the magnetic length, $a = (D_t/D_z)^{1/2}$ is an anisotropy factor, $\delta = l^2/4D_t \tau_{in}$, D_t is the diffusion constant in the plane perpendicular to magnetic field *B*, and $F(\delta)$ is the Kawabata function.^{4,5}

The most essential difference between our result and the Kawabata theory is the anisotropy factor. The Kawabata theory predicts the following form for this factor

$$\alpha_K = (m_z/m_{\parallel})^{1/2}, \qquad (4)$$

where m_{\parallel} (m_z) is the electron mass in the direction parallel (perpendicular) to the layers ($m_{\parallel} = m^*$). In comparison, the SLWL theory yields an anisotropy factor

$$\alpha = \frac{v_F \hbar}{wa}, \qquad (5)$$

where v_F is the Fermi velocity in the plane perpendicular to the magnetic field, and $v_F \approx \text{const}$ for $2w \ll E_F$. For a superlattice, where the free-particle dispersion relation is not applicable, m_z is not well defined. For purposes of comparison, we use m_z at the bottom of the miniband. We have made a fit of the experimental data to the following formula for the total magnetoconductivity:

$$\Delta \sigma = \Delta \sigma_{\rm WL} - \beta B^2, \qquad (6)$$

where the second term accounts for the classical magnetoconductivity, other effects such as the Zeeman splitting part of the electron-electron interaction,⁷ and the cutoff uncertainty.⁸ Equation (6) contains the three fitting parameters: α , β , and γ , in which $\gamma = D_t \tau_{in}$. The second term in Eq. (6) is relatively small (maximum $\approx 15\%$ at the highest field, 0.03 T) compared to the weaklocalization term. The results of the fitting procedure are shown as the solid curves through the data in Fig. 2. The anisotropy factor α is shown in Fig. 3 to be temperature independent. Finally, we found the effect of temperature on the inelastic scattering time (Fig. 4). It reveals a tendency to obey a power law: $\tau_{in} \propto T^{-p}$ with p=2, in the temperature region above ≈ 1 K. This value of p is characteristic of electron-electron scattering in a pure regime. This regime is also indicated by the parameter $k_F \lambda > 10$ (λ is elastic mean free path and k_F is the Fermi wave vector). Below ≈ 1 K the inelastic time begins to saturate. Similar effects have been observed in a wide variety of systems^{9,10} and ascribed to scattering by residual magnetic impurities and/or decoupling of the electron gas from the thermal bath. In our case the lack of a T^{-3} term in the inelastic time, which is characteristic of electron-phonon scattering, makes the electron heating effect a possible explanation.

In the following analysis we demonstrate the effects of superlattice structure on weak localization in parallel transport that favor the SLWL model.

(i) The anisotropy factor derived from our data, $\alpha = 19 \pm 3$, agrees reasonably well with the prediction from our theoretical model, $\alpha_{\text{theory}} = 15.3$, where α depends directly on the parameters of the superlattice. The most uncertain parameter is the height of the barrier, equivalent to the conduction-band offset. We adopted the value $\Delta E_c / \Delta E_g = 0.63$.¹¹ If this value is changed to



FIG. 4. Inelastic scattering time vs temperature. The dashed line represents quadratic behavior of the inelastic scattering rate as a function of temperature, viz., $\tau_{in} = CT^{-2}$, where C = 220 ps K².

0.85 (the value accepted several years ago but updated recently), we would obtain a theoretical value of $\alpha = 24.5$. The measured value of $\alpha = 19$ implies a band offset of $\Delta E_c/\Delta E_g = 0.72$. One other possible source of the small discrepancy of α could be the simplifying assumption about the isotropic scattering time. Although the scattering may be considered isotropic on a microscopic scale, the averaging of the processes over scatterers produces, nevertheless, anisotropy due to a nonuniform distribution in the superlattice direction.

(ii) The anisotropic 3D theory by Kawabata gives $\alpha = 2.5$. With this value of α one finds an enormous discrepancy (at least a factor of 5) between the experimental data and the theory.

(iii) The analysis of our data using a theory of weaklocalization corrections in two dimensions¹² is not justified. This approach requires treating our sample as a system of thirty independent, noninteracting layers. We can only obtain a fit if we multiply the theoretical result by an arbitrary factor $\eta = n/N = 0.46$ at T = 0.1 K, where n and N are the effective and real number of layers, respectively. This analysis suggests the existence of less than fifteen active layers, in contrast to the known geometry of the sample. Depletion of several top and bottom layers due to pinning of the Fermi level caused by the deep Cr⁺ impurity levels in the substrate and by midgap surface states at the cap layer can reduce the number of the active layers by not more than five (an estimate analogous to that made by Störmer et al.²). But this effect, by no means, can account for the whole discrepancy. The same problem appeared in a recent work on positive magnetoresistance by Moyle, Cheung, and Ong,³ who obtained $\eta = n/N = 0.3 - 0.42$. The fact that η is significantly less than unity works strongly against a simple picture of a stack of n effective, independent, two-dimensional layers.

(iv) Experimental results for zero-field conductance shown in Fig. 1 reveal a linear variation of the conductivity with temperature. By comparison, weak-localization theory for 2D systems predicts a logarithmic temperature dependence of the zero-field conductivity, and has been confirmed in numerous experiments.^{13,14} The only exceptions are systems in which dephasing processes and thus weak localization are dominated by temperature-independent scattering from magnetic impurities or localized spins.¹⁵ Under these circumstances other mechanisms may determine changes of zero-field conductivity with temperature, yet a negative magnetoresistance will result from weak-localization effects. In the case of GaAs/AlGaAs heterostructures, contributions from magnetic impurities and/or localized spins are known to be negligible.¹³ This situation is especially true in our case since the dephasing time obtained from the magnetoconductivity data is strongly temperature dependent when T > 1 K using either a 3D analysis (Fig. 4) or a 2D one [see Sec. (vi)]. This implies that inelastic processes like electron-electron scattering exceed those of a spin-flip character.

Therefore, a 2D weak-localization theory cannot explain our data. The application of the 3D SLWL model is more successful. In particular, the linear variation of the zero-field conductivity $\sigma_{WL}(0)$ implies that the inelastic scattering rate (τ_{in}^{-1}) is a quadratic function of temperature as interpreted by Eq. (1). Exactly the same temperature dependence of the inelastic scattering time is obtained from the magnetoconductivity data, $\Delta \sigma_{WL}(B)$.

(v) The 3D analysis reveals a self-consistency which goes beyond the qualitative arguments presented above. An analysis of the inelastic time as a function of temperature at higher temperatures (outside the region of saturation) yields the coefficient $C = 220 \text{ ps} \text{K}^2$ (see Fig. 4). Substituting this value together with the anisotropy factor $\alpha = 19 \pm 3$ and the diffusion constant $D_t = 0.01$ m^2s^{-1} (derived from the Hall mobility and carrier density at 0.1 K) into Eq. (1) results in a linear temperature coefficient of zero-field conductivity equal to 100 ± 15 $Sm^{-1}K^{-1}$. From the experimental data (Fig. 1) we find this coefficient to be 128 ± 4 Sm⁻¹K⁻¹. The rather close agreement between these values suggests that even if a Coulomb impurity scattering mechanism^{16,17} is operational in our sample, its contribution is small. Also it is important to note that a classical Coulomb scattering mechanism cannot account for the observed positive magnetoconductivity.

(vi) A 2D analysis, by comparison, is subjected to the following *inconsistency*. From the best 2D fit to the magnetoconductivity data, τ_{in} was found to be 3.7 ps at T=7 K and 52.8 ps at T=0.1 K. This yields a change in conductivity of $\sigma(T=7 \text{ K}) - \sigma(T=0.1 \text{ K}) = 63.8 \mu\text{S}$ at B=0 (using the standard formula for 2D weak-localization corrections of the conductivity). This value is to be compared with the experimentally measured change of zero-field conductivity per layer, which amounts to 18.7 μ S. This large discrepancy strongly suggests that a 2D approach is inadequate, unless one introduces a new effect which cancels the logarithmic temperature dependence of the conductivity in weakly localized 2D samples.

In summary, the negative magnetoresistance has been observed in a superlattice and properly interpreted for the first time. The observations have been explained by weak-localization corrections to the electronic transport properties of the system. In particular, the influence of the periodic structure of the superlattice on the behavior of the parallel conductivity can be understood through the application of a recent weak-localization theory specialized to superlattices.

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- ^(a)Permanent address: Department of Physics, Jagellonian University, Cracow, Poland.
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