Coherent Pair Creation in Linear Colliders

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Coherent pair creation during the collision of e^+e^- beams in linear colliders is examined. This includes contributions from both real and virtual photons, where the real photons are predominantly from beamstrahlung. The pair-creation probability is shown to be orders of magnitude larger than the corresponding incoherent processes. The energy spectrum is also calculated where the effective threshold energy is shown to be inversely proportional to the beamstrahlung parameter Υ. Implications of this effect on future e^+e^-, $e\gamma$, and $\gamma\gamma$ linear colliders are discussed.

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It is generally recognized that to avoid severe synchrotron-radiation loss in storage rings, future high-energy e^+e^- colliders will necessarily be linear.[1] To compensate for the much lower collision rates in linear colliders, one is forced to collide much tighter beams. This, however, generates its own kind of radiation loss problem. Each particle during collision would be bent by the strong collective macroscopic electromagnetic field provided by the oncoming beam, and radiate. This phenomenon, called beamstrahlung, has been a subject under intensive study in recent years.[2]

Recently, it was recognized[3] that the generally high-energy beamstrahlung photons, which have to travel through the same collective field in the remainder of the oncoming beam, have a high probability of turning into e^+e^- pairs. Being lower in energy, these e^+e^- pairs will be deflected more severely than the high-energy primary particles, and will potentially cause background problems for high-energy physics experiments.

The problem of pair creation in a magnetic field is not new. Klepikov[4] first calculated this problem in a uniform magnetic field. Several others[5] have reexamined the problem with different formalisms. In the lowest-order approximation of perturbation theory, the matrix element for the pair-creation process is essentially the same as that for beamstrahlung, except that now the initial electron momentum k_μ has to be replaced by $-k_\mu$ in the cross channel. The probability of pair creation per unit time can then be obtained to be[5] ($\hbar = c = 1$)

$$\frac{dn}{dt} = \frac{8\alpha}{3\pi^2}\frac{m}{\omega\chi}\int_0^\infty du\int_0^\infty dv \cosh^3 u\{K_{1/3}^2(\eta)+\cosh^2 u(2\cosh^2 v-1)[K_{1/3}^2(\eta)+K_{2/3}^2(\eta)]\}, \quad (1)$$

where $\chi \equiv (\omega/m)B/B_c$, $\eta \equiv (4/3\chi)\cosh^2 v\cosh^3 u$; $\cosh^2 u = 1+\gamma^2\theta^2$, $\cosh^2 v = \omega^2/[4\epsilon(\omega-\epsilon)]$; α, the fine-structure constant; B, the local magnetic field strength; $B_c \equiv m^2c^3/e\hbar \sim 4.4\times 10^{13}$ G, the Schwinger critical field; ω and ϵ, the energies of the photon and the pair-created particle (either e^+ or e^-), respectively; and θ, the angle between the photon and the $(\mathbf{v},\dot{\mathbf{v}})$ plane of the secondary particle. The integral can be carried out analytically in the asymptotic limits:

$$\frac{dn}{dt} = \begin{cases} 0.23\dfrac{\alpha m^2}{\omega}\chi\exp\left(\dfrac{-8}{3\chi}\right), & \chi \ll 1, \\ 0.38\dfrac{\alpha m^2}{\omega}\chi^{2/3}, & \chi \gg 1. \end{cases} \quad (2)$$

For the entire range of χ, dn/dt can be well approximated by the following expression:[6]

$$\frac{dn}{dt} \simeq \frac{4}{25}\frac{\alpha m^2}{\omega}K_{1/3}^2\left(\frac{4}{3\chi}\right). \quad (3)$$

We see that $\chi \sim 1$ corresponds to the threshold condition for finite probability, below which the pair-creation rate is exponentially suppressed. This condition can be appreciated by the following intuitive arguments: Consider the boosted frame where the e^+e^- pair is created at rest. In this frame, there is an electric field which is $E' = (\omega/2m)B$. At the threshold, the created particle with unit charge e should acquire enough energy within one Compton wavelength to supply its rest mass. Thus, the threshold condition is $eE'\lambda\!\!\!^{-}_c \sim m$, or $\chi \sim 1$.

In accelerators, the particles in a high-energy beam are normally in Gaussian distributions, with standard deviations σ_x, σ_y, and σ_z. For the sake of simplicity in discussions, one may consider the beam particles as being distributed uniformly within an elliptical cylinder with dimensions $2\sigma_x$, $2\sigma_y$, and $2\sqrt{3}\sigma_z$ instead. The local field strength certainly varies in the x-y plane. But is has been shown[7] that an effective field strength can be assigned to the entire beam, upon which the various beamstrahlung phenomena can be faithfully described. In association with the constant effective field, the *beamstrahlung parameter* for beam particles with energy $\mathscr{E}$ is defined as[7]

$$\Upsilon = \frac{5}{6} \frac{\gamma r_e^2 N}{\alpha \sigma_z \sigma_y (1+R)}, \tag{4}$$

where $\gamma = \mathscr{E}/m$, $R \equiv \sigma_x/\sigma_y$ is the beam *aspect ratio*, and N is the total number of particles in the bunch. The coefficient $\frac{5}{6}$ is empirical.

It is useful to express the pair-creation probability in terms of the primary particles, instead of the intermediate photons. For pair creation through the real beamstrahlung photons, the number of pairs per primary particle after collision is

$$n_b = \frac{4\sqrt{3}}{25\pi} \left[\frac{\alpha \sigma_z}{\gamma \lambda\!\!\!^{-}_c} \Upsilon \right]^2 \Xi(\Upsilon), \tag{5}$$

where

$$\Xi(\Upsilon) \equiv \frac{1}{2\Upsilon^2} \int_0^1 \left[\int_q^\infty K_{5/3}(p)\,dp + \frac{y^2}{1-y} K_{2/3}(q) \right] K_{1/3}^2 \left[\frac{4}{3y\Upsilon} \right] \frac{dy}{y} \simeq \begin{cases} 0.5\exp(-16/3\Upsilon), & \Upsilon \ll 1, \\ 2.6\Upsilon^{-2/3}\ln\Upsilon, & \Upsilon \gg 1, \end{cases} \tag{6}$$

where $y \equiv \omega/\mathscr{E}$. Here, the synchrotron-radiation spectrum is used with the beamstrahlung parameter defined in Eq. (4) and $q \equiv (2/3\Upsilon)y/(1-y)$. The trident cascade through virtual photons, on the other hand, has been studied in the past.[8,9] For the sake of comparison, we express it as

$$n_v = \frac{4\sqrt{3}}{25\pi} \left[\frac{\alpha \sigma_z}{\gamma \lambda\!\!\!^{-}_c} \Upsilon \right] \Omega(\Upsilon), \tag{7}$$

where, according to Ritus,[9]

$$\Omega(\Upsilon) \simeq 2.6\alpha \ln\Upsilon, \quad \Upsilon \gg 1. \tag{8}$$

The auxiliary functions $\Xi(\Upsilon)$ and $\Omega(\Upsilon)$ are introduced following the spirit of Erber.[6] The $\Upsilon \ll 1$ limit for the trident cascade was omitted in Eq. (8), due to the subtlety of its being a negative probability as derived by Ritus. However, Ritus argues that this caused only a minor suppression to the real process, while the total probability of $n_b + n_v$ is still positive definite, and satisfies the unitary condition. The clarification of this issue is beyond the scope of this paper, and will be published in a separate effort.

A numerical plot of the two functions is given in Fig. 1. Notice that the crossover between the two functions occurs at $\Upsilon \sim 10^3$. In addition to the different scalings between Ξ and Ω, the beamstrahlung pair creation increases quadratically with the quantity $(\alpha\sigma_z/\gamma\lambda\!\!\!^{-}_c)\Upsilon$, while the trident cascade scales linearly. This is simply because the former necessarily involves a real intermediate process and thus a double integration in time.

It turns out that in e^+e^- linear colliders, the quantity $(\alpha\sigma_z/\gamma\lambda\!\!\!^{-}_c)\Upsilon$ cannot be arbitrary. It has been shown[7,10] that the average energy loss δ of the primary particles due to beamstrahlung behaves differently in the *classical* ($\Upsilon \lesssim 0.1$), the *transition* ($0.1 \lesssim \Upsilon \lesssim 100$), and the *quantum* ($\Upsilon \gtrsim 100$) regimes:

$$\delta \approx \begin{cases} \dfrac{3}{10\sqrt{\pi}} \dfrac{\alpha\sigma_z}{\gamma\lambda\!\!\!^{-}_c} \Upsilon, & 0.1 \lesssim \Upsilon \lesssim 100, \\ \dfrac{6}{5\sqrt{\pi}} \dfrac{\alpha\sigma_z}{\gamma\lambda\!\!\!^{-}_c} \Upsilon^{2/3}, & \Upsilon \gtrsim 100. \end{cases} \tag{9}$$

In designing linear colliders, one usually chooses a reasonable value of δ as a constraint to the choices of other beam parameters, such that the energy resolution of the colliding beam is adequate for meaningful high-

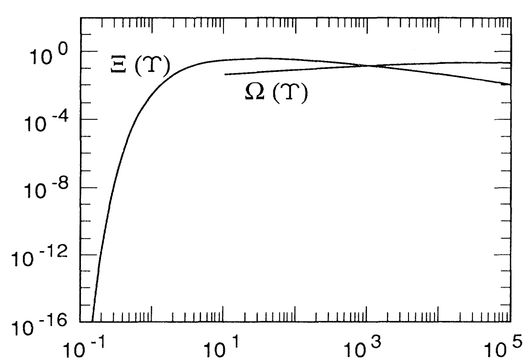

FIG. 1. Auxiliary functions Ξ and Ω of the real and virtual coherent-pair-creation probability, respectively.

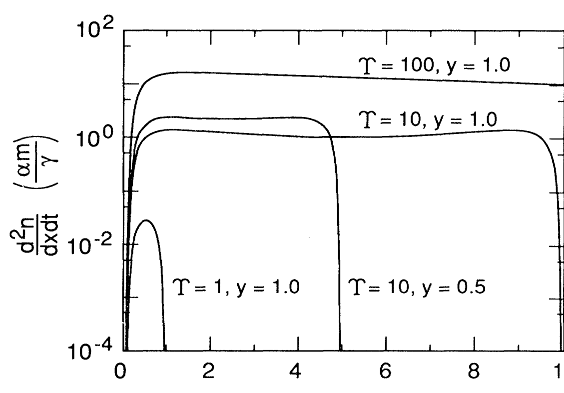

FIG. 2. Normalized spectrum for coherent pair creation for different values of Υ and y, in units of $\alpha m/\gamma$, as a function of $x\Upsilon$.

energy experiments. Combining Eqs. (6), (8), and (9), we find that

$$\frac{n_b}{n_v} \sim \frac{5\sqrt{\pi}}{6\alpha} \delta \Upsilon^{-1/3}, \quad \Upsilon \gtrsim 100. \tag{10}$$

Take, for example, $\delta \sim 0.2$; then the quantity $(\alpha\sigma_z/\gamma\bar{\lambda}_c) \times \Upsilon$ is of order unity, and $n_b \sim 10^{-2}$ for the most part of the transition regime ($\Upsilon \gtrsim 1$). The typical number of particles in each bunch is of order 10^{10}, so one expects to find $\sim 10^8$ e^+e^- pairs per collision. In the extreme quantum regime, if we again fix $\delta \sim 0.2$, then $n_b/n_v \sim 40\Upsilon^{-1/3}$, and is of order unity at $\Upsilon \sim 10^4$.

For our purpose, it is important to study the energy spectrum of the pair-created secondary particles. From Eq. (1), we find that[11]

$$\frac{d^2n}{dx\,dt} = \frac{1}{2\sqrt{3}\pi}\frac{\alpha m}{\gamma}\frac{1}{x(y-x)}\left\{(1+\tanh^2 v)K_{2/3}(a) + \frac{1}{2\cosh^2 v}\int_a^\infty dx\,K_{1/3}(z)\right\},$$

$$= \frac{1}{4\sqrt{3}\pi}\frac{\alpha m}{\gamma} \times \begin{cases} \frac{\sqrt{\pi}}{2}(3\Upsilon)^{1/2}\frac{3+(1-2x/y)^2}{[xy(y-x)]^{1/2}}\exp\left\{-\frac{2}{3\Upsilon}\frac{y}{x(y-x)}\right\}, & y\Upsilon \ll 1, \\ \Gamma(\tfrac{2}{3})(3\Upsilon)^{2/3}\frac{1+(1-2x/y)^2}{[xy^2(y-x)]^{1/3}}, & y\Upsilon \gg 1, \end{cases} \tag{11}$$

where $x \equiv \epsilon/\mathcal{E}$ is the fractional energy of the secondary particle, and $a \equiv 8\cosh^2 v/3y\Upsilon$. For $\Upsilon \ll 1$, the pair tends to equally share the photon energy; while for $\Upsilon \gg 1$, the spectrum becomes much broader, with two maxima located at x and $1-x \sim 1.6/\Upsilon$, respectively. A numerical plot of the spectral function is shown in Fig. 2, where the pair-created particle energy is normalized as $x\Upsilon$. For a given value of Υ, the threshold energy $x_{\rm th}$ is independent of the intermediate photon energy y. In addition, for different values of Υ's, $x_{\rm th}$ scales as $1/\Upsilon$. This can be explained by the following qualitative arguments.

In the Lorentz frame, where the pair is created at rest, the invariant mass of the system is $W = 2eE'\bar{\lambda}_c$. The Lorentz factor for the boost is obviously the photon energy ω divided by the invariant mass. Thus, we have $W^2 = 2eB\omega\bar{\lambda}_c$. On the other hand, from the final state we have $W^2 = \omega^2 m^2/\epsilon_+\epsilon_-$, where ϵ_+, ϵ_- are the energies of the pair particles. In the case where one particle is at very low energy, e.g., $\epsilon_+ \ll \epsilon_- \sim \omega$, we have $W^2 \sim \omega m^2/\epsilon_+$. Thus, $\epsilon_{\rm th} \sim \gamma m/2\Upsilon$, or $x_{\rm th} \sim 1/2\Upsilon$, and is independent of the photon energy. Indeed, we see from Fig. 2 that the falloff of the spectrum starts universally around $1/2\Upsilon$. The "half-maximum" value, however, tends to be located at a smaller value of x. For practical purposes, it should be reasonable to assume the minimum energy to be $x_{\min} \sim 1/5\Upsilon$.

The low-energy secondary particles, once created, will be strongly deflected by the same macroscopic collective field. The nature of the deflection differs between the pair. The secondary particle with opposite charge to the oncoming beam sees a focusing field, while the like charge sees a defocusing field. Since both particles are generally low in energy, the opposite charge tends to be confined by the potential and oscillate on its way out, whereas the like charge would be deflected without bound. In general, for *flat* beams, i.e., $R \gg 1$, the most effective unbound deflection occurs in the vertical direction. This is because for flat beams the vertical field extends fairly uniformly to a distant $\sim 2\sigma_x \gg 2\sigma_y$, with a strength $E+B \sim 2eN/\sqrt{3}\sigma_x\sigma_z$, whereas the horizontal field strength increases only linearly to the same value at $2\sigma_x$. Define the diagonal angle of the field to be $\theta_d \equiv 2\sigma_x/\sqrt{3}\sigma_z$; then the vertical deflection angle for a like charge with fractional energy x can be shown to be approximately

$$\theta_y = \begin{cases} \frac{\sqrt{3}Nr_e\sigma_z}{\gamma x\sigma_x^2}\theta_d, & \theta_y \le 2\theta_d, \\ \left[\frac{2\sqrt{3}Nr_e\sigma_z}{\gamma x\sigma_x^2}\right]^{1/2}\theta_d, & \theta_y \ge 2\theta_d. \end{cases} \tag{12}$$

If an e^+e^- collider is designed such that the beams are colliding head on, then the above consideration imposes a severe constraint on its design. The typical distance between the final focusing magnet and the interaction point (IP) is $\sim 10^2$ cm, while the aperture of the final focusing magnet is $\lesssim 10^{-1}$ cm. This means that any particle with outcoming angle larger than ~ 1 mrad will necessarily hit the magnet and generate backgrounds.

One obvious way to alleviate the problem is to sufficiently suppress these *coherent* processes. In principle, this can be achieved by imposing a constraint on the value of Υ through the requirement $n_b N \simeq 1$. From Fig. 1, the condition is realizable only if $\Upsilon \lesssim 0.3$, where the pair-creation rates are exponentially suppressed to $n_b \sim 10^{-10}$. However, both beamstrahlung photons and virtual photons can also turn into e^+e^- pairs through individual scattering, or incoherent, processes. These are the well-known Bethe-Heitler and Landau-Lifshitz processes. At 1+1 TeV, they are $\sim 5\times 10^{-26}$ and 3×10^{-26} cm^2, respectively. For high-energy physics purposes, a collider at this energy range would have luminosity per collision $\mathcal{L} \sim 10^{31}$ cm^{-2}. Thus, the number of incoherent pairs is $\sim 10^6$ per collision, which is about 2 orders of magnitude smaller than the coherent ones if $\Upsilon \gtrsim 1$, and is equivalent to the coherent yields at $\Upsilon \sim 0.4$.

So, it does not help to suppress the coherent events entirely. For colliders where Υ is inevitably large beyond the 1-TeV range, such that the coherent probability is finite and the spectrum is broad, the situation is evidently difficult.

It has been proposed[12] that the high-energy experiments be performed through either $\gamma\gamma$ or $e\gamma$ collisions. As a hindsight, this idea may have the advantage over the e^+e^- scheme, since the direct beam-beam macroscopic field may be reduced. As it turns out, however, the $\gamma\gamma$ and $e\gamma$ schemes are not free from the influence of coherent pair creation.

In the $\gamma\gamma$ collision scheme of Ref. 12, each of the two primary electron beams first collides with a low-energy photon beam and converts into a high-energy photon beam through Compton backscatterings. The scattered electron beams are then swept aside by bending magnets. Although there is no direct collective field in $\gamma\gamma$ collisions, photons may still be influenced by the residual field of the swept beams and create e^+e^- pairs.

Let the conversion efficiency be κ; then the number of electrons that have not been Compton scattered is $N' = N(1-\kappa)$. Let the location of the conversion be a distance d upstream from the IP, and the horizontal separation between the electron beam and the photon beam at the IP be x_0. It can be verified that the effective χ which the photon beam experiences at the IP due to the residual field of the electron beam is

$$\chi = \frac{2\gamma r_e^2(1-\kappa)N}{\sqrt{3}\alpha\sigma_z x_0}, \tag{13}$$

where the Compton-scattered electrons are not contributing, since they are much softer and assumed to be bent farther away. The luminosity in this case is[12] $\mathcal{L}_{\gamma\gamma} = f\kappa^2N^2/4\pi a_\gamma^2$, where a_γ is the spot size of the photon beam at the IP and f is the collision rate. The spot size is dominated essentially by the stochastic nature of Compton scattering, which has a typical opening angle $\sim 1/\gamma$, and thus $a_\gamma \sim d/\gamma$. On the other hand, d and x_0 are related by $x_0/d^2 = eB_e/2\gamma m$, where B_e is the external bending field with length d.

For our concerns, we would like to limit the number of e^+e^- pairs per photon during collision. This determines χ through Eq. (3), which, in turn, determines x_0 from Eq. (13). For multi-TeV colliders, we take the $\chi \gg 1$ limit of Eq. (3), and eventually get

$$\mathcal{L}_{\gamma\gamma} \simeq \frac{fN}{4\pi r_e^2}\frac{B_e}{B_c}\left(\frac{\kappa^2}{1-\kappa}\right)\left(\frac{n^3\gamma^3\lambda\!\!\!^-_c}{\sqrt{3}\alpha\sigma_z}\right)^{1/2}. \tag{14}$$

Consider, for example, a (5+5)-TeV $\gamma\gamma$ collider, with $f=1000$, $N=10^{10}$, $\sigma_z = 0.1$ μm, $\kappa=0.5$, and $B_e = 3$ T. If we further choose the pair-creation probability to be $n=0.1$, then we find $\chi \sim 4.5\times 10^4$, and the attainable luminosity is $\sim 6\times 10^{34}$ cm^{-2}sec^{-1}.

For $e\gamma$ collisions, the primary contribution to coherent pair creation comes from the direct interaction of the high-energy photon beam with the collective field in the electron beam, where $\chi \sim \gamma r_e^2 N/\alpha\sigma_z\sigma_x$ as in Eq. (4). The luminosity is now $\mathcal{L}_{e\gamma} = f\kappa N^2/4\pi\sigma_x a_\gamma$. Through similar arguments for $\gamma\gamma$ collisions, with a minimal possible horizontal separation $x_0 \gtrsim \sigma_x$, we find

$$\mathcal{L}_{e\gamma} \lesssim \frac{f\kappa}{\sqrt{2}\pi\alpha r_e^2}\left(\frac{N\gamma B_e}{B_c}\right)^{1/2}\left(\frac{n^3\gamma\lambda\!\!\!^-_c}{\sqrt{3}\alpha\sigma_z}\right)^{3/4}. \tag{15}$$

With the same parameters as above, we find $\mathcal{L}_{e\gamma} \lesssim 8\times 10^{33}$ cm^{-2}sec^{-1}. If one insists on larger e-γ separation, e.g., $x_0 \gtrsim 5\sigma_x$, then the luminosity reduces to $\mathcal{L}_{e\gamma} \lesssim 3.5\times 10^{33}$ cm^{-2}sec^{-1}, and is about 20 times smaller than that of the $\gamma\gamma$ case.

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