Relativistic One-Boson-Exchange Model and Elastic Electron-Deuteron Scattering at High Momentum Transfer

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A relativistic covariant analysis of the elastic electromagnetic form factors of the deuteron is carried out using the one-boson-exchange model, including the $\rho\pi\gamma$ and $\omega\epsilon\gamma$ mesonic-exchange-current contributions. The theoretical predictions are compared with the recent experimental data at high momentum transfer.

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Experimental data[1-3] have recently become available on the elastic electromagnetic (em) form factors of the deuteron at high momentum transfer, which may help in discriminating between the predictions of various dynamical theories of the nuclear interaction. In particular, they can serve as a testing ground for the limits of validity of the conventional meson description in this region. In this picture, mesonic exchange currents (MEC's) are shown to play an important role in elastic electron scattering on nuclei.[4] The so-called pair term, being of relativistic origin, and true MEC graphs like the $\rho\pi\gamma$ and $\omega\epsilon\gamma$ graphs yield important contributions at intermediate momentum transfer.[5-7] However, for the pair term it has been argued in the case of the deuteron using a relativistic one-boson-exchange (OBE) model[8] that a consistent relativistic treatment of both the nucleon-nucleon dynamics and the em interaction is needed.[9] In particular, the pair term contribution to the magnetic deuteron form factor is grossly canceled by a correction from the nucleon-nucleon dynamics,[9] resulting in a disparity with the experimental data at moderately large momentum transfer.[2]

Usually the MEC graphs are calculated in a nonrelativistic framework. Moreover, only leading orders in the kinetic motion of the nucleons and momentum transfer are kept in the calculation of the effective two-body em operators.[6] At high momentum transfer these approximations are certainly not expected to be reliable anymore. In this Letter we present results of a fully relativistic analysis of the MEC contributions to the em form factors using the OBE model. The model was studied previously[10] in the relativistic impulse approximation (IA). The calculation we report on has clearly the virtue that both the nucleonic and mesonic contributions to the em current are carried out within the same dynamical model. As noted previously, at the level of the IA, the pair term is already automatically included in this formalism, since it is done in a fully relativistic covariant way.

Relativistic calculations of the MEC contributions are done starting from the deuteron current matrix elements in the Breit frame. From these the em form factors can immediately be deduced. For a detailed description of this formalism we refer to Ref. 10. Denoting the relative momenta of the final and initial states as p' and p, we have

$$\langle P',M'|J_\mu(q)|P,M\rangle = \frac{1}{2M_d(2\pi)^7}\int d^4p\int d^4p'\tilde{\Phi}_d^{(M')}(p';P')S_2(p',P')\Gamma_\mu(q,p',P',p,P)S_2(p,P)\Phi_d^{(M)}(p;P)\,, \tag{1}$$

where $\Phi_d^{(M)}$ represents the deuteron vertex function with total four-momentum P and polarization M. Furthermore, $S_2(p,P)$ is the free two-nucleon Green's function and Γ_μ describes the em vertex. The deuteron vertex function $\Phi_d^{(M)}$ is calculated in the two-nucleon cm system, using the helicity formalism. It is assumed to satisfy a relativistic quasipotential equation, where both nucleons are treated on equal footing; i.e., the relative energy is set to zero.[11] In our considered dynamical model the interaction is assumed to be described by the exchange of ρ, ω, π, ϵ, η, and δ mesons. The corresponding phase parameters predicted by this model are in good agreement with the experimental ones up to 250-MeV laboratory energy. For the deuteron properties and low-momentum-transfer behavior of the elastic em form factor, predictions in the relativistic IA are of similar quality as those found for realistic potentials such as the Reid soft core (RSC).[12] To determine $\Phi_d^{(M)}$ in Eq. (1), the operator for boost transformations $\Lambda(\mathcal{L})$ for spin-$\frac{1}{2}$ particles is used to relate it to the calculated deuteron vertex function.

In a theoretically meson-based description of the nuclear force, we have to also consider, in the analysis of the em form factors of the deuteron, besides the IA graph, the additional contributions arising from meson exchange currents. In the calculation of the $\rho\pi\gamma$ and the $\omega\epsilon\gamma$ graph (Fig. 1) we take the em Lagrangians[5,13]

$$\begin{aligned} L_{\rho\pi\gamma} &= -e\frac{g_{\rho\pi\gamma}}{2m_\rho}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}(\rho^\gamma\cdot\partial^\delta\boldsymbol{\pi})\,,\\ L_{\omega\epsilon\gamma} &= -\frac{eg_{\omega\epsilon\gamma}}{m_\omega}F_{\alpha\beta}\omega^\alpha\partial^\beta\epsilon\,. \end{aligned} \tag{2}$$

For the em form factors of the $\rho\pi\gamma$ and $\omega\epsilon\gamma$ currents we

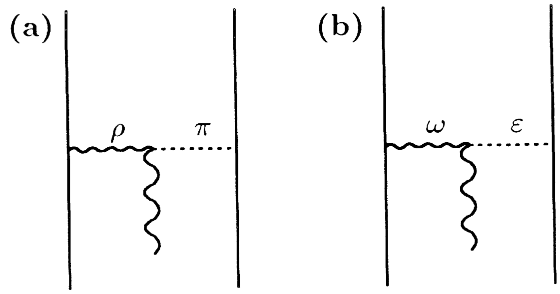

FIG. 1. The $\rho\pi\gamma$ and $\omega\epsilon\gamma$ meson-exchange-current diagrams.

assume vector dominance. Moreover, the adopted forms of the em Lagrangians are such that gauge invariance is satisfied. Taking for the meson-nucleon vertices the same as in Ref. 8, we get in the Breit system for the charge operators

$$\rho^{\rho\pi\gamma} = -C_{\rho\pi\gamma}\left[\gamma+\frac{g^T_{\rho NN}}{4M_N}(\gamma\not{k}_\rho-\not{k}_\rho\gamma)\right]q\gamma_5\not{k}_\pi\tau_1\cdot\tau_2\,,$$
$$\rho^{\omega\epsilon\gamma} = -C_{\omega\epsilon\gamma}\gamma_0 q\cdot k_\epsilon\,, \tag{3}$$

where $\gamma = i(k^1_\pi\gamma^2 - k^2_\pi\gamma^1)$. To determine the whole set of the deuteron em form factors we need also the current component $J_+ = J_1 + iJ_2$. It has the form

$$J^{\rho\pi\gamma}_+ = -C_{\rho\pi\gamma}\left[\gamma^0+\frac{g^T_{\rho NN}}{4M_N}(\gamma^0\not{k}_\rho-\not{k}_\rho\gamma^0)\right]\times q(k^1_\rho+ik^2_\rho)\gamma_5\not{k}_\pi\tau_1\cdot\tau_2\,,$$
$$J^{\omega\epsilon\gamma}_+ = C_{\omega\epsilon\gamma}\left[(\gamma^1+i\gamma^2)q\cdot k_\epsilon - \not{q}(k^1_\epsilon+ik^2_\epsilon) + \frac{\not{k}_\omega \mathbf{q}^2}{m^2_\omega}(k^1_\epsilon+ik^2_\epsilon)\right]. \tag{4}$$

In Eqs. (3) and (4) $C_{\rho\pi\gamma}$ and $C_{\omega\epsilon\gamma}$ contain the various strong and em couplings and the meson propagators. It is given by

$$C_{\rho\pi\gamma} = e g^V_{\rho NN} g^{PV}_{\pi NN}\frac{g_{\rho\pi\gamma}}{m_\rho}F_\pi(k_\pi)F_\rho(k_\rho)F_{\rho\pi\gamma}(q)\Delta_\pi(k_\pi)\Delta_\rho(k_\rho)\,,$$
$$C_{\omega\epsilon\gamma} = e g^V_{\omega NN} g_{\epsilon NN}\frac{g_{\omega\epsilon\gamma}}{m_\omega}F_\epsilon(k_\epsilon)F_\omega(k_\omega)F_{\omega\epsilon\gamma}(q)\times\Delta_\epsilon(k_\epsilon)\Delta_\omega(k_\omega)\,, \tag{5}$$

with F_π, F_ϵ, and F_ρ the strong meson form factors and $F_{\rho\pi\gamma}$ and $F_{\omega\epsilon\gamma}$ the $\rho\pi\gamma$ and $\omega\epsilon\gamma$ form factors. In these expressions we have set the relative energy to zero. The effective em operator is defined as the charge and current operator between Dirac spinors. Using these operators and the helicity basis, the two-loop integrals in Eq. (1) are carried out. Because of rotational symmetry Eq. (1) reduces to a fivefold integral. The tedious but straightforward Dirac algebra has been done using the algebraic program REDUCE.[14]

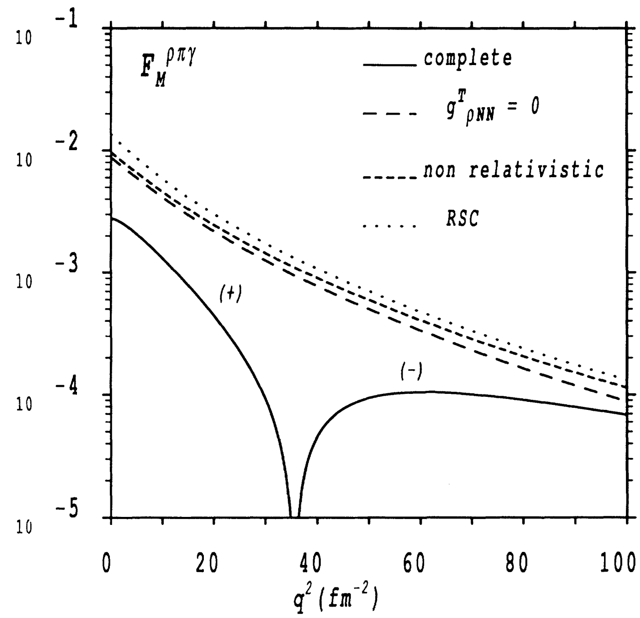

FIG. 2. The $\rho\pi\gamma$ MEC magnetic form factors. The long-dashed line is the γ^0 contribution as defined in Eq. (5). For comparison the Gari and Hyuaga (Ref. 6) results are also shown (dotted line).

In the nonrelativistic limit an expansion in p/M, p'/M, and q/M is made of this effective operator and only the leading-order term is kept. In so doing, the em operator from Ref. 6 is recovered. It is local, resulting in a considerable simplification of the matrix element. In our calculations the recent experimental value[15] $g_{\rho\pi\gamma} = 0.56$ is used, while for the meson-nucleon coupling constants and the strong form factors we have taken the parameters as obtained in the OBE model of Ref. 8. The most dramatic effect of the recoil corrections is found in the magnetic form factor. In Fig. 2 are shown the results for the magnetic form factor of the $\rho\pi\gamma$ graph with the complete effective operator and the nonrelativistic limit of this operator. In the nonrelativistic reduction of the current operator Eq. (4) the term from the tensor part $g^T_{\rho NN}$ of the ρN interaction is usually neglected. The major difference between the fully relativistic and nonrelativistic effective em operators can be ascribed to this term. The long-dashed line in Fig. 2 shows the result for the relativistic em operator but without the $g^T_{\rho NN}$ term. Considering the loop integrals we find that there are still significant contributions at high relative momenta of the order of 1 GeV/c. In this momentum region the $g^T_{\rho NN}$ term is comparable to the γ^0 contribution, but of opposite sign. This cancellation enlarges the effects of the recoil corrections, leading to a dip in the exchange magnetic form factor. Apart from this large correction the magnitude of the additional recoil and relativistic effects are similar as found for the electric form factors, i.e., of the order of 25%. The contributions from the boost

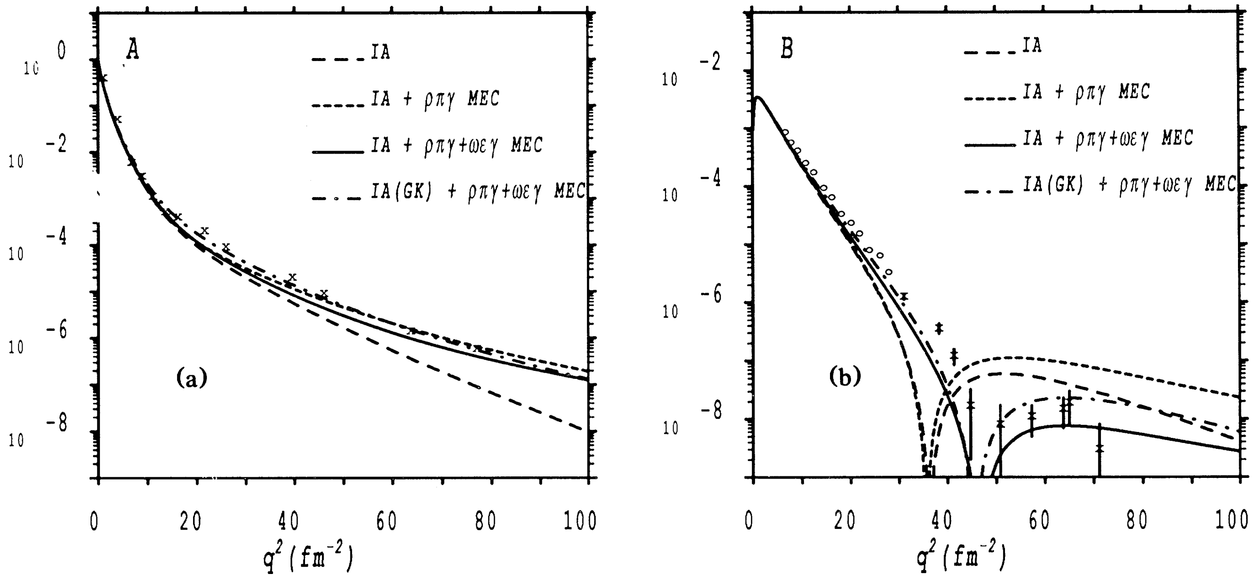

FIG. 3. The electric and magnetic form factors A and B of the deuteron, including the MEC contribution, with Höhler *et al.* form factors and relativistic wave function. The dot-dashed line is with Gari-Krümpelmann nucleon form factors. The data in (a) are from Ref. 1 and the data in (b) labeled by ○ and × are from Refs. 2 and 3.

transformations and negative spinor states are found to be small, similarly as in the IA.

For the $\omega\epsilon\gamma$ coupling constant no experimental data are available. We take $g_{\omega\epsilon\gamma}=-0.56$, as suggested from a study of a relativistic quark model by Chemtob, Moniz, and Rho,[5] where it was found that $g_{\rho\pi\gamma}$ and $g_{\omega\epsilon\gamma}$ coupling constants are the same in magnitude, but of opposite sign. This value of the coupling constant is also consistent[5] with the experimental upper limit set by the decay $\omega\rightarrow\pi^0\pi^0\gamma$. Apart from these em couplings we need the meson parameters and strong meson-nucleon form factors, which are taken from our OBE model. In particular, the mass of the ϵ is given by the value of 570 MeV typically used in OBE models. It should be noted that other isoscalar exchange currents exist such as $\omega\eta\gamma$ and $\rho\delta\gamma$. Because of the smaller meson couplings to the nucleons and larger meson masses it is expected that these contributions are smaller in magnitude. They have therefore not been considered, also in view of the uncertainty of the strength of $g_{\omega\epsilon\gamma}$.

The above results for the MEC contributions can now be combined with the relativistic IA calculation, obtained with the same OBE wave function and Höhler *et al.*[16] one-nucleon form factors. Corrections due to the MEC modify only slightly the em properties at low momentum transfer. In particular, the MEC contributions to the magnetic and quadrupole moment are found to be 1.71×10^{-4} and 1.07×10^{-3}, respectively. In Fig. 3 are shown the measured electric and magnetic deuteron form factors A and B, together with the theoretical calculations. In both em form factors the IA predictions are lower as compared to the data. In particular, the dip in the magnetic form factor is at too low momentum transfer. Similar results are found in calculations with Bethe-Salpeter wave functions.[10] The MEC contributions shift the dip to the right position and an overall good agreement is found. As can be seen from the figure, most of the effect comes from the $\omega\epsilon\gamma$ graph. Also, in the case of the electric form factor A, the inclusion of the MEC contributions yield a substantial improvement. This is mainly due to the $\rho\pi\gamma$-graph contribution. We see also that the $\rho\pi\gamma$ graph increases the IA prediction of A, whereas the $\omega\epsilon\gamma$ contribution lowers it.

In view of the large uncertainty in the experimental neutron form factor and the known sensitivity of the deuteron em form factors on this,[17] we have also considered the use of the nucleon form factors of Gari and Krümpelmann.[18] As is seen from Fig. 3 for both form factors A and B the agreement with the experimental data is somewhat better than with the Höhler *et al.* form factors. This is especially true at moderate momentum transfer. It should, however, be noted that smaller additional corrections, like MEC contributions from the virtual excitations of heavier mesons and Δ degrees of freedom, have not been considered and this clearly leaves room for the small deviations of the predictions from the data found.

Finally, in Fig. 4 is shown the calculated tensor polarization of the deuteron. The relativistic IA result[20] is close to the nonrelativistic RSC prediction, in agreement with Frankfurt *et al.*,[21] but in contrast to the finding of Dymarz and Khanna.[22] Inclusion of the MEC contributions do not have an effect at low momentum transfer, where the agreement with the existing experiments is good. At higher momentum transfer the MEC corrections are significant, thereby shifting the zero in t_{20} to

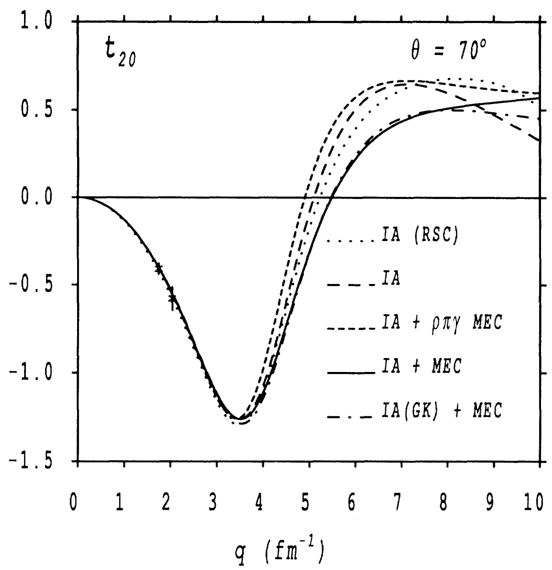

FIG. 4. The deuteron tensor polarization t_{20} for the relativistic OBE wave function with Höhler *et al.* one-nucleon form factors. The dot-dashed curve is with Gari-Krümpelmann form factors. The solid and dot-dashed lines contain the $\rho\pi\gamma$ and $\omega\epsilon\gamma$ MEC. For comparison, the RSC wave-function result (dotted curve) is also shown. The data are from Ref. 19.

higher momentum transfer. In this case use of the Höhler *et al.* or the Gari-Krümpelmann form factors do not lead to large differences, except at very large momentum transfer.

In summary, we have calculated in a relativistic one-boson-exchange model the elastic em form factors of the deuteron. The same meson-nucleon coupling constants and form factors, employed in the description of the nucleon-nucleon interaction, are used to evaluate the em MEC contributions. In particular, the recoil and relativistic effects are found to have a substantial effect on the MEC graphs and the contribution of the $\omega\epsilon\gamma$ graph to the deuteron magnetic form factor is significant at high momentum transfer. Theoretical predictions are well in agreement with the experimental data. We have to emphasize, however, that the results depend on the chosen em coupling constants, which are additional constants and some of them, such as the $\omega\epsilon\gamma$ coupling, are not well known and do not follow from our OBE model.

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