

## Instanton-Induced Quark-Quark Interactions in Two-Baryon Systems

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We study effects of instanton-induced quark-quark interactions in baryon spectroscopy and in the baryon-baryon interaction. The quark-cluster model is applied to calculating baryon-baryon interactions. Results are compared with those of the one-gluon-exchange interaction. We propose a new phenomenological model which contains both the instanton-induced interaction and the one-gluon-exchange potential.

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Quark models of hadrons are remarkably successful in explaining the spectrum and static properties of hadrons.<sup>1</sup> There are two crucial roles of quark dynamics: (1) confinement which makes quark wave functions localized and gives a single-particle energy or an effective constituent quark mass of a few hundred MeV, and (2) the short-range color-magnetic interaction [ $\propto (\sigma_i \cdot \sigma_j)(\lambda_i \cdot \lambda_j)$ ] which causes the hyperfine structure of the spectrum. In the bag model the former is manifested as a boundary condition at the surface of a bag of radius  $R_c \sim 1 \text{ fm} \sim 1/\Lambda_{\text{QCD}}$ , and the latter as a gluon exchange between the quarks. The color-magnetic interaction is in fact so significant that a typical hyperfine splitting,  $M_\Delta - M_N \sim 300 \text{ MeV}$ , is of the same order of magnitude as the constituent quark mass. However, we encounter a difficulty in determining the gluon coupling constant  $\alpha_s$ . To reproduce the hyperfine splittings,  $\alpha_s$  must be roughly twice as large as that naively expected in QCD. With a typical  $\alpha_s \sim 1.6$ , it is hard to justify the perturbative truncation of multigluon exchanges.

Several years ago, Shuryak<sup>2</sup> proposed a new model of the QCD vacuum based on a nonperturbative property of QCD, instanton liquid model. He suggested a new scale of the QCD vacuum, the size of the instanton,  $\rho_c \sim \frac{1}{3} \text{ fm}$ , which is considerably smaller than the confinement size  $R_c$ . He showed then that the coupling of light quarks to instantons breaks chiral symmetry at the scale  $\rho_c$ , and that the light-quark ( $u$ ,  $d$ , and  $s$ ) -instanton coupling provides an effective (constituent) quark mass of a few hundred MeV and an effective constituent quark size  $\sim \rho_c$ . The model also suggests a short-range quark-quark interaction mediated by instantons, which can be expressed as a flavor determinant of  $N_f$  ( $=3$ ) bilinear quark fields, and therefore yields a three-body interaction of flavor-singlet three quarks.<sup>3,4</sup> The determinant form is a manifestation of an explicit  $U(1)$ -symmetry breaking due to the quark-instanton coupling.<sup>3</sup> A two-body quark-quark interaction arises when the QCD vacuum has a  $q\bar{q}$  condensate owing to the chiral-symmetry breaking.

The meson and baryon spectra in this model have been studied by Kochelev<sup>5</sup> and by Shuryak and Rosner.<sup>6</sup> They showed that the instanton-induced quark mass is consistent with the quark model and that the spin dependence of the instanton-induced  $qq$  interaction  $V_{\text{ins}}$  explains the hyperfine structure of hadron spectrum equally as well as the one-gluon-exchange interaction  $V_{\text{OGE}}$ . This is no surprise because, as we will see later, they have the same spin dependence. If we phenomenologically fit the strengths of  $V_{\text{ins}}$  and  $V_{\text{OGE}}$ , then the two interactions give the same hyperfine structure. This may not be the case for multibaryon systems because the color dependence of the two interactions is different. For three-quark systems, the difference only changes the overall strength, because the color-singlet state is unique (totally antisymmetric). For six- or more-quark systems, the color-singlet state has a nontrivial color symmetry and therefore the color dependence plays a significant role.

The purpose of this paper is to study effects of the instanton-induced  $qq$  interaction in six-quark systems. We employ the quark-cluster model,<sup>7</sup> which consists of two clusters of constituent quarks whose interaction is given as a potential. (Although the bag model is useful in describing the single hadron, it is extremely hard to treat the multihadron problem except for the limit of spherical multiquark bag states.) The quark-cluster model imposes the Fermi statistics of quarks, and thus the exchange force between two clusters is fully taken into account. The motion of quarks is assumed to be nonrelativistic so that the motion of the clusters is treated fully dynamically. The quark-cluster model has been applied to various two-baryon systems ( $NN$ ,  $\Lambda N$ ,  $\Sigma N$ , etc.) and has succeeded in describing the short-range behavior of their interaction. It has been shown that the exchange force induced by the color-magnetic one-gluon exchange plays a dominant role. The strong repulsive  $NN$  force at short distance, for instance, is almost entirely attributable to the color-magnetic exchange interaction. However, one finds the same difficulty as seen in

the bag model; i.e., a large fine-structure constant,  $\alpha_s \sim 1.6$ , is necessary to reproduce the baryon spectrum.

Many attempts to reproduce hadron masses in lattice QCD simulation show that in the quenched approximation, where inner quark loops are suppressed, the spin splittings, for instance,  $N\text{-}\Delta$  or  $\pi\text{-}\rho$ , are predicted to be too small, roughly  $\frac{1}{2}$  of the observed values.<sup>8</sup> If we postulate that only a half of the splittings comes from gluon exchanges and that the rest is due to a nonperturbative  $q\bar{q}$  condensate, then we should not be surprised that the quenched lattice calculation underestimates the splitting. The latter could be understood as a result of the instanton-induced interaction where  $q\bar{q}$  condensate is necessary to reduce the three-body force into the two-body one.

We therefore propose a model in which a significant part ( $\sim \frac{1}{2}$ ) of the hyperfine splitting is due to the instanton-induced interaction and thus the phenomenological fine-structure constant  $\alpha_s$  is reduced accordingly. We take the form of the instanton-induced interaction according to the instanton liquid model, but determine its strength and other quark-model parameters (quark mass, etc.) phenomenologically. Our conclusions in this model are the following: (1) The instanton-induced interaction consists of a spin-independent attractive force and a color-magnetic interaction. (2) The ground-state baryon spectrum is only shifted by the mixing of the instanton-induced interaction if the overall strength is fitted to the observed hyperfine splitting,  $N\text{-}\Delta$ , for instance. (3) The SU(3)-breaking effect is also unchanged if we take the ratio of  $u\text{-}d$  and  $u(d)\text{-}s$  interactions to be  $\sim m_u/m_s \sim 0.6$ . (4) Baryon-baryon interactions have a strongly attractive direct force due to the color-spin-independent potential. The direct interaction has a longer range than the exchange color-magnetic interaction and therefore is dominant at long distance. (5) The exchange force due to the color-magnetic interaction remains and thus the  $NN$  force is repulsive at short distance.

We consider the following quark-potential-model Hamiltonian:

$$H = K + V_{\text{conf}} + V_{\text{OGE}} + V_{\text{ins}}, \quad (1)$$

where  $K$  is the kinetic energy,  $V_{\text{conf}}$  a two-body color-dependent quark confinement potential, and  $V_{\text{OGE}}$  the static part of the one-gluon-exchange potential.<sup>7</sup> The tensor and spin-orbit terms of  $V_{\text{OGE}}$  are omitted in this study.  $V_{\text{ins}}$  is the newly introduced instanton-induced potential:

$$V_{\text{ins}} = - \sum_{i < j} \frac{1}{2} W_{ij} (1 - P_{ij}^f) [1 - \frac{1}{5} (\sigma_i \cdot \sigma_j)] \delta(\mathbf{r}_{ij}), \quad (2)$$

which is derived by taking the static and nonrelativistic limit of the effective Lagrangian<sup>3,4</sup> for light quarks in the vacuum with small instantons (see Ref. 9 for details). It acts only on quark pairs which are antisymmetric under flavor exchange, as is expected from the

determinant form of the effective interaction, and therefore is proportional to  $1 - P_{ij}^f$ , where  $P_{ij}^f$  is the exchange operator of flavor. We have neglected the instanton size in choosing a zero-range  $[\delta(\mathbf{r})]$  force. Although this constraint can be removed by introducing a range parameter ( $\sim$  instanton size), we employ the simplest form for our qualitative discussion in this paper. We also omit noncentral forces which arise as relativistic corrections.<sup>9</sup> The coefficient  $W_{ij}$ , which is related to the  $q\bar{q}$  condensate, is proportional to the inverse of the effective quark masses,  $\propto 1/m_i m_j$ .<sup>2,5</sup>  $V_{\text{ins}}$  can be expressed equivalently in terms of the color and spin operators by using  $P_{ij}^f = -P_{ij}^c P_{ij}^s$ :

$$V_{\text{ins}} = - \sum_{i < j} \frac{1}{2} W_{ij} \left[ \frac{16}{15} + \frac{1}{10} (\lambda_i \cdot \lambda_j) + \frac{3}{10} (\lambda_i \cdot \lambda_j) (\sigma_i \cdot \sigma_j) \right] \delta(\mathbf{r}_{ij}). \quad (3)$$

Here the last two terms in the brackets are color-octet exchange terms, proportional to  $\lambda_i \cdot \lambda_j$ , and contribute in the quark-cluster model only in the exchange interaction. The first term corresponds to a color-singlet exchange, giving a direct interaction as well as an exchange force.

From Eq. (3) we immediately see that the spin dependence of  $V_{\text{ins}}$  is identical to that of  $V_{\text{OGE}}$ , which leads us to conclusion (2) above; i.e., the ground-state baryon spectrum cannot distinguish  $V_{\text{ins}}$  and  $V_{\text{OGE}}$ . In fact, the  $N\text{-}\Delta$  and  $\Lambda\text{-}\Sigma$  mass differences are totally independent of the mixing because they are given by

$$M_{\Delta} - M_N = 3(u + w), \quad (4)$$

$$M_{\Sigma} - M_{\Lambda} = 2(1 - \xi)(u + w),$$

where  $u$  and  $w$  are, respectively, the contributions of  $V_{\text{OGE}}$  and  $V_{\text{ins}}$ ,  $u = \langle 8\pi\alpha_s \delta(\mathbf{r}) / 9m_u^2 \rangle$  and  $w = \langle 4W_{ud} \times \delta(\mathbf{r}) / 5 \rangle$ , and  $\xi$  is the ratio of  $m_u$  and  $m_s$ :  $\xi = m_u/m_s$ . The  $\Sigma\text{-}\Lambda$  mass difference requires  $\xi \sim 0.6$ , which is consistent with the ratio of the effective quark masses. Other mass differences may change by the mixing only up to a few MeV. The spin-independent terms of  $V_{\text{ins}}$  give an overall shift of baryon masses.

For two-baryon systems, we first consider a limit in which all six quarks are in a single confining potential, or a bag. The ground state is the state with all the quarks in the lowest single-particle orbit. In the SU(3) limit,  $V_{\text{ins}}$  for such states is given by

$$\langle V_{\text{ins}} \rangle_{6q} = \frac{w}{4} \left[ -45 + S(S+1) + 3 \left\langle \sum_{i < j} P_{ij}^f \right\rangle \right]. \quad (5)$$

We obtain  $\langle V_{\text{ins}} \rangle = -15w/2$  ( $-17w/2$ ) for the  $^1S_0$  ( $^3S_1$ )  $NN$  state. The matrix element  $w$  estimated approximately in the single baryon by (4) gives  $w \approx (M_{\Delta} - M_N)/3 - u \sim 100 \text{ MeV} - u$ . Thus  $V_{\text{ins}}$  gives a strong attraction mostly due to the first term of (5), or the color-singlet term of (3), while the spin dependence is similar to  $V_{\text{OGE}}$ .

Now we employ the quark-cluster model to study the

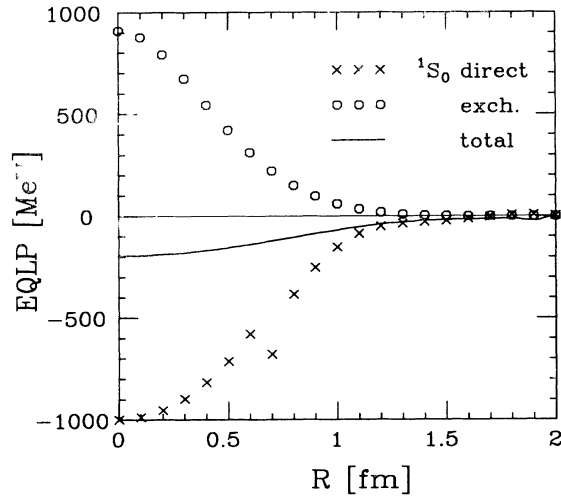


FIG. 1. Equivalent local potentials calculated for  $^1S_0$  NN scattering at 20 MeV, in the quark-cluster model in the full instanton limit. For definition of the equivalent local potential, see Ref. 7. The solid curve is the total potential, while  $\times$  ( $\circ$ ) shows the direct (exchange) contributions.

baryon-baryon interactions at arbitrary separation. Because of quark antisymmetrization, exchange interactions as well as the direct one are induced between two clusters (baryons). The color-octet terms in (3) contribute only to the exchange force and therefore the interaction must be short range. The spin-isospin exchange factor,  $\frac{1}{27}$  for the NN S wave, suppresses the  $\lambda_i \cdot \lambda_j$  contribution, while the color-magnetic term,  $(\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j)$ , plays a significant role as it does in the one-gluon exchange. By contrast, the color-singlet term induces an attractive direct interaction whose range is longer than the exchange interaction.

Figure 1 shows the equivalent local potentials calculated in the quark-cluster model in the limit that  $V_{ins}$  gives the full  $N-\Delta$  mass difference without  $V_{OGE}$ , i.e.,  $\alpha_s = 0$  and  $W_{ud} = 920 \text{ MeV fm}^3$ . Other parameters are chosen as follows: the effective quark mass  $m_q = 313 \text{ MeV}$ , the strength of the confining potential  $a = 43.8 \text{ MeV/fm}$ , and the baryon-size parameter  $b = 0.62 \text{ fm}$ . The results confirm the above arguments. The direct potential due to the color-singlet term is strongly attractive, while the exchange contribution due mostly to the color-magnetic interaction is strongly repulsive and has a shorter range. The total potential due to  $V_{ins}$  is weakly attractive. In this limit, there exists an NN bound state in both  $^3S_1$  and  $^1S_0$ .

If we postulate that only a half of the hyperfine splitting of hadrons is due to the one-gluon exchange, and that the other comes from the instanton-induced interaction, we may mix these two with about an equal weight. By fixing the overall strength of the spin-dependent term by the  $N-\Delta$  mass difference, we obtain  $\alpha_s = 1.657/2 = 0.834$  and  $W_{ud} = 460 \text{ MeV fm}^3$ . Figure 2 shows the

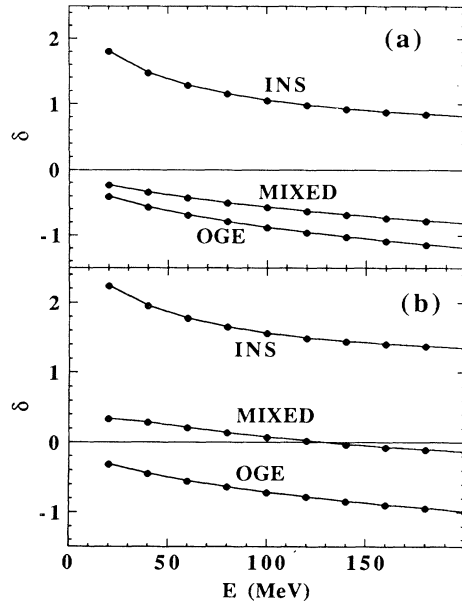


FIG. 2. (a)  $^1S_0$  and (b)  $^3S_1$  NN scattering phase shifts for the full instanton, INS, pure one-gluon exchange, OGE, and the mixed case, MIXED.

calculated NN scattering phase shifts for three cases: (1) one-gluon exchange only (OGE),  $\alpha_s = 1.657$ , and  $W_{ud} = 0$ ; (2) instanton-induced interaction only (INS),  $\alpha_s = 0$ , and  $W_{ud} = 920 \text{ MeV fm}^3$ ; and (3) the mixed case (MIXED). One sees that the mixing enhances the short-range repulsion from the color-magnetic interaction and thus the total equivalent local potential (Fig. 3) is repulsive inside and weakly attractive outside.

It is remarkable that qualitative features, i.e., short-

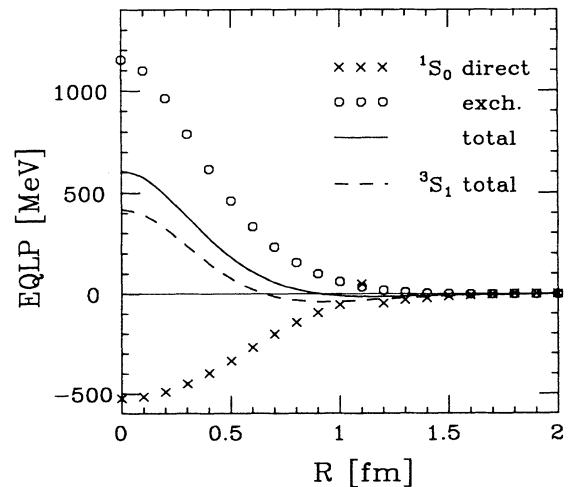


FIG. 3. Equivalent local potentials in the mixed case. The notation is the same as Fig. 1. The dashed curve is the total potential for  $^3S_1$ .

range repulsion and medium-range attraction, of the nuclear force are reproduced in the present model. It is not surprising that the present calculation does not reproduce realistic  $NN$  phase shifts, because it does not yet incorporate another essential component of the nuclear force, meson exchanges, which will provide long-range interactions.<sup>7</sup> We do not attempt here to describe the complete nuclear force, because the present model is qualitative.

In this paper, we applied a new type of quark-quark interaction induced by light-quark-instanton coupling to the study of the baryon spectrum and the short-range part of the baryon-baryon interaction. We found that the instanton-induced potential contains a color-magnetic term, which is essential in the hyperfine splittings of the baryon and the short-range exchange force between two baryons. A color-spin-independent attraction between quarks is also found and shown to give a strong long-range attraction between two baryons. We have proposed a model in which the one-gluon-exchange and the instanton-mediated interaction share the role in the hyperfine splitting of the hadron. An  $NN$  potential with a short-range repulsion and a medium-range attraction was obtained by this model. The strong attractive force becomes significant in other two-baryon systems, too. Especially, the  $\Lambda\Lambda$ - $N\Xi$ - $\Sigma\Sigma$  system with  $J=0$  and some other systems with strangeness are known to have a nonrepulsive exchange force and therefore a further attraction by  $V_{\text{ins}}$  makes dibaryon bound or resonance states likely.<sup>10</sup> Here we employed the simplest form of the instanton-induced interaction, which does not include finite-size effects nor noncentral forces. Further effort is

necessary for a more realistic study.<sup>9</sup>

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