Comment on "Charging Effects and Quantum Coherence in Regular Josephson Junction Arrays"

In a recent Letter, Geerligs *et al.*[1] reported that the transition to global superconductivity in artificially structured two-dimensional Josephson junction arrays occurs at a critical value of $x \equiv E_C/E_J \simeq 1.5$. Here, $E_C = eV_g$ and $E_J = h\Delta_0/8e^2R_N$, where V_g and R_N are the normal-state Coulomb gap and junction resistance, respectively, and Δ_0 is the zero-temperature BCS gap. We expect that the theoretical $x=1$ threshold mentioned by Geerligs *et al.*, which is appropriate for islands having only a self-capacitance, is inapplicable to the arrays in question, which are probably dominated by the nearest-neighbor mutual capacitance, C_1. Mean-field theory then leads to a threshold coupling strength[2(a)] $g \equiv C_1E_J/4e^2 = 1/2z^2 = 1/32$, with $z=4$ the square-lattice coordination number. From a generalization of the familiar resistance-network problem,[3] the effective capacitance between two neighboring islands in the array is $C_{\text{eff}} = 2C_1$, so that $E_C = e^2/2C_{\text{eff}}$ and the mean-field threshold is $x = e^2/4C_1E_J = 1/16g = 2$. The first two terms[2(b)] of a $1/z$ expansion[4] of the modification of the threshold by quantum fluctuations, $2z^2g = 1 + 6/5z = 1.3$, yield the phase boundary $x \simeq 1.54$, as shown by the dashed horizontal line in Fig. 1. However, Geigenmüller and Schön[5] have obtained $C_{\text{eff}}/C_1 = \pi/(\pi-2)$, which would move this line down to $x \simeq 1.1$.

As noted by Geerligs *et al.*,[1] such experiments are approaching the miniaturization necessary to observe effects associated with the virtual-quasiparticle-tunneling capacitance, which is believed[2,6] to dominate the geometrical capacitance in ultrathin granular films.[7] Including $\Delta C = 3h/64\Delta_0R_N$ in parallel with C_1 yields the modified phase boundary

$$1.3 = \frac{2}{x} + 3\gamma\left(\frac{h}{8e^2R_N}\right)^2,$$

shown by the solid curve in Fig. 1. Our estimate[2(b)] of the numerical correction factor resulting from the frequency and phase dependence of ΔC is $\gamma \simeq 0.6$. Note that x is still defined in terms of the normal-state determination of the capacitance. The small-grain limit, $x \to \infty$, yields the universal threshold,[2(b)] $R_N \simeq 3.8$ kΩ, shown by the vertical dashed line. The globally superconducting portion of the phase diagram of Fig. 1 is the shaded region labeled "*S*" below the horizontal dashed line, if capacitance renormalization is neglected. Including this effect of virtual quasiparticle tunneling reduces the normal region labeled "*N*" and enlarges the superconducting region to include the doubly crosshatched area, as indicated by the vertical arrow labeled "ΔC." The open and solid circles in Fig. 1, which exhibit the reported[1] low-temperature zero- and high-resistance behavior, respectively, are evidently consistent with the theoretical phase boundary. But further miniaturization is needed to establish unambiguously the role of capacitance renormalization.

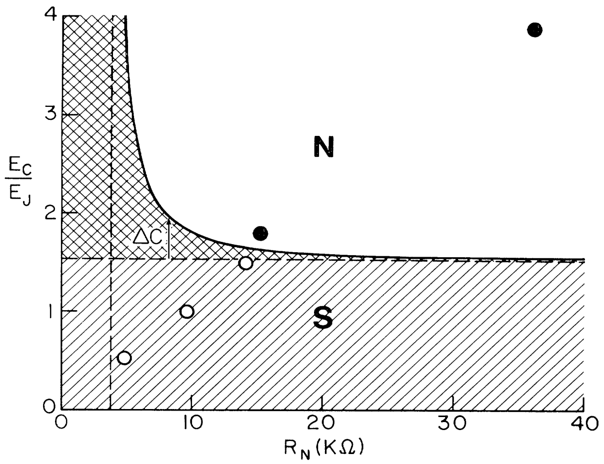

FIG. 1. Theoretical scaled-charging-energy vs normal-state-resistance phase diagram of a regular two-dimensional Josephson junction array at zero temperature.

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