Crossover to Strong Intensity Correlation for Microwave Radiation in Random Media

N. Garcia and A. Z. Genack
Department of Physics, Queens College of City University of New York, Flushing, New York 11367
(Received 17 January 1989)

We report measurements of microwave intensity statistics in random configurations of polystyrene spheres contained in a cylinder with reflecting walls. As the sample thickness increases measurements of the intensity distribution and the intensity correlation function with frequency shift reveal a crossover from uncorrelated to correlated photon diffusion.

PACS numbers: 42.20.−y, 78.20.Dj, 78.70.Gq, 84.40.Cb

Microwave experiments offer wide latitude with which to probe the universal character of wave propagation. Here we report measurements of first- and second-order intensity statistics of microwave radiation in random configurations of polystyrene spheres contained in a cylindrical tube with highly reflecting copper walls. By confining the radiation, the probability that a photon passing through a point within the medium also passes through another point or returns to the first point is enhanced. This leads to an enhancement of intensity correlation which increases further as the sample length, L, increases. The degree of correlation is exhibited in the distribution of intensities and in the intensity-intensity correlation function with incident radiation frequency at a point in the sample.

In the weak scattering limit $kl \gg 1$, where $k=2\pi/\lambda$ and l is the transport mean free path, the intensity-intensity correlation function has been calculated by Stephen and Cwilich[1] and by Feng, Kane, Lee, and Stone.[2] It can be expressed as an expansion in the parameter $1/g$, where $g=G/(e^2/h)$ is the dimensionless conductance. In the absence of inelastic processes $g \approx N_t l/L$, where N_t is the number of transverse input modes at the excitation frequency. The leading term in the expansion corresponds to uncorrelated diffusion and is directly related to the time-of-flight distribution of photons.[3] When g is large this term dominates the local intensity correlation function. Higher-order terms include corrections due to correlation between different random paths in the medium. Though these corrections dominate fluctuations in total transmission and conductance, they have not previously been directly observed in local statistics. Measurements of optical fluctuations have been in the regime of large g ($>10^4$).[3-5] In electronic experiments[6] the requirement of subwavelength resolution has not been achieved. These restrictions are overcome in the present microwave experiments because intensity measurements at a point are possible and g can be made small by the use of reflecting walls to limit the sample's transverse dimensions.

In the weak scattering limit, g can be identified with the Thouless number δ (Ref. 7) which is the ratio of the level width to the level spacing, $\delta = \delta E/(dE/dN)$, for eigenstates of the Schrödinger equation of a random potential.[8] For classical waves δ can be defined using the correspondence principle as the dimensionless frequency $\delta = \delta\nu/(d\nu/dN)$. The level width $\delta\nu$ can be identified with the half-width, $\delta\nu_E$, of the field-field correlation function.[3] Thus δ can be determined experimentally from measurements of intensity fluctuations with frequency and is a natural parameter with which to describe electromagnetic propagation.

We measure the scale dependence of the average intensity, $T(L)$, the spectral cumulant intensity correlation function, $C^I(\Delta\nu;L)$, and the distribution of intensities, $P(I/\langle I\rangle;L)$, of the transmitted radiation. The sample consists of uniform, one-half inch polystyrene spheres with filling fraction $f=0.56$. The diameter of the copper tube is 7.3 cm and the typical wavelength is ~ 1 cm. The spheres uniformly fill the tube but are still loosely enough packed to move significantly when the tube is tumbled. The wave is launched from a horn positioned 20 cm in front of the sample. The frequency is set by a computer-generated voltage and the sample is tumbled by a computer-controlled motor. The intensity of microwave radiation is detected at the point of contact of a wire probe with a Schottky diode. The diode is mounted on a plunger which defines the sample length. The front of the plunger is covered with a microwave-absorbing material to minimize reflection back into the sample. The time average of intensity at a point as a function of L, which is measured as the sample is tumbled, is proportional to the configuration average of transmission versus thickness. The tumbling can be stopped and the frequency dependence of the transmitted intensity for a particular configuration of the sample can be studied by scanning the microwave frequency. The spectra at a given thickness are normalized to the average of all the spectra at that thickness to eliminate the influence of the frequency variations in the instrumental response. A typical spectrum of intensity fluctuations at $L=30$ cm is shown in Fig. 1(a). The intensity correlation function is computed for each sample configuration and the ensemble average is obtained by averaging the correlation function for several hundred configurations, all with the same number of scatterers. The points in Fig. 1(b) are the results obtained for $L=30$ cm. The distribution of intensities is obtained from values of intensity in the

 © 1989 The American Physical Society

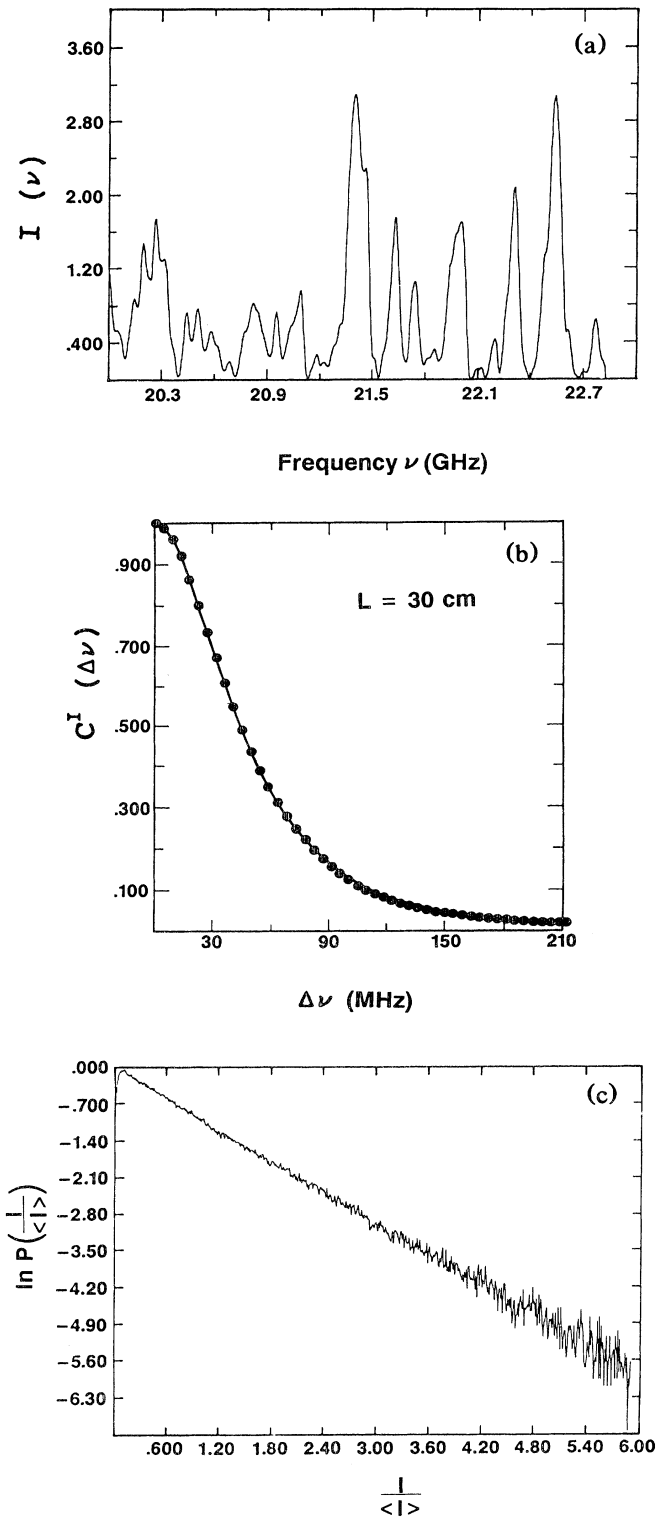

FIG. 1. (a) Intensity fluctuations of microwave radiation transmitted through a sample of thickness $L=30$ cm. (b) Average of the cumulant autocorrelation function for 500 different sample configurations. The solid line is a least-squares fit of Eq. (2) to the data points. (c) Distribution of intensities.

spectrum for all frequencies and sample configurations probed at each sample thickness. Thus we are able to measure propagation in samples of different configuration and length but with statistically equivalent disorder. This is the paradigm of ensemble averaging which is not readily achieved in electronic studies except for the case of substitutional disorder.

We have measured the thickness dependence of the transmitted intensity $T(L)$ at several frequencies between 18 and 24 GHz. We find that the measurements of $T(L)$ are described by the expression for transmission given by the photon-diffusion model:[3,5] $T(L) \sim \sinh(\alpha a)/\sinh(\alpha L)$. Here $\alpha = 1/L_a = 1/(D\tau_a)^{1/2}$ is the extinction coefficient, D is the diffusion coefficient, τ_a is the photon absorption time, and $a = 5l/3$. A fit of the expression for $T(L)$ to the data gives $L_a = 25 \pm 1$ cm.

The field-field correlation function with frequency shift, $G^E(\Delta\nu) \sim \langle E(\nu)E^*(\nu+\Delta\nu)\rangle$, is the Fourier transform of the photon time-of-flight distribution, $I(t;L)$.[3] Using the result of the photon-diffusion model for $I(t;L)$ gives

$$G^E(\Delta\nu;L) \sim \mathrm{Re}[\sinh(q_0 a)/\sinh(q_0 L)]\,, \qquad (1)$$

where q_0 is the root of $q^2 = \alpha^2 + 2\pi i \Delta\nu/D$ with negative imaginary part.[3] In the absence of long-range spatial correlation, the intensity correlation function,

$$G^I(\Delta\nu;L) = \langle E(\nu)E^*(\nu)E(\nu+\Delta\nu)E^*(\nu+\Delta\nu)\rangle\,,$$

can be evaluated by factorizing the fields,

$$G^I(\Delta\nu;L) = |\langle E(\nu)E^*(\nu+\Delta\nu)\rangle|^2 + \langle E(\nu)E^*(\nu)\rangle$$

(Refs. 1 and 9). The cumulant intensity correlation function is then given by[3]

$$C^I(\Delta\nu;L) \sim |\sinh(q_0 a)/\sinh(q_0 L)|^2\,. \qquad (2)$$

The half-width of the intensity correlation function $\Delta\nu_I$ as determined by Eq. (2) depends on D, L_a, and L. A fit of the values of $\Delta\nu_I$ predicted by Eq. (2) to the measured values using D as a fitting parameter gives $D = (3.1 \pm 0.2)\times 10^{10}$ cm^2/s. For small thicknesses the functional dependence of the entire correlation function agrees with Eq. (2). The solid line in Fig. 1(b) is a least-squares fit of Eq. (2) to the measured correlation function, shown as the points, using $L_a = 25$ cm and D as an adjustable parameter. The value of D obtained from the fit is $(2.9 \pm 0.2)\times 10^{10}$ cm^2/s. The mean velocity, v, in the medium can be estimated from the relation[10] $1/v = nf/c + (1-f)/c$, where n is the index of refraction of the spheres and f is the filling fraction. For polystyrene in the K band, $n = 1.59$. This gives $v = 2.3\times 10^{10}$ cm/s. Using the Boltzmann relation, $D = vl/3$, the transport mean free path is $l = 4.0$ cm and $kl = 25$.

The distribution of intensity values in 500 intensity spectra with 625 points each is shown in Fig. 1(c). The data show that $P(I/\langle I\rangle) = \exp(-\beta I/\langle I\rangle)$, where $\beta = 1 \pm 0.01$ and $\langle I\rangle$ is the average intensity. This is in

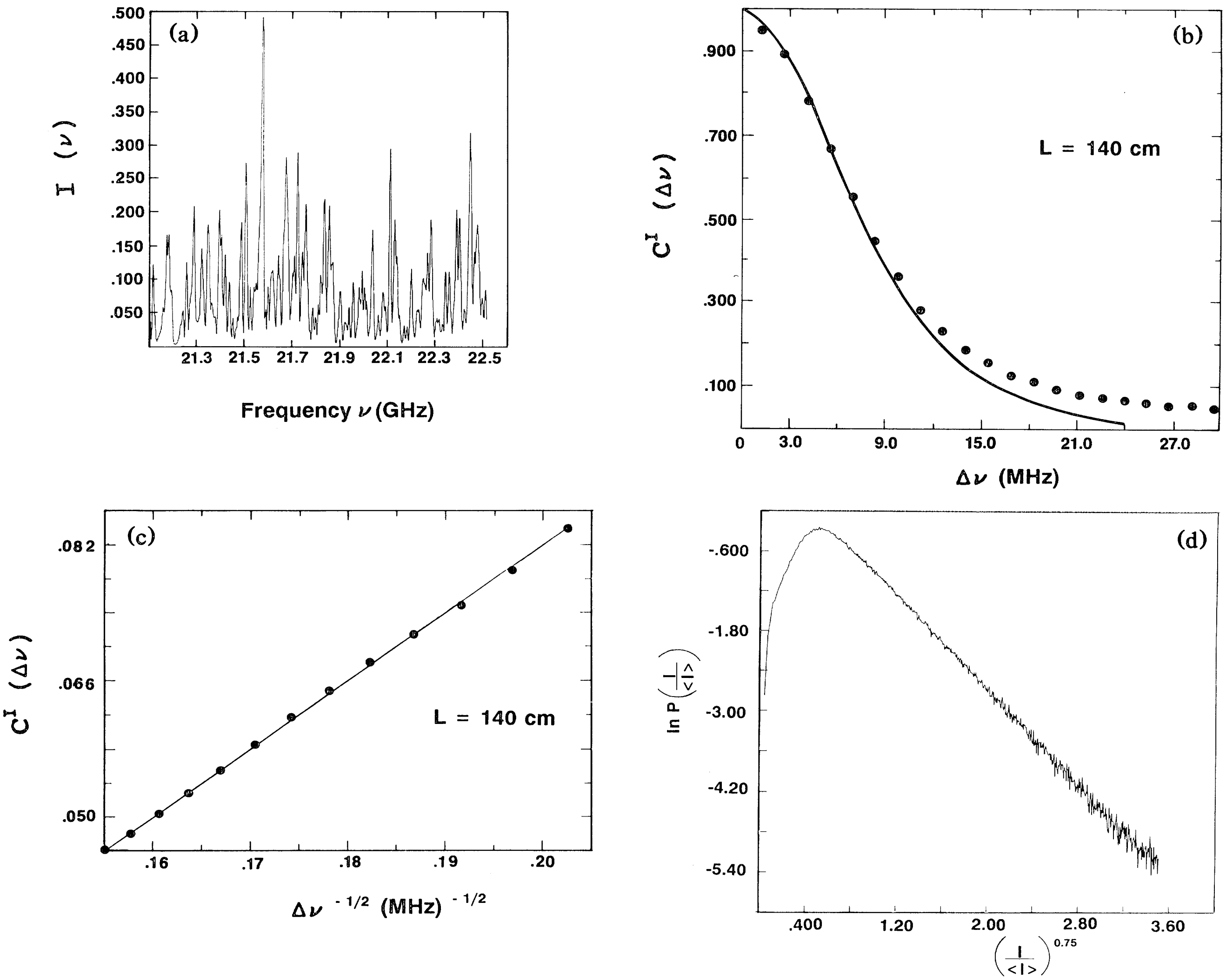

FIG. 2. (a) Spectral intensity fluctuations for $L=140$ cm. (b) Measured values (points) of $C^I(\Delta\nu)$ for $L=140$ cm. The solid line is a plot of Eq. (2) using the measured values of D and L_a. (c) The tail of the measured $C^I(\Delta\nu)$ shown in Fig. 1(b) vs $\Delta\nu^{-1/2}$. (d) Intensity distribution function vs $(I/\langle I\rangle)^{0.75}$.

excellent agreement with negative-exponential statistics, $P(I/\langle I\rangle)=\exp(-I/\langle I\rangle)$, expected in the absence of correlation between the amplitudes of different paths.[11]

As L increases, however, we find that there are increasing deviations from the uncorrelated-diffusion model. They are most pronounced for the largest value of L at which measurements were made, $L=140$ cm. In typical measurements of local intensity fluctuations at $L=140$ cm, the spectrum seems to float above the baseline as seen in Fig. 2(a). The virtual absence of low intensity values is seen in $P(I/\langle I\rangle)$, shown in Fig. 2(d). For larger values of I, $P(I/\langle I\rangle)$ does not fall exponentially but is best described as a stretched exponential, $P(I/\langle I\rangle)=\exp[-(I/\langle I\rangle)^\gamma]$ with $\gamma=0.75\pm0.05$. The prominence of large values of I suggests that large fluctuations play an increasingly important role as L increases. $C^I(\Delta\nu)$ is shown as the points in Fig. 2(b). The solid line is a plot of Eq. (2) using $D=3.1\times10^{10}$ cm^2/s and $L_a=25$ cm. Though $\Delta\nu_I$ is consistent with the prediction of the uncorrelated-diffusion model, the tail of $C^I(\Delta\nu)$ no longer decays exponentially but as $\Delta\nu^{-1/2}$ [see Fig. 2(c)]. This is the functional dependence of the first-order correction to $C^I(\Delta\nu)$ in $1/g$ or $1/\delta$ predicted by Stephen and Cwilich[1] and by Feng, Kane, Lee, and Stone.[2] It is the same falloff as predicted for the cross-correlation function of intensities at different points of the sample and is associated with long-range correlation resulting from the long-range nature of diffusion.[1,2] The same slow falloff is predicted for the correlation function of universal conductance fluctuations[12] which are dominated by long-range intensity correlation.

The change of intensity statistics as L increases is chartered in Table I. The predictions of the uncorrelated-diffusion model are $(I/\langle I\rangle)_m=0$, where $(I/\langle I\rangle)_m$

TABLE I. Tabulation of the change of intensity statistics as L increases and δ decreases. Terms are defined in the text.

L (cm)	$(I/\langle I\rangle)_m$	γ	$\Delta C^I/C^I$	δ
10	$\approx 0.02 \pm 0.02$	1.0 ± 0.05	≈ 0	668
20	0.08 ± 0.02	1.0 ± 0.05	≈ 0	172
30	0.10 ± 0.02	1.0 ± 0.05	≈ 0	112
40	0.14 ± 0.02	0.95 ± 0.05	≈ 0	87
60	0.15 ± 0.02	0.85 ± 0.05	0.4 ± 0.2	65
80	0.18 ± 0.02	0.85 ± 0.05	1.0 ± 0.2	56
100	0.20 ± 0.02	0.80 ± 0.05	1.3 ± 0.2	52
140	0.24 ± 0.03	0.75 ± 0.05	2.1 ± 0.2	47

is the value of $I/\langle I\rangle$ at which $P(I/\langle I\rangle)$ has its maximum value, and $\gamma = 1$. The increasing deviation of the cumulant of the measured intensity autocorrelation function, C^I_{expt}, from the predictions of the uncorrelated-photon-diffusion model in Eq. (2), C^I_{unc}, is exhibited in the parameter $\Delta C^I/C^I = (C^I_{\text{expt}} - C^I_{\text{unc}})/C^I_{\text{unc}}$ evaluated at a frequency shift $\Delta\nu = 3\Delta\nu_I$.

A natural parameter with which to describe the degree of correlation in the sample is the number of independent basis states represented in a wave at frequency ν. This is given by the product of the density of states and the level width, $(dN/d\nu)\delta\nu_E$, which is the parameter δ. The density of states for waves confined by reflecting side walls in a medium of cross-sectional area A is $dN/d\nu = 4k^2AL/\pi v$. $\delta\nu_E$ is obtained from Eq. (1) using the experimental values of D and L_a. The calculated values of δ for our sample are listed in Table I. We find that the enhancement of correlation as L increases is associated with the decrease in the value of δ. The observation of strong correlation occurs for large enough values of δ that D is not significantly renormalized.

The reduced probability of low intensity values may be associated with the transition from a wave with three-dimensional to one-dimensional characteristics as L increases. Calculations of local intensity statistics have so far only been carried out in the regime in which δ is large. Therefore, it is not possible at present to make a detailed comparison with theory.

In conclusion, we have shown that microwave studies can provide a complete statistical picture of propagation in random samples. Our results show the increasing influence of intensity correlation upon local intensity statistics as the dimensionless level width δ decreases. These results are not a special case related to the sample's geometry, $L \gg A^{1/2}$. They are universal characteristics which hold for any sample with the same value of δ.

We thank E. Kuhner for construction of the microwave sample holder, L. Ferrari for valuable discussions, and W. Polkosnik and J. Zhu for numerical calculations. This work was supported by Professional Staff Congress–City University of New York Research Awards and the Exxon Research Foundation.

[1]M. J. Stephen and G. Cwilich, Phys. Rev. Lett. **59**, 285 (1987).

[2]S. Feng, C. Kane, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. **61**, 834 (1988).

[3]A. Z. Genack and J. M. Drake (to be published); A. Z. Genack, in *Classical Wave Localization,* edited by P. Sheng (World Scientific, Singapore, 1989).

[4]P. E. Wolf, G. Maret, E. Akkermans, and R. Maynard, J. Phys. (Paris) **49**, 63 (1988); M. Kaveh, M. Rosenbluh, I. Edrei, and I. Freund, Phys. Rev. Lett. **57**, 2049 (1986).

[5]A. Z. Genack, Phys. Rev. Lett. **58**, 2043 (1987).

[6]R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985).

[7]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[8]D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977).

[9]B. Shapiro, Phys. Rev. Lett. **57**, 2168 (1986).

[10]P. Sheng and Z. Q. Zhang, Phys. Rev. Lett. **57**, 1879 (1986).

[11]J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985).

[12]P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).