Inner-Horizon Instability and Mass Inflation in Black Holes

E. Poisson and W. Israel
Canadian Institute for Advanced Research Cosmology Program, Theoretical Physics Institute, University of Alberta, Edmonton, Alberta, Canada T6G 2J1
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Gravitational collapse with rotation leaves a slowly decaying radiative tail which becomes infinitely blueshifted at the inner horizon of the resulting black hole. We study the gravitational effects of this on the inner structure of the hole, using a simple spherical model. In the presence of outflow from the collapsing star, the gravitational-mass parameter and the curvature are inflated at and within the inner horizon to values which, classically, are unlimited. Implications of this result are briefly discussed.

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In its external aspects, the end point of a gravitational collapse is rigidly specified by general relativity.[1] Subject to one unproven but plausible hypothesis (cosmic censorship), it is established that the external field relaxes to a Kerr-Newman black hole, and the radiative perturbations die out with advanced time according to an inverse-power law.

Hope that theory might prove equally informative about internal conditions in the hole and the ultimate fate of the collapsing material faded after Penrose[2] noted twenty years ago that the inner horizon which is expected to appear in a collapse with angular momentum has pathological features which present a barrier to predictability. In the stationary (Kerr-Newman) state to which the external geometry tends asymptotically, the inner horizon is an intersecting pair of lightlike three-sheets inside the event horizon. The ingoing sheet, corresponding to constant (and infinite) advanced time, acts as a Cauchy horizon—a boundary beyond which the future evolution of physical fields is no longer uniquely determinable from initial data prescribed at the onset of collapse. What is more, the Cauchy horizon is highly unstable to time-dependent exterior perturbations. A test field in the form of an initially uniform train of waves propagating into a Kerr-Newman black hole has its crests crowded together and magnified by gravitational and Doppler blueshifts that grow without bound at the Cauchy horizon. Such perturbative results strongly suggest (though they do not prove) that inside a black hole formed in a generic collapse, an observer falling toward the inner horizon should be engulfed in a wall of (classically) infinite density immediately after seeing the entire future history of the outer universe pass before his eyes in a flash.

Repeated confirmation that radiative perturbations diverge to linear order at the Cauchy horizon has come from a long series of investigations.[3] But to date there has been no attempt to analyze the perturbations beyond linear order, or to examine how their growth deforms the background geometry through gravitational effects. This Letter reports briefly on a first attempt to tackle both of these questions.

The problem becomes tractable if one considers a simple model: A charged, spherical (Reissner-Nordström) black hole perturbed by crossflowing radial streams of infalling and outgoing lightlike particles. This model is very idealized, but there are good reasons for believing that it captures the essential physics. In the first place, the causal and horizon structures of the Reissner-Nordström and Kerr black holes are known to be very similar.[1] Secondly, the large blueshift of infalling gravitational waves means that high-frequency components will dominate near the Cauchy horizon, so that Isaacson's "effective stress-energy" description[4] for the waves (in effect, the "optical," graviton approximation) should be an adequate approximation.

We begin by setting up the basic equations of the problem. The field equations for spherical symmetry are most conveniently expressed in a form covariant under arbitrary transformations of the coordinates x^a $(a,b,\ldots=0,1)$ in a "radial" two-space $(\theta,\varphi)=$const.[5] The spacetime metric is

$$ds^2=g_{ab}dx^a dx^b+r^2 d\Omega^2,$$

where g_{ab} and r are functions of x^a. We now define functions $f(x^a)$, $m(x^a)$ by

$$1-2m/r+e^2/r^2=f=g^{ab}(\partial_a r)(\partial_b r), \tag{1}$$

where e can be identified with the electric charge within radius r; here we take it for simplicity to be a constant (no electric current).

The Einstein field equations are then contained in

$$\partial_a m=4\pi r^2 T^b_a \partial_b r, \quad r_{;a}{}^b+\kappa\delta^b_a=-4\pi r T^b_a, \tag{2}$$

which imply the conservation law $(r^2 T^b_a)_{;b}=0$. We have defined

$$-\kappa(r,m)=\tfrac{1}{2}\,\partial_r f(r,m)=(m-e^2/r)/r^2,$$

which resembles a Newtonian gravitational force; the semicolon denotes covariant differentiation with respect to the two-metric g_{ab}. T^b_a is the nonelectron part of the energy density and flux which we assume to be traceless. It is to be supplemented by the electric stress-energy $(T_\alpha^\beta)_{\rm el}=(e^2/8\pi r^4){\rm diag}(-1,-1,1,1)$ [which is separately conserved by virtue of (2)] to form the total energy-momentum tensor.

Equations (2) yield two-dimensional wave equations for the key variables:

$$\Box r \equiv r_{;a}{}^{a} = -2\kappa, \quad \Box m = -r^{-1}(4\pi r^2)^2 T^{ab}T_{ab}. \tag{3}$$

In this form the equations can be (formally) integrated: The general prescription is that $\Box\psi=\varphi(x^a)$ has the solution

$$\psi_A = -\tfrac{1}{2}\int_{ABCD}\varphi\, dS + \psi_C + \psi_D - \psi_B \tag{4}$$

for the value of ψ at any point A. The integration is over the interior of a lightlike rhombus in radial two-space whose vertices are A,B,C,D (in clockwise order, beginning at noon).

Let us now turn to the specification of our physical model. Inside a black hole formed in a realistic collapse the space between the horizons is exposed to a shower of infalling gravitational radiation, whose externally measured intensity falls off like v^{-p} when the advanced time $v\to\infty$, where $p=4(l+1)$ for a multipole of order l.[1] Simultaneously, the inner (Cauchy) horizon is irradiated by a stream of electromagnetic and gravitational radiation, in part flowing directly out of the collapsing star and in part backscattered from the infall.

In our simplified model, we consider a spherisymmetric black hole having a fixed charge e and subjected to radial ingoing (flowing toward the left in Fig. 1) and going (rightward) lightlike fluxes $T_L^{ab}=\rho_L l^a l^b$ and $T_R^{ab}=\rho_r n^a n^b$, respectively. The radial lightlike vectors l^a, n^a each satisfy relations of the form $l^a l_a = l^a{}_{;b} l^b = l^a{}_{;a} = 0$, and are still arbitrary up to scale factors conserved along their rays. We suppose for simplicity that the two streams interpenetrate without interacting, so that T_R^{ab}, T_L^{ab} are separately conserved.

Accretion due to the inflow makes the mass of the hole, as measured in our universe, an increasing function $m_L(v)$ of advanced time with asymptotic limit m_0 (say), so that $m_0 - m_L(v) \sim v^{-(p-1)}$ $(v\to\infty)$. We introduce

$$r_0 = m_0 - (m_0^2 - e^2)^{1/2},$$
$$\kappa_0 = (m_0^2-e^2)^{1/2}/r_0^2, \quad V = -e^{-\kappa_0 v},$$

the radius, surface gravity, and Kruskalized advanced time associated with the inner horizon of the static Reissner-Nordström black hole which the external field is approaching.

As coordinates with reasonable behavior in a neighborhood of the Cauchy horizon we select V and a Kruskalized retarded time U (equal to $-\infty$ on the event horizon) based on the outer horizon of the black hole in its initial static configuration (mass m_1, surface gravity κ_1) prior to the moment (U_1,V_1) when the fluxes were

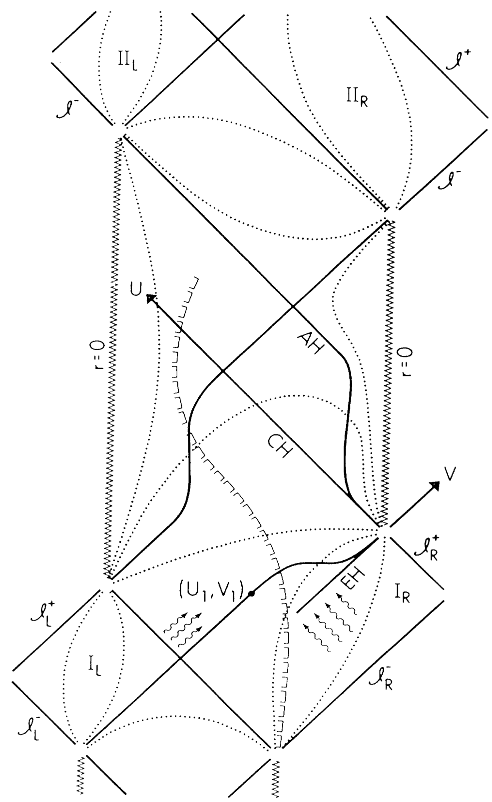

FIG. 1. Analytically extended exterior geometry of collapsing star, with radiative influx and outflux. Section to left of star's boundary (stapled curve) bears not resemblance to physical spacetime. Shown are the event horizon (EH), Cauchy horizon (CH), inner apparent horizon (AH), and some curves of constant r (dotted). To make the figure legible, the degree of deflation of AH has been toned down, and the outflux turned on for just a short interval.

turned on. If we imagine the exterior manifold analytically extended to the left (ignoring the presence of the collapsing star), U would be related to advanced time $\bar v$ in the asymptotically flat sector marked I$_L$ in Fig. 1 by $U-U_1=e^{\kappa_1 \bar v}$.

The conservation laws imply that $r^2\rho_R$ and $r^2\rho_L$ depend only on U and V, respectively. Choosing $n_a = -\partial_a U$, $l_a = -\partial_a V$, we can set

$$4\pi r^2 \rho_L = L_L(V) = \left(\frac{dv}{dV}\right)^2 \frac{dm_L(v)}{dv},$$
$$4\pi r^2 \rho_R = L_R(U) = \left(\frac{d\bar v}{dU}\right)^2 \frac{dm_R(\bar v)}{d\bar v}. \tag{5}$$

The quantity $m_R(\bar v)$ can be interpreted (within its domain of definition) as the gravitational mass of the hole as measured[6] in universe I$_L$, growing with advanced time $\bar v$ because of radiative inflow from $\mathcal{J}_L^-$.

We begin by setting $\rho_R=0$ and considering a pure inflow which continues up to the Cauchy horizon $V=0$, and then turns off. In this case, the metric takes a simple explicit form[7] in terms of Eddington coordinates (v,r):

$$ds^2=2\,dr\,dv-f\,dv^2+r^2d\Omega^2$$

with f given by (1) and $m=m_L(v)$. From (5)

$$L_L(V)=\dot m_L(v)(\kappa_0 V)^{-2}$$
$$\sim|\ln(-V)|^{-p}V^{-2}\quad (V\to 0-)\,. \qquad (6)$$

This implies that a physical observer crossing the Cauchy horizon encounters an energy density and flux diverging like (6).

Despite this divergence, the perturbation of the metric, δg_{UV}, remains bounded and falls to zero at the Cauchy horizon like $m_0-m_L(v)\sim|\ln(-V)|^{-(p-1)}$. This is not a trivial conclusion. The verification is straightforward but lack of space precludes a sketch of it here. It is at least obvious that the second V derivative of the perturbed metric does indeed reproduce curvatures of order (6).

It needs to be emphasized that this seemingly plausible and routine result is actually something of a miracle. The black hole is absorbing radiation blueshifted to arbitrarily large energies (6), and the integrated energy absorbed diverges. One should have expected gravitational effects of this energy to boost the gravitational-mass parameter $m(U,V)$ to arbitrarily large values, entailing a gross divergence of the metric. Of course, observers in our universe I_R cannot become aware of any such mass inflation, since we are divorced from causal contact with the Cauchy horizon on which it occurs. However, it should be observable (and, indeed, extractible) in asymptotically flat universes (such as $\mathrm{II}_L,\mathrm{II}_R$ in Fig. 1) which succeed ours in the analytically extended Reissner-Nordström lattice, and in which m reacquires its direct operational meaning as gravitational mass. Nevertheless, we actually have $m=m_0$ for $V>0$ and there is no sign of mass inflation. But this is merely an accident of our specialized model of pure inflow tapering off at late times. In this case the Cauchy horizon is static and coincident with the associated apparent horizon (defined generally, in the context of spherical symmetry, as the three-space $f=0$ on which a spherical light wave propagating leftward is momentarily stationary). A static surface of this kind serves in two capacities: as infinite blueshift surface for our universe and infinite redshift surface for the succeeding universe. Conservation of gravitational mass is thus achieved in this special instance by a fortuitous cancellation of infinite blueshift and redshift. However, there is no general law or principle enforcing conservation of gravitational mass when general relativity is applied to a latticelike extension of a conventional universe in which spatial sections are alternately open and closed. The gravitational field inside a black hole, like the Newtonian field, is a bottomless source of energy.

In the presence of a concurrent outflow of energy, $\rho_R>0$, the situation is radically altered. Irradiation of the Cauchy horizon gravitationally focuses its generators, inducing contraction. The apparent horizon contracts faster (in fact, it deflates catastrophically) and peels off from the Cauchy horizon. The latter remains an infinite blueshift surface for material entering the hole from our universe, but its role as infinite redshift surface for the succeeding universe is now taken over by the apparent horizon in its final, deflated stationary configuration (Fig. 1). The blueshift at the Cauchy horizon can no longer be neutralized by a redshift, and mass inflation will occur.

We proceed to the mathematical description of this phenomenon. We suppose the influx and outflux turned on at times V_1 and U_1, respectively, so that $m(U,V)$ reduces to $m_L(v)$ for $U\le U_1$ and to $m_R(\bar v)$ for $V\le V_1$, with $m_L(v)=m_R(\bar v)=m_1$ in the initial static state preceding (U_1,V_1). Since our basic concern is with effects near the Cauchy horizon rather than $\mathcal{I}_R^+$, we may conveniently assume $U_1>0$, i.e., that the outflux turns on after the event horizon is crossed.

We apply (3) and (4) to a lightlike rhombus whose outermost past and future vertices are (U_1,V_1) and an arbitrarily point (U,V) near the Cauchy horizon. In our coordinates, the two-metric has a null form, $-2e^{2\sigma}\times dU\,dV$, so we find

$$m(U,V)=\int_{U_1}^{U}\int_{V_1}^{V}(r'e^{2\sigma'})^{-1}L_R(U')L_L(V')\,dU'\,dV'$$
$$+m_L(v)+m_R(\bar v)-m_1\,, \qquad (7)$$

with obvious abbreviations $\sigma'=\sigma(U',V')$, etc.

It is clear from (7) and (6) that, for positive outflux $L_R(U)$, $m(U,V)$ becomes unbounded as $V\to 0$ unless $\psi\equiv re^{2\sigma}$ happens to approach infinity rapidly enough in this limit. But it is easy to show this cannot happen: From $\Box\sigma=-\frac{1}{2}\,{}^{(2)}R=-\partial_r\kappa(r,m)$ and (3) we find $\Box(\ln\psi)=(3e^2-r^2)/r^4$, so that

$$\ln\psi(U,V)=\ln[\psi_L(U_1,V)\psi_R(U,V_1)/\psi_L(U_1,V_1)]$$
$$-\tfrac{1}{2}\int\!\!\int\psi'(3e^2-r'^2)r'^{-5}\,dU'\,dV'\,.$$

Now, $\psi_L(U_1,V)$ is bounded by finiteness of the pure-inflow metric at the Cauchy horizon. The contribution of the second (integral) term, if it were not bounded, would have to be negative, since it would then by dominated by values of the integrand near the Cauchy horizon, where $r'<r_0<|e|$, so that the integrand is positive. Therefore the only possibility is that the left-hand side is bounded above.

Thus we have shown that, in the presence of an outflux, the radiative tail (6) generates unbounded mass inflation at the Cauchy horizon: $m(U,V)\to\infty$ for $V\to 0$, $U>U_1$.

Equation (7) provides a ready estimate of the growth rate. If the influx (measured far from the Cauchy horizon) and the outflux are assumed small, we can expand in powers of the perturbations L_L, L_R about the static Reissner-Nordström background with mass m_0, neglecting terms beyond the bilinear order $L_L L_R$. This yields the crude estimate

$$m(U,V) \sim m_0 \epsilon^2 \delta(U) e^{\kappa_0 v} (v/m_0)^{-12},$$

in which ϵ is a dimensionless quadrupole moment of the collapsing star, and $\delta(U)$ is the fraction of its rest mass radiated outward between the moment it enters the event horizon and the retarded time U. (We assume $|e| \sim m_0$, so that inner and outer static horizons are not vastly different in size.) Given this characteristic growth time, $\Delta v \sim \kappa_0^{-1} \sim m_0$, the curvature m/r^3 must reach Planckian values (as one approaches the Cauchy horizon at fixed U) in scarcely more than $\sim 10^2 G m_0/c^3$ sec of external time after the collapse, even on the most conservative estimates. In sharp contrast to the naive picture, in which one expects the curvature to revert to moderate values m_0/r_0^3 after one has passed through the superdense wall at the Cauchy horizon, mass inflation implies that this rise of curvature is irreversible.

We have drawn far-reaching conclusions from a highly idealized, spherical model. However, our qualitative discussion shows that the mechanism of mass inflation basically just depends on two quite general features: the infinite blueshift at the Cauchy horizon, and the separation of Cauchy and apparent horizons under irradiation by a transverse flux. We therefore see little reason to doubt that our conclusions should remain qualitatively valid for a generic, rotating black hole formed in a collapse.

Mass inflation forces e/m and J/m to approach zero near the Cauchy horizon, since charge and angular momentum J are conserved. Thus, the classical geometry becomes of Schwarzschild-type, and speculations[8] about the "nuclear region," $r \lesssim (\hbar G^2 c^{-5} m)^{1/3}$, of Schwarzschild black holes become pertinent, though now at the enormously larger radius of the inner horizon. The (classical) proper time required to reach $r = 0$ from the Cauchy horizon is, however, only about a Planck time. Even if he were impregnable to enormous blueshifts, our infalling observer witnessing the end of our universe would himself be (at least according to classical theory) only 10^{-43} sec from his own end.

In its simplest schematic form, mass inflation emerges from the Dray–'t Hooft–Redmount[9] (DTR) relations for the collision of two lightlike shells near a horizon. In fact, it was through a generalized form of these relations that this phenomenon—one of the most remarkable manifestations of the nonlinear aspects of general relativity—first suggested itself to us. Shortly afterward, Blau kindly informed us of his very interesting independent and parallel work,[10] in which the DTR relations are employed to elucidate Eardley's[11] scenario for the "death" of white holes. We are indebted to him for discussions.

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[6] Although mere mathematical fictions, sectors like I_L in the analytic extension of the exterior manifold are conceptually useful for clarifying paradoxical features of the internal physics of the hole; for example, how radiation flowing out of the star can cause the gravitational mass to grow between the two horizons, even though the star is losing mass.

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