Comment on "Theory of Electronic Diamagnetism in Two-Dimensional Lattices"

The Letter by Hasegawa *et al.*[1] (HLRW) shows two results. First, it calculates numerically the density of states (DOS) of tight-binding electrons in two dimensions with a magnetic field, for simple values of the magnetic flux $\Phi = p/q$ in a plaquette. Then, it gives numerical evidence that, for a fixed electron filling ν, the total energy E_T of the electrons has an absolute minimum when $\Phi = \nu$. However, the DOS and the total energy have been computed for a very restricted number of values of the flux. We show here that E_T can be evaluated very simply and that it exhibits local cusplike minima whenever $\nu = M + N\Phi$.

The DOS has been calculated analytically by Wannier, Obermair, and Ray.[2] When $\Phi = p/q$, it can be written as

$$\rho(E) = \frac{1}{2\pi^2 q}\left|\frac{dF}{dE}\right| K'\left(\frac{F}{4}\right), \tag{1}$$

where $K'(k) = K(1-k^2)^{1/2}$ and K is the complete elliptic integral of the first kind. $F(E)$ is a polynomial of order q. It can be written as

$$F = \det\begin{bmatrix} M_1 & -1 & & 0 & +1 \\ -1 & M_2 & \ddots & & \\ & & & & \\ 0 & & -1 & M_{q-1} & -1 \\ +1 & 0 & & -1 & M_q \end{bmatrix}, \tag{2}$$

with $M_n = -E - 2\cos 2\pi n\Phi$. The DOS exhibits a logarithmic singularity at each half-filled subband (Fig. 1).

When q increases, the DOS in each subband is more and more peaked so that it can be approximated by a δ function. For large q, the DOS is $\rho(E) = \sum_{n=1}^{q} \delta(E - E_n)/q$, where the E_n are solutions of Eq. (2). We use this form to calculate E_T. For small values of q, the exact formulation must be used, as it has been done by HLRW. Figure 2 shows E_T vs Φ at a fixed value of the filling ν. On the same figure the seven points computed by HLRW are plotted. The present result fully confirms

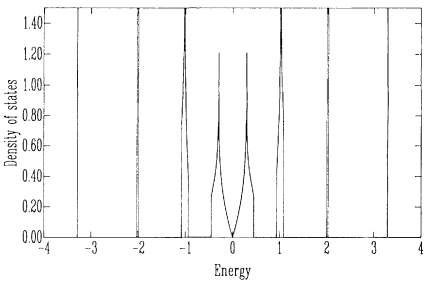

FIG. 1. The DOS $\rho(E)$ when $\Phi = \frac{1}{8}$.

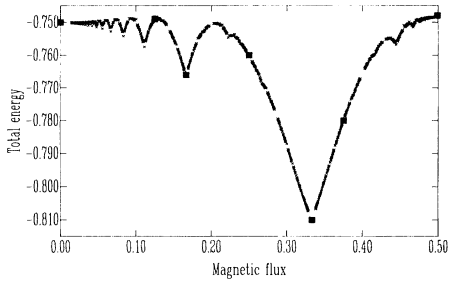

FIG. 2. $E(\Phi)$ for $\nu = \frac{1}{3}$. Only the points with $8 < q < 70$ are presented.

that there is an *absolute* minimum of E_T when $\Phi = \nu$. This minimum is *cusplike*. Figure 3 shows the variation of this minimum versus ν. It is always lower than the zero-field energy. I also find local cusplike minima whenever $\nu = M + N\Phi$, $M, N \in \mathbb{Z}$. These minima result from the hierarchical gap structure of the spectrum.[3]

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[2] G. H. Wannier, G. M. Obermair, and R. Ray, Phys. Status Solidi (b) **93**, 337 (1979).
[3] G. H. Wannier, Phys. Status Solidi (b) **88**, 757 (1978).

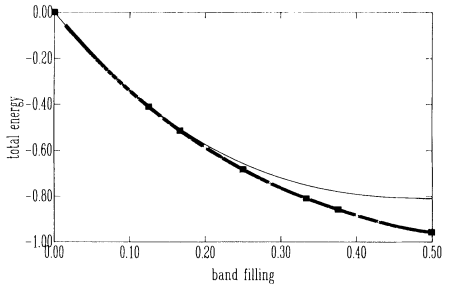

FIG. 3. Variation with the band filling ν of the minimum of the total energy obtained when $\Phi = \nu$. The squares are the points found by HLRW. The thin line is the total energy $E(\nu)$ in zero field $\Phi = 0$.

