Kapitza Resistance and Thermal Transport across Boundaries in Superfluid ^{3}He

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We calculate the temperature and normal-fluid velocity of superfluid ^{3}He between parallel plates in the presence of a stationary heat flow normal to the plates. The system is modeled by a Landau-Boltzmann equation and a diffuse scattering mechanism at the boundaries. In the hydrodynamic regime the temperature jump at the wall turns out to be small. The three Onsagar surface coefficients proposed recently by Grabinski and Liu are determined. In the Knudsen regime the thermal boundary resistance is found to increase exponentially with decreasing temperature.

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There has been considerable interest in recent years in the flow of quantum liquids in restricted geometries.[1] For one, the interpretation of flow experiments in terms of bulk properties of the liquid requires knowledge of the effect of boundaries. Second, the nature of the interaction of a quantum liquid with a solid surface is an interesting problem in its own right, which is still largely unexplored. One may distinguish two different situations, depending on whether the influence of the interaction of the particles in the liquid with the boundary disturbs the thermodynamic equilibrium state of the liquid over a shorter or longer range. For flow channels of extension large compared to the mean free path l of thermal excitations a hydrodynamic description supplemented by appropriate macroscopic boundary conditions is adequate. In the opposite limit, the so-called Knudsen regime, a microscopic description in terms of a distribution function for thermal excitations becomes necessary.

While transverse flow has been studied extensively, including the effect of different types of interaction of the thermal excitations with the channel walls for both normal and superfluid quantum liquids,[2,3] flow in the form of thermal counterflow in a superfluid has not yet attracted similar attention. In this Letter we calculate for the first time the macroscopic boundary conditions and the thermal boundary resistance in the hydrodynamic regime, as well as the thermal counterflow in the Knudsen regime, for the simplest model of a surface, the diffuse scattering model.

It was shown recently by Grabinski and Liu[4] that even in the hydrodynamic regime the macroscopic boundary conditions for the case of thermal flow are more complex than previously thought.[5] Two related aspects of this are (i) that the entropy flow f in a superfluid is the sum of a convective and a diffusive component $f = sv_n - (\kappa/T)dT/dx$ (s entropy density, κ thermal conductivity) and (ii) that in the static limit, the temperature $T(x)$ and normal velocity $v_n(x)$ in the presence of a stationary entropy flow f vary according to (linearized) hydrodynamics as[6] $T - T_\infty = T_s^- e^{x/\lambda} + T_s^+ e^{-x/\lambda}$ and $sv_n = \bar{\kappa}(T - T_\infty) + f$. Here λ is a characteristic length associated with counterflow, given by $\lambda^2 = \alpha\kappa/s^2 T$, where $\alpha = \frac{4}{3}\eta - 2\zeta_1\rho + \zeta_2 + \zeta_3\rho^2$ is a combination of viscosity coefficients,[5] ρ is the mass density, and $\bar{\kappa} = \kappa/T\lambda$.

Accordingly, the temperature profile across a solid wall shows a jump at the wall (within the Knudsen layer of thickness $\sim l$), followed by an exponential decay region of width λ ("λ regime"). A quick estimate of λ shows that $\lambda \simeq l$ for superfluid ^{4}He, but $\lambda \simeq (T_F/T) l \gg l$ for superfluid ^{3}He.

Thus, for ^{4}He the λ regime cannot be separated from the Knudsen regime and it is perfectly reasonable to introduce a single temperature jump as done usually.[5] In the case of ^{3}He, on the other hand, the question is how the entropy flow is divided up into the convective and diffusive parts and which fraction of the total temperature jump (and even its sign) is due to the exponentially varying parts ($T_s^\pm$). This is determined by the boundary conditions and has been calculated here for diffuse scattering of quasiparticles at the wall.

The general hydrodynamic boundary condition derived by Grabinski and Liu[4] relate the entropy flow f_0 and the superfluid mass flow j_s (in the rest frame of the normal fluid) at the boundary to the temperature jump ΔT and the normal-fluid velocity gradient (dv_n/dx) is

$$\begin{pmatrix} f_0 \\ j_s \end{pmatrix} = \begin{pmatrix} a & ca/\rho \\ c & ba/\rho \end{pmatrix} \begin{pmatrix} \Delta T \\ dv_n/dx \end{pmatrix}, \tag{1}$$

where $j_s = -\rho v_n$ for an impenetrable wall. The form of the boundary condition (1) will be derived below from microscopic theory and the surface Onsager coefficients a,b,c will be calculated below for an isotropic Fermi superfluid such as ^{3}He-B.

This requires, in principle, a microscopic description of the interaction of the particles of the superfluid with the boundary. At present a microscopic theory on the atomic scale is not feasible. However, we expect simple models of the surface such as the diffuse scattering model or the specular scattering model to account for the qualitative effects of the surface. In the following we will use the diffuse scattering model, but our calculations may be readily extended to other scattering laws at the boundary (see, e.g., Ref. 3). In the superfluid state, the interaction of (Bogoliubov) quasiparticles with a surface may be strongly affected by the possible distortions of the gap parameter near the surface.[7] These effects will be neglected here for simplicity.

The dynamics of a weakly excited superfluid on length scales large compared to the coherence length are described by a linearized Landau-Boltzmann equation for the distribution function of quasiparticles f_p or rather its deviation from equilibrium, $\delta f'_p = f_p - f^0_p - (\partial f^0_p/\partial E_p)\delta E_p$. In the single-relaxation-time approximation for the collision integral one has in a stationary state (flow in the x direction)

$$v_{px}\frac{d\delta f'_p}{dx} = -\frac{1}{\tau}\left[\delta f'_p + \left(\frac{\partial f^0_p}{\partial E_p}\right)\left(p_x v_n + E_p\frac{\delta T}{T}\right)\right]. \quad (2)$$

Here E_p, $\mathbf{v}_p = \nabla_p E_p$, f^0_p, and τ are the quasiparticle energy, group velocity, equilibrium distribution function, and collision time, respectively. For a start, we adopt the simplest boundary condition on $\delta f'_p$, i.e., quasiparticles emitted by the wall are assumed to be in local equilibrium with the wall. The corresponding distribution function is given by $\delta f^{(+)}_p(x_w) = -(df^0_p/dE_p)(E_p/T)\delta T_w$, with δT_w the temperature of the wall. It follows from (2) that $\delta f'_p$ is completely determined by the velocity and temperature fields $v_n(x)$ and $\delta T(x)$, and given by

$$\delta f'_p = \begin{cases} e^{-\lambda_p(x+L/2)}\delta f^{(+)}_p(-L/2) + \int^x_{-L/2}dx' e^{-\lambda_p(x-x')}Q_p(x'), & v_{px}>0, \\ e^{-\lambda_p(x-L/2)}\delta f^{(+)}_p(L/2) + \int^x_{L/2}dx' e^{-\lambda_p(x-x')}Q_p(x'), & v_{px}<0, \end{cases} \quad (3)$$

where

$$Q_p(x) = -\lambda_p(\partial f^0_p/\partial E_p)(p_x v_n + E_p\delta T/T)$$

and $\lambda_p = (v_{px}\tau)^{-1}$.

The variables v_n and δT, in turn, obey a set of two coupled linear integral equations, obtained from the conservation laws for momentum and energy,

$$\frac{d}{dx}\sum_p p_x v_{px}\delta f'_p(x) = 0, \quad \frac{d}{dx}\sum_p E_p v_{px}\delta f'_p(x) = 0, \quad (4)$$

which for the parallel-plate geometry $(-L/2 \le x \le L/2)$ take the form

$$\begin{aligned} &\left[K_1\left(x+\frac{L}{2}\right) - K_1\left(\frac{L}{2}-x\right)\right]v_n\left(-\frac{L}{2}\right) + \int^{L/2}_{-L/2}dx'\left[K_1(|x-x'|)\frac{dv_n(x')}{dx'} - M_0(|x-x'|)\frac{\delta T(x')}{T}\right] \\ &\qquad = \left[M_1\left(x+\frac{L}{2}\right) - M_1\left(\frac{L}{2}-x\right)\right]\frac{\delta T_L}{T}, \\ &\left[L_1\left(x+\frac{L}{2}\right) + L_1\left(-x+\frac{L}{2}\right) - 2L_1\left(\frac{L}{2}\right)\right]\frac{1}{T}\left[\delta T\left(-\frac{L}{2}\right) - \delta T_L\right] \\ &\qquad + \int^{L/2}_{-L/2}dx'\left\{[L_1(|x-x'|) - L_1(|x'|)]\frac{1}{T}\frac{dT(x')}{dx'} - [M_0(|x-x'|) - M_0(|x'|)]v_n(x')\right\} = 0. \end{aligned} \quad (5)$$

Here the integral kernels are defined by

$$\begin{pmatrix} K_n \\ M_n \\ L_n \end{pmatrix} = \sum_{\substack{p \\ v_{px}>0}} \left(-\frac{\partial f^0_p}{\partial E_p}\right) v_{px}(v_{px}\tau)^{n-1} e^{-\lambda_p x} \begin{pmatrix} p_x^2 \\ p_x E_p \\ E_p^2 \end{pmatrix}. \quad (6)$$

Temperature changes are defined with respect to the midpoint temperature $(x=0)$, and $2\delta T_L$ is the difference in the temperature of the walls at $x=-L/2$ and $x=L/2$ (note, $x_w = \pm L/2$).

We have solved (3) numerically for an isotropic Fermi superfluid, where $E_p^2 = v_F^2(p-p_F)^2 + \Delta^2$, for a range of values of the ratio L/l. In Fig. 1 the temperature profile is shown for the three regimes described above. The exponential decay of δT for $L \gg \lambda$ is clearly seen. For $l \ll L \ll \lambda$ the x dependence is almost linear. Only in the Knudsen regime $(L \ll l)$ does a discontinuity ΔT at the wall develop.

In order to determine the temperature dependence one needs to know the relaxation time τ as a function of T.

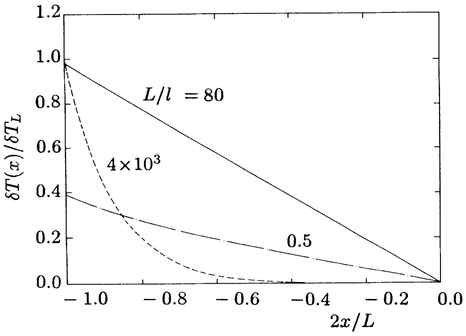

FIG. 1. Temperature profile of superfluid ^{3}He-B at $T = 0.5T_c$ in a slab of width L for three values of L/l (l is the mean free path).

We have used the following approximate expressions for $\tau(T)$ and $\Delta(T)$ given in Ref. 8,

$$\tau(T) = \tau(T_c)\exp[\Delta(T)/T]\,,$$

$$\Delta(T) = \Delta(0)\tanh\left\{\frac{\pi}{\delta_{sc}}\left[\frac{2}{3}\frac{\Delta C}{C_N}\left(\frac{T_c}{T}-1\right)\right]^{1/2}\right\},$$

with the weak coupling values $\delta_{sc} \equiv \Delta(0)/T_c = 1.76$ and $\Delta C/C_N = 1.43$. By fitting the numerical results in the limit $L/l \gg 1$ to the hydrodynamic result calculated employing the boundary condition (1) we have determined the Onsager coefficients a,b,c. The resulting values for a, $b\sigma^2$, and $c\sigma$ in units of $\frac{1}{2}sv_F/T_F$ are plotted as a function of $\Delta(T)/T$ in Fig. 2 ($\sigma = s/\rho$ is the entropy per mass). The coefficient a is seen to dominate at all temperatures. It is found to increase with decreasing temperature as $T^{-3/2}$. The coefficient $b\sigma$ is small and positive, whereas $c\sigma^2$ is of similar magnitude and negative. This leads to (i) the temperature jump at the wall being negligible compared to the amplitude $T_s^{+,-}$ of the exponential decay in the λ layer and (ii) the boundary condition at the wall being $f \cong -(\kappa/T)dT/dx$.

The thermal resistance $R_T = 2\delta T_L/Tf$ across a slab of thickness $L \gg l$ is then found as $R_T = (2\lambda/\kappa)\tanh(L/2\lambda)$. In the limit of wide slabs, $L \gg \lambda$, we find $R_T = (2/s) \times (a/\kappa T)^{1/2}$; i.e., the dependence on the mean free path l cancels out of the ratio a/κ. The T dependence of R_T is seen to be dominated by the entropy density s, which leads to an exponentially growing thermal resistance at low T,

$$R_T = \tfrac{2}{3}(\pi^3/2)^{1/2}[T_c/C_N(T_c)](T\Delta)^{-1/2}(a/\kappa T)^{1/2}e^{\Delta/T}\,, \tag{7}$$

where $C_N(T_C)$ is the specific heat in the normal state at T_c and $(a/\kappa T)^{1/2} \sim T_F/T_c v_F$ is a T-independent limiting value.

In the intermediate regime defined by $l \ll L \ll \lambda$ the thermal resistance becomes proportional to L and inversely proportional to the thermal conductivity, $R_T = L/\kappa$. Since κ (according to theory) is approximately proportional to $1/T$ in the whole temperature range, we predict $R_T \propto T$ in this regime. In other words, the thermal resistance R_T is expected to decrease with decreasing temperature, until the mean free path increases sufficiently to become comparable with L.

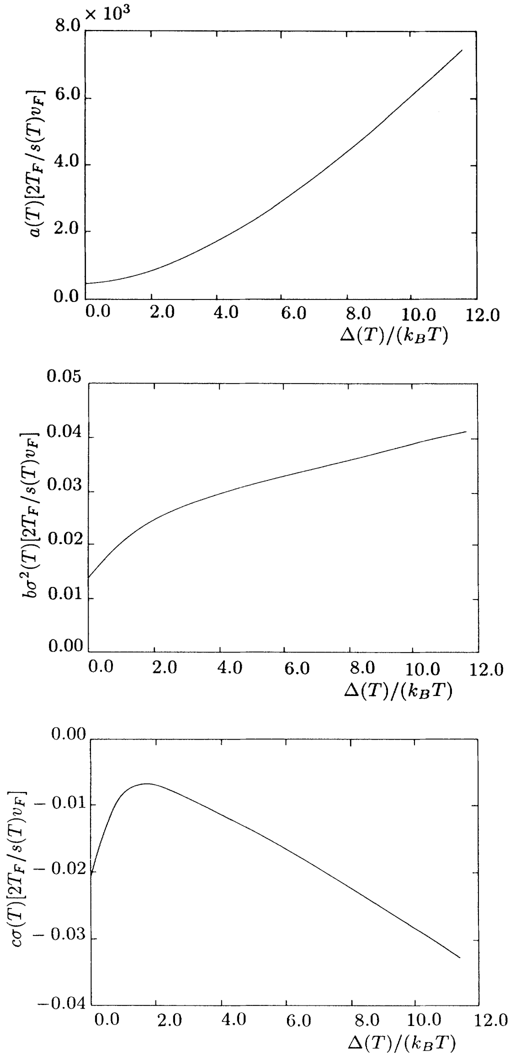

FIG. 2. Surface Onsager coefficients a,b,c as defined in (1) as a function of temperature. Plotted are the normalized quantities $a(2T_F/sv_F)$, $b\sigma^2(2T_F/sv_F)$, and $c\sigma(2T_F/sv_F)$ vs $\Delta(T)/T$.

For $L \ll l$, in the Knudsen regime, the hydrodynamic description breaks down. In Fig. 3, $\ln R_T$ is shown as a function of T_c/T for a slab of width $L = 10l(T_c)$. In the neighborhood of T_c we find $R_T \propto T$ as discussed above. Below the crossover temperature $T_x \simeq 0.5T_c$ into the Knudsen regime [defined by $L = l(T_x)$] we find $R_T \propto \exp(\Delta/T)$.

Also shown in Fig. 3 is the total thermal resistance for

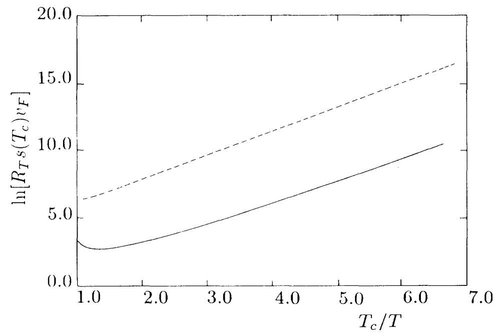

FIG. 3. Thermal resistance R_T across a slab of width $L = 10l(T_c)$ (solid line) and boundary resistance for a single surface bounding a half-space volume of ^{3}He-B (broken line). Plotted is the logarithm of $R_T s(T_c) v_F$ vs T_c/T.

a single boundary. The result is indistinguishable from a straight line with slope $\Delta(T=0)/T_c$, i.e., $R_T \propto \exp[\Delta(T=0)/T]$, over the complete temperature range.

It is also interesting to discuss the proper Kapitza resistance (for a review, see Ref. 9) associated with the temperature discontinuity ΔT at the boundary, which is usually defined as $R_K \equiv \Delta T/T f$. Since ΔT turned out to be small except in the Knudsen regime, R_K will, in general, be small. In terms of the surface Onsger coefficients a,b,c one may express R_K as $R_K \simeq (aT)^{-1}$, using $a \gg b\sigma^2, c\sigma$.

Experimental data[10,11] appear to be consistent with $R_T \propto \exp(\Delta/T)$, although an accurate comparison is difficult due to the complexity of the geometries employed, the uncertainties in the determination of the boundary surface area, and the possible effects of a suppression of the gap at the boundary.[12] More data on the thermal boundary resistance for slabs of different thickness are needed in order to test the predictions of our theory in detail. A more complete account of our work will be presented elsewhere.

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