# Nucleus as a Color Filter in QCD: Hadron Production in Nuclei

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The data on hadron production in nuclei exhibit two striking regularities which are not readily explained by conventional hadron dynamics: (1) The mass-number dependence $A^{\alpha(x_F)}$ of inclusive production cross sections has a universal power $\alpha(x_F)$, which is independent of the produced hadron. (2) The $A$ dependence of $J/\psi$ production in nuclei has two distinct components: an $A^1$ contribution at low $x_F$ and an anomalous $A^{2/3}$ contribution which dominates at large $x_F$. We show that both phenomena can be understood in QCD as a consequence of the nucleus filtering out small, color-singlet Fock-state components of the incident-hadron wave function.

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*(1) The nucleus as a color filter.*—In high-energy hadron-nucleus collisions the nucleus may be regarded as a "filter" of the hadronic wave function.[1] The argument, which relies only on general features such as time dilation, goes as follows: Consider the equal-time Fock-state expansion of a hadron, in terms of its quark and gluon constituents. For example, for a meson,

$$|h\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \cdots . \tag{1}$$

The various Fock components will mix with each other during their time evolution. However, at sufficiently high hadron energies $E_h$, and during short times $t$, the mixing is neglible. Specifically, the relative phase $\exp[-i(E-E_h)t]$ of a given term in Eq. (1) is proportional to the energy difference

$$E - E_h = \left( \sum_i \frac{m_i^2 + \mathbf{p}_{Ti}^2}{x_i} - M_h^2 \right) \Big/ 2E_h , \tag{2}$$

which vanishes for $E_h \to \infty$. Hence the time evolution of the Fock expansion (1) is, at high energies, diagonal during the time $\sim 1/R$ it takes for the hadron to cross a nucleus of radius $R$.

The diagonal time development means that it is possible to describe the scattering of a hadron in a nucleus in terms of the scattering of its individual Fock components of Eq. (1). Here we shall explore the consequences for typical, soft collisions characterized by momentum transfer $q_T \simeq \Lambda_{\text{QCD}}$. The partons of a given Fock state will then scatter independently of each other if their transverse separation is $r_T \gtrsim 1/\Lambda_{\text{QCD}}$; i.e., if the state is of typical hadronic size. Conversely, the nuclear scattering will be coherent over the partons in Fock states having $r_T \ll 1/\Lambda_{\text{QCD}}$ since $e^{iq_T r_T} \simeq 1$. For color-singlet clusters, the interference between the different parton amplitudes interacting with the nuclear gluonic field is destructive. Thus the nucleus will appear nearly transparent to small, color-singlet Fock states.[2]

The momenta of the produced secondary hadrons depend on how the Fock state scatters. A large Fock state will tend to produce slow hadrons, since its momentum is shared by the partons which scatter, and hence also fragment, independently of each other. A small, color-singlet Fock state can transport the entire hadron momentum through the nucleus, and then convert back to one, or several, fast hadrons. In an experiment detecting fast secondary hadrons the nucleus indeed serves, then, as a filter that selects the small Fock components in the incident hadrons.

For ordinary, light hadrons the small Fock components typically constitute some fraction of the valence-quark state [i.e., of $|q\bar{q}\rangle$ in (1)]. However, if the hadron has an intrinsic heavy-quark Fock state,[3,4] then this nonvalence state can be important in processes with fast, heavy hadrons in the final state. Consider the intrinsic charm state $|u\bar{d}c\bar{c}\rangle$ of a $|\pi^+\rangle$. Because of the large charm mass $m_c$, the energy difference (2) will be minimized when the charm quarks have large $x$, i.e., when they carry most of the longitudinal momentum. Moreover, because $m_c$ is large, the transverse momenta $p_{Tc}$ of the charm quarks range up to $O(m_c)$, implying that the transverse size of the $c\bar{c}$ system is $O(1/m_c)$. Hence, provided only that the $c\bar{c}$ forms a color singlet, it can penetrate the nucleus with little energy loss. In effect, the nucleus is transparent to the heavy-quark pair component of the intrinsic state. The light-quark pair of the intrinsic state typically is of hadronic size and thus is absorbed by the nucleus.

*(2) Universal A dependence of hadroproduction.*—The experimental results on particle production in hadron-nucleus collisions show a remarkable regularity.[5] When the $A$ dependence is parametrized as

$$\frac{d\sigma}{dx_h}(p+A \to h+X) = A^{\alpha} \frac{d\sigma_N}{dx_h} , \tag{3}$$

where $d\sigma_N/dx_h$ is independent of $A$, it is found that the exponent $\alpha(x_h)$ is the same for all hadrons $h=\pi^{\pm},K^{\pm}p,n,\Lambda,\bar{\Lambda}$. Thus, at a given momentum fraction $x_h$, the ratios of the production of the various types of hadrons $h$ are independent of the nucleus (and also of the beam energy). The exponent $\alpha$ decreases smoothly from $\alpha(x=0.1)\simeq 0.7$ to $\alpha(x=0.9)\simeq 0.45$.

It is perhaps even more remarkable that a parametrization of the form (3) gives an $x_h$ dependent $\alpha$, even in the case of charm production ($h=D,\Lambda_c,J/\psi,\ldots$). According to the hard-scattering picture of QCD, $\alpha=1$ for all $x_h$ would be expected. In the Drell-Yan process of large-mass muon-pair production $\alpha\simeq 1$ for all $x_h$ is indeed observed.[6] However, several experiments on open charm production show[7] that $\alpha(x\gtrsim 0.2)\simeq 0.7$–$0.8$. For small $x_h$, an indirect analysis[8] comparing different measurements of the total charm-production cross section indicates $\alpha(x\simeq 0)\simeq 1$. More detailed data on the nuclear dependence of charm production are available from the hadroproduction of $J/\psi$. Here a decrease of $\alpha$ from $\alpha(x\simeq 0)\simeq 1$ to $\alpha(x\simeq 0.8)\simeq 0.8$ has been seen by several groups.[9] Particularly interesting from our present point of view is the analysis of Badier *et al.*[9] They noted that the production of $J/\psi$ at large Feynman $x_h$ (up to $x_h\simeq 0.8$) cannot be explained only by the gluon and light-quark fusion mechanisms of perturbative QTD, due to the anomalous $A$ dependence. However, their $\pi^- A\to J/\psi+X$ data were well reproduced if, in addition to hard QCD fusion (with $\alpha=0.97$), they included a "diffractive" component of $J/\psi$ production having $\alpha=0.77$. Using the measured $A$ dependence to extract the "diffractive" component, they found that (for a pion beam) it peaks at $x\simeq 0.5$ and dominates the hard scattering $A^1$ component for $x\gtrsim 0.6$.

*(3) Hadroproduction by penetrating Fock states.*—We consider two cases.

*(a) Light-hadron production in nuclei.*—The simple qualitative features of the data on light-hadron production in nuclei follow in a straightforward way from the picture of a nuclear filter described above. The fast hadrons are fragments of the small, color-singlet, penetrating valence-quark Fock states. Due to time dilation, the Fock state fragments only after passing through the nucleus.[10] Since it carries the quantum numbers of the beam hadron, it is natural that the ratios of the $x_h$ distributions of the various secondary hadrons $h$ in (3) will be independent of the size of the nuclear target.

To illustrate our ideas, let us assume that a penetrating Fock state suffers an energy loss in the nucleus which is proportional to its transverse area,

$$\frac{dE}{dl}=-\rho r_T^2 E\,, \tag{4}$$

where $\rho$ is an effective nuclear density. Thus in the average nuclear distance $\frac{4}{3}R$ the state retains a fraction $z$ of its energy,

$$z(r_T)=E_{\text{out}}/E_{\text{in}}=\exp(-\tfrac{4}{3}R\rho r_T^2)\,. \tag{5}$$

The inclusive hadron distribution (3) derived from the penetrating state is then

$$\frac{d\sigma}{dx_h}=\pi R^2\int d^2r_T\,|\psi(r_T)|^2 z^{-1}f_h(x_h/z)\,. \tag{6}$$

If we parametrize the incident-hadron wave function $\psi(r_T)$ by a Gaussian,

$$|\psi(r_T)|^2=(\pi\langle r_T^2\rangle)^{-1}\exp(-r_T^2/\langle r_T^2\rangle)\,, \tag{7}$$

and describe the inclusive fragmentation function $f_h(x)$ of the final Fock state into hadrons $h$ as

$$f_h(x)=\frac{C_h}{x}(1-x)^n\,, \tag{8}$$

then the inclusive cross section (6) is

$$\frac{d\sigma}{dx_h}=\frac{3\pi C_h R}{4\rho\langle r_T^2\rangle x_h}\int_{x_h}^{1}\frac{dz}{z}z^{\beta/R}\left[1-\frac{x_h}{z}\right]^n\,, \tag{9}$$

where $\beta=3/4\rho\langle r_T^2\rangle$.

The $A$ dependence of $d\sigma/dx_h$ now follows from $R\propto A^{1/3}$. For $x_h\simeq 1$ we have $z\simeq 1$ in the integral (9) and, consequently, $d\sigma/dx_h\propto A^{1/3}$ for all values of $n$, i.e., independently of the shape of the fragmentation function $f_h(x)$ in (8). For $x_h\simeq 0$ the integral in (9) is seen to give $d\sigma/dx_h\propto A^{2/3}$, again for all values of $n$. These scaling laws follow from our general picture of the nucleus as a filter of the incident Fock states, and are thus independent of the specific model considered here. They are also in good accord with the trend of the data.[5]

For intermediate values of $x_h$ the effective power $\alpha(x_h)$ in (3) can be estimated from

$$\alpha(x_h)=\frac{1}{3}R\frac{d}{dR}\left[\frac{d\sigma}{dx_h}\right]\Big/\frac{d\sigma}{dx_h}\,. \tag{10}$$

The value of $\alpha(x_h)$ depends, in our model, on the parameter $\beta/R$, and also on $n$. In practice the $n$ dependence can be relatively weak. For example, taking $\beta/R=10$ we find that the $\alpha(x_h)$ calculated from (10) differs from the experimental parametrization of Barton *et al.*[5] by less than 0.07 as $n$ ranges from 2 to 8, for all $x_h$ between 0.1 and 0.8. At this value of $\beta/R$, a penetrating Fock state with $r_T^2=\langle r_T^2\rangle$ loses, according to (5), 10% of its energy in the nucleus.

At very small $x_h$ our independent Fock-state scattering picture breaks down. The hadronization begins to occur already inside the nucleus, resulting in a hadronic cascade. A simple empirical characterization[11] of the $A$ dependence of soft-hadron production is $d\sigma/dx\simeq\frac{1}{2}(1+\bar{\nu})\sigma\propto A^1$, where $\bar{\nu}\propto A^{1/3}$ is the mean number of collisions and $\sigma\propto A^{2/3}$ is the geometric cross section.

*(b) Heavy-quarkonium production in nuclei.*—In heavy-quark production on nuclei, the experimental evidence that the exponent $\alpha$ in Eq. (3) is $x_h$ dependent requires a nonperturbative contribution to charm production. The usual QCD factorization formula always gives

an $x_h$ independent $\alpha$ in the scaling (energy independent) region, regardless of the form of the nuclear structure function.[12] In fact, the $A$ dependence indicated by the data on open charm,[7,8] and also measured in $J/\psi$ production,[9] can be readily understood if the incident hadron has Fock states with intrinsic charm.[3]

According to our earlier discussion, the $c\bar{c}$ pair in the intrinsic charm Fock state carries most of the momentum and has a small transverse extent, $\langle r_T \rangle \sim 1/m_c$. For such separations the nucleus is practically transparent, i.e., $z \simeq 1$ in (5). Thus the $c\bar{c}$ color-singlet cluster in the incident hadron passes through the nucleus undeflected; it can then evolve into charmonium states after transiting the nucleus.[13] The remaining cluster of light quarks in the intrinsic charm Fock state is typically of hadronic size and will interact strongly on the front surface of the nucleus. Consequently, the $A$ dependence of the cross section (6) is given by the geometrical factor, $\alpha \simeq \frac{2}{3}$. This justifies the analysis of Badier *et al.*,[9] in which the perturbative and nonperturbative charm-production mechanisms were separated on the basis of their different $A$ dependence ($\alpha = 0.97$ and 0.77 for a pion beam, repectively). The effective $x_h$ dependence of $\alpha$ seen in charm production is explained by the different characteristics of the two production mechanisms. Hard, gluon fusion production dominates at small $x_h$, due to the steeply falling gluon structure function. The contribution from intrinsic charm Fock states in the beam peaks at higher $x_h$, due to the large momentum carried by the charm quarks.

An important consequence of our picture is that all final states produced by a penetrating intrinsic $c\bar{c}$ component will have the same $A$ dependence. Thus, in particular, the $\psi(2S)$ radially excited state will behave in the same way as the $J/\psi$, in spite of its larger size. The nucleus cannot influence the quark hadronization which (at high energies) takes place outside the nuclear environment.

Quarkonium production due to the intrinsic heavy-quark state will fall off rapidly for $p_T$ greater than $M_Q$, reflecting the fast-falling transverse momentum dependence of the higher Fock-state wave function. Thus we expect the conventional fusion contributions to dominate in the large-$p_T$ region. The data are, in fact, consistent with a simple $A^1$ law for $J/\psi$ production at large $p_T$. The CERN experiment of Badier *et al.*[9] finds that the ratio of nuclear cross sections is close to additive in $A$ for all $x_F$ when $p_T$ is between 2 and 3 GeV. The data of the Fermilab experiment of Katsanevas *et al.*[9] show consistency with additivity for $p_T$ ranging from 1.2 to 3 GeV.

The probability for intrinsic heavy-quark states in a light-hadron wave function is expected[3,14] to scale with the heavy-quark mass $M_Q$ as $1/M_Q^2$. This implies a production cross section proportional to $1/M_Q^4$. The total rate of heavy-quark production by the intrinsic mechanism therefore decreases with quark mass, compared to the perturbative cross section which is proportional to $1/M_Q^2$. At large $x$ the intrinsic production should still dominate, however, implying a nuclear dependence in this region characterized by $\alpha \simeq 0.7$–$0.8$ in Eq. (3). Experimental measurements of beauty hadroproduction in nuclei over the whole range of $x$ will be essential for unraveling the two components of the cross section.

*(4) Coherence and hadron production in nuclei.*—The coherent scattering of quark systems has largely been neglected in earlier treatments of hadroproduction on nuclei, as, for example, in the additive quark model.[15] For light-hadron production, the $x_h$ dependence of $\alpha$ in Eq. (3) has often been assumed[16] to result from a dominantly peripheral production mode for fast hadrons. In such a picture, the nucleus is taken to be nearly opaque to hadrons, which consequently lose most of their momentum in central collisions. However, if this were the case it would also imply that $\alpha < 1$ in the Drell-Yan process: The incoming hadron could not interact as effectively with the quarks on the back side of the nucleus. The experimental proof[6] that $\alpha \simeq 1$ in large-mass muon-pair production requires the nucleus to be nearly transparent to the individual quarks of the beam hadron. The coherence of the hadronic wave function is nevertheless destroyed by the nuclear interactions—only the small Fock components can penetrate coherently and produce fast hadrons even in central collisions.

Our Fock-state picture will cease to be useful at low energies, when the Fock states no longer evolve independently over nuclear distances. According to Eq. (2), the required beam energy is higher for heavy-quark states and, more generally, for states with small transverse size. At low energies, hadrons will form, and may reinteract, inside the nucleus. This implies breakdown of Feynman scaling, which could thus be used as an experimental signal for the transition to the low-energy region.

In conclusion, we have found that the qualitative characteristics of both light- and heavy-particle production on nuclei can be understood in terms of the nucleus acting as a filter for the incident Fock states. The picture we have presented, which is consistent with the general principles of gauge theory, immediately accounts for the gross features of the data. By contrast, it is difficult to find simple explanations of those features in other models. For charm production, there is no way of understanding the $x_h$ dependence of $\alpha$ purely within perturbative QCD.

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