Short-Range Correlations and the Intermittency Phenomenon in Multihadron Rapidity Distributions

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We show that the increase of bin-averaged factorial moments with decreasing size of the rapidity bin δy (the so-called "intermittency" phenomenon) can be understood on the basis of conventional short-range correlations and a simple linked-pair *Ansatz* for higher-order correlations.

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Large numbers of hadrons can be produced at high center-of-mass energies available in contemporary accelerators. Collisions of individual hadrons (pions, protons, antiprotons, etc.) typically produce some tens of final hadrons (mostly pions) while several hundred can be made in nucleus-nucleus collisions. Since the momenta transverse to the collision axis are fairly small, the main focus of the analysis of the final state is currently on the longitudinal momentum distributions, and correlations. It has become customary to measure the longitudinal hadron distributions in terms of the "rapidity" variable $y=\frac{1}{2}\ln[(E+p_z)/(E-p_z)]$. The information thus obtained sheds light on the dynamical and statistical aspects of production mechanisms. In particular, the increased precision of recent data on rapidity distributions in hadronic collisions[1,2] allow one to ask whether these irregular histograms could indicate fractal,[3-5] or "intermittent," behavior. Bialas and Peschanski[6] have suggested that intermittency can be observed as a power behavior of the bin-averaged factorial multiplicity moments as δy is decreased, i.e., $\langle F_i\rangle \sim (\Delta Y/\delta y)^{\alpha_i}$. Recent experimental data on these moments[1,7-10] could be fitted with the power law even though a leveling off is seen for extremely small δy. The real question is whether "new physics" is implied by this phenomenon. The experimental results have inspired a host of models[11-14] (typically branching and self-similar cascading) as well as other means of analysis.[13,15,16]

In this paper we propose that the observed behavior is a straightforward manifestation of the short-range correlations. As the rapidity bin size δy increases from a value less than the correlation length ξ, the moments (being integrals over the correlation functions) naturally decrease in the way required to explain the data. The results presented here depend on the following key assumptions: (1) The two-particle correlation function is taken as input; (2) we restrict the analysis to the central region of rapidity so that stationarity (translation invariance) can be used; (3) we write all formulas for identical particles; and (4) we evaluate the higher-order correlations using products of connected two-particle correlations. Given these assumptions we obtain an accurate quantitative description of the moment functions $F_n(\delta y)$, $n=2,3,4,5$. The existence[17] of plausible cluster decay models of the correlation function suggest that extraordinary phenomena are not required to describe the moment data.

The hierarchy of rapidity density correlation functions is defined by

$$\rho_1(y_1)=\left\langle \sum_i \delta(y_1-y_i)\right\rangle = \frac{1}{\sigma_I}\frac{d\sigma}{dy_1}, \tag{1}$$

$$\rho_2(y_1,y_2)=\left\langle \sum_{i\neq j}\delta(y_1-y_i)\delta(y_2-y_j)\right\rangle = \frac{1}{\sigma_I}\frac{d^2\sigma}{dy_1\,dy_2}, \tag{2}$$

$$\rho_3(y_1,y_2,y_3)=\left\langle \sum_{i\neq j\neq k\neq i}\delta(y_1-y_i)\delta(y_2-y_j)\delta(y_3-y_k)\right\rangle = \frac{1}{\sigma_I}\frac{d^3\sigma}{dy_1\,dy_2\,dy_3},\ \cdots . \tag{3}$$

Here the labels i,j,k label the rapidity values and the angular brackets denote the event ensemble average over these points. σ_I is the inelastic cross section and $d\sigma/dy$, $d^2\sigma/dy_1dy_2$, etc., are inclusive cross sections. Integration of the correlation functions over a domain Ω of the rapidity space gives the well-known identities

$$\int_{\Omega_1} dy_1\,\rho_1(y_1)=\langle n\rangle_{\Omega_1}, \tag{4}$$

$$\int_{\Omega_2} dy_1\,dy_2\,\rho_2(y_1,y_2)=\langle n(n-1)\rangle_{\Omega_2}, \tag{5}$$

$$\int_{\Omega_3} dy_1\,dy_2\,dy_3\,\rho_3(y_1,y_2,y_3)=\langle n(n-1)(n-2)\rangle_{\Omega_3},\ \cdots . \tag{6}$$

Dynamical correlations are best expressed in terms of the cumulant moments, defined as[18]

$$K_2(y_1,y_2)=\rho_2(y_1,y_2)-\rho_1(y_1)\rho_1(y_2), \tag{7}$$

$$K_3(y_1,y_2,y_3)=\rho_3(y_1,y_2,y_3)-\rho(y_1)\rho_2(y_2,y_3)-\rho_1(y_2)\rho_2(y_3,y_1)-\rho_1(y_3)\rho_2(y_1,y_2)+2\rho_1(y_1)\rho_1(y_2)\rho_1(y_3),\ \cdots . \tag{8}$$

Cumulants vanish when any variable becomes statistically independent of the others. As a consequence only "linked" components occur in decompositions of the K_n, as elaborated below.

Equations (7) and (8) allow the ordinary moments to be expressed in terms of the cumulants. This is most conveniently done using reduced moments r_n and reduced cumulants k_n defined by

$$r_2(y_1,y_2)\equiv\rho_2(y_1,y_2)/\rho_1(y_1)\rho_1(y_2)\,, \tag{9}$$

$$r_3(y_1,y_2,y_3)\equiv\rho_3(y_1,y_2,y_3)/\rho_1(y_1)\rho_1(y_2)\rho_1(y_3)\,, \tag{10}$$

$$\vdots$$

$$k_2(y_1,y_2)\equiv K_2(y_1,y_2)/\rho_1(y_1)\rho_1(y_2)\,, \tag{11}$$

$$k_3(y_1,y_2,y_3)\equiv K_3(y_1,y_2,y_3)/\rho_1(y_1)\rho_1(y_2)\rho_1(y_3)\,. \tag{12}$$

In terms of "reduced" densities we have

$$r_2(y_1,y_2)=1+k_2(y_1,y_2)\,, \tag{13}$$

$$r_3(y_1,y_2,y_3)=1+\sum k_2(y_i,y_j)+k_3(y_1,y_2,y_3)\,, \tag{14}$$

$$r_4(y_1,y_2,y_3,y_4)=1+\sum k_2(y_i,y_j)+\sum k_2(y_i,y_j)k_2(y_m,y_l)+\sum k_3(y_i,y_j,y_k)+k_4(y_1,y_2,y_3,y_4)\,, \tag{15}$$

$$\begin{aligned}r_5(y_1,y_2,y_3,y_4,y_5)=1&+\sum k_2(y_i,y_j)+\sum k_2(y_i,y_j)k_2(y_m,y_l)+\sum k_3(y_i,y_j,y_k)k_2(y_m,y_l)\\&+\sum k_3(y_i,y_j,y_k)+\sum k_4(y_i,y_j,y_k,y_l)+k_5(y_1,y_2,y_3,y_4,y_5)\,.\end{aligned} \tag{16}$$

The sums are to be taken over distinct values of the y_i, disregarding terms equivalent by permutation of the labels (note that r_n and k_n are symmetric). Thus in (14) we have three terms in the sum, in (15) there are, respectively, 6, 3, and 4 terms in the sums, while in (16) there are 10, 15, 10, 10, and 5 terms in the summations.

Now we invoke translational invariance, i.e., $\rho_2=\rho_2\times(y_2-y_1)$ and integrate Eq. (5) over a strip (Fig. 1) of width δy and length ΔY; to compare with experimental data we chose ΔY to be M units of δy. Passing to coordinates $\lambda=(y_1+y_2)/2$, $\zeta=y_2-y_1$, we express Eq. (15) as

$$\Delta Y\int_0^{\delta y}d\zeta\rho_2(\zeta)=\langle n(n-1)\rangle_{\Omega_2}\,. \tag{17}$$

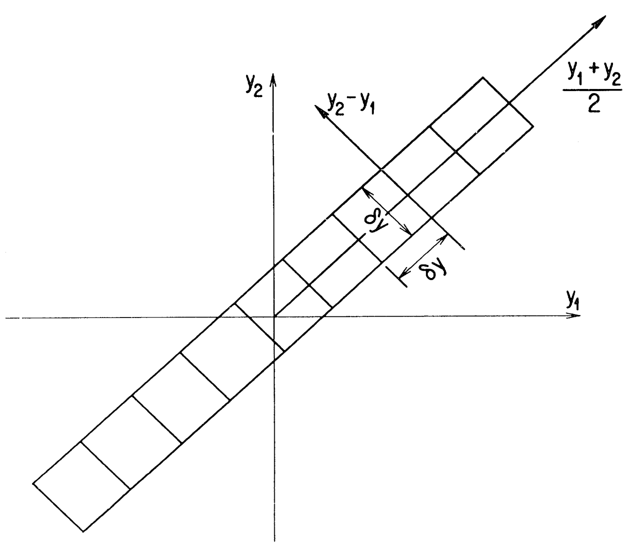

FIG. 1. The domain of integration Ω for the two-particle correlation integral. The contributing points must have relative coordinate less than δy; the c.m. of the points runs over $\Delta Y=M\,\delta y$ and corresponds to M rapidity bins of width δy.

Ω_2 consists of M squares Ω_k of area $(\delta y)^2$. Clearly this corresponds to λ sweeping through M bins of width δy. Since y_1 and y_2 are confined to Ω_k, so are the contributing points y_i,y_j [see Eq. (2)] and therefore

$$\langle n(n-1)\rangle_{\Omega_2}=\sum_{k=1}^{M}\langle n(n-1)\rangle_{\Omega_k}\,, \tag{18}$$

i.e., the sum over bin moments. This technique is easily generalized to n-dimensional tubes, giving

$$\Delta Y\int_0^{\delta y}d\zeta_1\cdots\int_0^{\delta y}d\zeta_{p-1}\rho_p(\zeta_1,\ldots,\zeta_{p-1})=\sum_{k=1}^{M}\xi_p^{(k)}\,, \tag{19}$$

where the nth-order factorial moment for bin k is[19]

$$\xi_p^{(k)}=\langle n(n-1)\cdots(n-p+1)\rangle_{\Omega_k} \tag{20}$$

and the ζ_i are appropriate pair coordinates.

Current data are presented in terms of the moments[6,20]

$$F_p=M^{-1}\sum_{k=1}^{M}\xi_p^{(k)}/\bar{n}^p\,. \tag{21}$$

Here $\bar{n}$ is the average number of particles per bin. Taking $\rho_1(y_1)=\rho_1(y_2)=\cdots=\bar{n}/\delta y$ and $\Delta Y=M\,\delta y$ we can rewrite Eqs. (19)-(21) in the form

$$F_p=(\delta y)^{1-p}\int_0^{\delta y}d\zeta_1\cdots\int_0^{\delta y}d\zeta_{p-1}r_p(\zeta_1,\ldots,\zeta_{p-1})\,. \tag{22}$$

Two-particle correlation data[21,22] are well fitted in the central region by the exponential formula

$$r_2(y_2-y_1)=1+\gamma_2\exp(-|y_1-y_2|/\xi)\,. \tag{23}$$

k_2 and ξ are slowly varying with c.m. energy: For NA22 we have $(k_2,\xi)\simeq(0.4,1.8)$[23] and for UA5 (900 GeV)

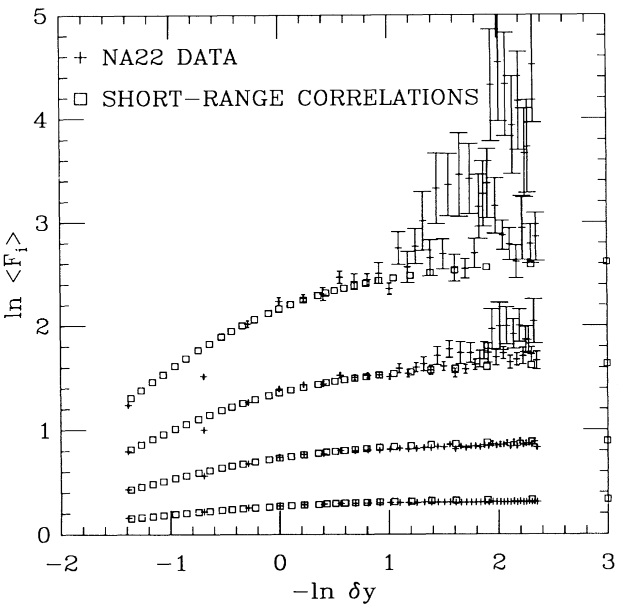

FIG. 2. NA22 data (Ref. 1) are compared with the theoretical results based on Eqs. (24) and (26). The four curves (from the bottom to the top) correspond to $\ln\langle F_2\rangle$, $\ln\langle F_3\rangle$, $\ln\langle F_4\rangle$, and $\ln\langle F_5\rangle$.

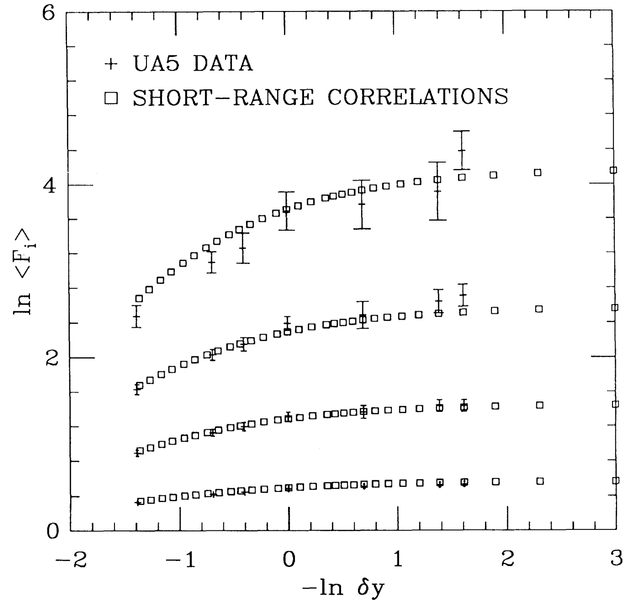

FIG. 3. This figure is calculated in the same way as Fig. 2, except that different correlation length ξ and coefficient γ_2 were used. The four curves (from the bottom to the top) correspond to $\ln\langle F_2\rangle$, $\mathrm{Ln}\langle F_3\rangle$, $\ln\langle F_4\rangle$, and $\ln\langle F_5\rangle$. Theoretical results are compared with UA5 data (Ref. 2).

we have $(k_2,\xi)\simeq(0.76,2.8)$.[21,22]

Integration of (17) or (22) gives for the second moment

$$F_2=1+\gamma_2\xi(1-e^{-\delta y/\xi})/\delta y\,. \tag{24}$$

Using the empirical values for γ_2 and ξ we find excellent agreement with NA22 and UA5 data (Figs. 2 and 3).

We construct higher-order cumulants assuming a linked-pair structure of higher-order correlations. Writing $e_{ij}=\exp(-|y_i-y_j|/\xi)$, we have

$$\begin{aligned} k_3&=\tfrac{1}{3}(a_3\gamma_2)^2(e_{12}e_{23}+\text{perm.})\,,\\ k_4&=\tfrac{1}{12}(a_4\gamma_2)^3(e_{12}e_{23}e_{34}+\text{perm.})\,,\\ k_5&=\tfrac{1}{60}(a_5\gamma_2)^4(e_{12}e_{23}e_{34}e_{45}+\text{perm.})\,. \end{aligned} \tag{25}$$

Evaluation of (22) gives the moments

$$\begin{aligned} F_3&=1+3f(\delta y)+a_3^2f^2(\delta y)\,,\\ F_4&=1+6f(\delta y)+3f^2(\delta y)+4a_3^2f^2(\delta y)+a_4^3f^3(\delta y)\,,\\ F_5&=1+10f(\delta y)+15f^2(\delta y)+10a_3^2f^3(\delta y)\\ &\quad+10a_3^2f^2(\delta y)+5a_4^3f^3(\delta y)+a_5^4f^4(\delta y)\,, \end{aligned} \tag{26}$$

where the function $f(\delta y)$ is defined as $f(\delta y)\equiv\gamma_2\xi\times(1-e^{-\delta y/\xi})/\delta y$.

The constants a_3, a_4, and a_5 are most easily found by successively fitting F_3, F_4, and F_5 to their values as $\delta y\to 0$ for either the UA5 or NA22 experimental data. Then the full curves are successfully given for both experiments, as shown in Figs. 2 and 3. (Thus eight curves are quantitatively described by the theory.) The values of the parameters used in our calculations are summarized in Table I. In Fig. 2 the excess at small δy for F_4 and F_5 is not accounted for by our calculation. Presently we assume that it is just noise.

Although the experimental form of the two-particle correlation function [Eq. (23)] is natural in some models,[24] the apparent success of the linked-pair approximation should motivate work on its form and of course the values of the empirical constants a_3, a_4, and a_5. It should be noted that we used the same correlation length for every moment. Moreover the constants a_3, a_4, and a_5 are the same for greatly different energies. Therefore we believe that in the linked-pair approximation the "in-

TABLE I. Parameters used in our calculation are summarized in this table. Experimental values of γ_2 and ξ are taken from the two-particle correlation function parametrized as in Eq. (23). The coefficients a_3, a_4, and a_5 were obtained by fitting Eqs. (24) and (26) in the limit $\delta y\to 0$ with either the UA5 or NA22 data. Note that these coefficients are energy independent.

	$\sqrt{s}=22$ GeV	$\sqrt{s}=900$ GeV
γ_2	0.4	0.76
ξ	1.8	2.75
a_3	1.3	1.3
a_4	1.6	1.6
a_5	2.8	2.8

termittency phenomenon" at any energy is completely determined by two-particle correlations. Even in the absence of a theory we can show that the dominance of the linked pairs used here. By the nature of cumulants, unlinked pairs such as $k_2(y_1,y_2)k_2(y_3,y_4)$ cannot occur. The ring graph $k_2(y_1,y_2)k_2(y_2,y_3)k_2(y_3,y_1)$ involves three relative integrations and hence gives a contribution to F_3 smaller by $O(\xi/\Delta Y)$. The same is true for a fourth-order term like $k_2(y_1,y_2)k_2(y_1,y_3)k_2(y_1,y_4)$ relative to the structure used in Eq. (25).

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