Consistent Quantization of Massive Chiral Electrodynamics in Four Dimensions

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We discuss the quantization of a four-dimensional model in which a massive Abelian vector field interacts with chiral massless fermions. We show that, by introducing extra scalar fields, a renormalizable unitary S matrix can be obtained in a suitably defined Hilbert space of physical states.

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In the last few years several attempts have been performed in order to quantize, in a consistent way, theories with dynamical fields coupled to genuine anomalous currents.[1–3] In particular, as far as models in which chiral fermions are coupled to a vector gauge field are considered, the two-dimensional (one time and one space dimension) case has been thoroughly investigated and satisfactorily solved, due to the circumstance that the fermionic determinant can be explicitly computed.[4]

Two different approaches have been essentially followed, one by Jackiw and Rajaraman[2] in which unitary theory is eventually recovered in a particular gauge choice provided that the parameter a regularizing the fermionic determinant is larger than 1; the other based on restoring gauge invariance by means of an integration over the group parameter ϑ which plays the role of an independent dynamical field.[5] It should be noticed that the two-dimensional case is special as the nonperturbative procedure of bosonization directly provides the expected kinetic term for the ϑ field.

In the more realistic four-dimensional situation, the fermionic determinant cannot obviously be explicitly evaluated; moreover, the naive kinetic term $\partial_\mu\vartheta\,\partial^\mu\vartheta$ does not possess the required canonical dimension. As a consequence, the only viable approach could be based on a perturbative expansion, which, however, in the massless case of gauge theories seems hard to carry out owing to the degeneracy of the Wess-Zumino action.[1]

A different four-dimensional model, which nevertheless possesses interesting features, is the massive Abelian vector meson coupled to fermion fields. If the fermions are Dirac spinors, it has been known for a long time that the model is both unitary and renormalizable, thanks to the conservation of the Dirac current.

The nontrivial generalization we want to discuss is the coupling with chiral spinors. In the latter case the fermionic left current is no longer conserved at the quantum level, owing to the occurrence of the chiral anomaly.[6] Recent papers have appeared in which the anomaly is seemingly compensated by means of the introduction of a Wess-Zumino term;[7] unfortunately all of those attempts do not consider the possibility that radiative corrections generate counterterms which jeopardize unitarity, namely, that ghostlike degrees of freedom can be generated by quantum corrections. The purpose of this paper is to show that it is indeed possible to construct a model in which all of the usual physical requirements are fulfilled thanks to the introduction of a suitable set of scalar fields.

We consider the chiral massive electrodynamics which is described by the following classical Lagrangian

$$\mathcal{L}_0 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\partial\!\!\!/ + ieA\!\!\!/P_L)\psi + \tfrac{1}{2}m^2A_\mu A^\mu\,, \quad (1)$$

where $P_L = \frac{1}{2}(1+\gamma_5)$. The equations of motion are

$$\partial^\mu F_{\mu\nu} + m^2 A_\nu = -J_{\nu,L}\,, \quad (2a)$$

$$(\partial\!\!\!/ + ieA\!\!\!/P_L)\psi = 0\,, \quad (2b)$$

where J_L^ν has the classical expression $ie\bar{\psi}\gamma^\nu P_L\psi$. Taking the divergence of Eq. (2a), we find

$$m^2\partial_\nu A^\nu + \partial_\nu J_L^\nu = 0\,. \quad (3)$$

At the classical level the fermionic left current turns out to be conserved thanks to Eq. (2b) and thereby the bosonic degrees of freedom are transverse, i.e., $\partial^\mu A_\mu = 0$, implying that the longitudinal field $A_\mu^\parallel$ is free.

As is well known, at the quantum level, the left current is no longer conserved owing to the anomaly. Therefore, $A_\mu^\parallel$ is no longer free, but nevertheless Eq. (3), with a suitable quantum definition of the left current, is expected to hold. Actually, in order to quantize the vector field A_μ, we shall impose Eq. (3) as a constraint, in a path-integral approach for instance, by means of a Lagrange multiplier $b(x)$, by adding to $\mathcal{L}_0$ the Lagrangian

$$\mathcal{L}_b = -m^2 \partial_\mu b A^\mu + b\,\partial_\mu J_L^\mu . \tag{4}$$

If we now consider the effective action

$$W[A_\mu] = N\int \mathcal{D}\bar\psi \mathcal{D}\psi \mathcal{D}b \exp\left[i\int (\mathcal{L}_0 + \mathcal{L}_b) d^4x \right] , \tag{5}$$

the stationary solution

$$\frac{\delta W}{\delta A_\mu} = 0 \tag{6}$$

corresponds to the vanishing of $\Box b$ in an average sense, namely,

$$\int \mathcal{D}\bar\psi \mathcal{D}\psi \mathcal{D}b \Box b \exp\left[i\int (\mathcal{L}_0 + \mathcal{L}_b) d^4x \right] = 0 . \tag{7}$$

The next step is to show that the condition (3) survives after the quantization of the vector field A_μ and thereby the average b field still satisfies a free-field equation.

To this purpose we consider the functional

$$W[b] = \mathcal{N}\int \mathcal{D}A_\mu \mathcal{D}\bar\psi \mathcal{D}\psi \exp\left[i\int (\mathcal{L}_0 + \mathcal{L}_b) d^4x \right] , \tag{8}$$

and set $A_\mu^{\parallel} = \partial_\mu \vartheta$, $A_\mu = A_\mu^{\perp} + \partial_\mu \vartheta$. Then the Lagrangian can be rewritten as

$$\mathcal{L}_0 + \mathcal{L}_b = \mathcal{L}(A_\mu^{\perp}, \bar\psi, \psi) + \tfrac{1}{2} m^2 [\partial_\mu (b-\vartheta)]^2 - \tfrac{1}{2} m^2 (\partial_\mu b)^2 + (b-\vartheta)\partial_\mu J_L^\mu . \tag{9}$$

If we change the variable of integration ϑ as $\tilde\vartheta = b - \vartheta$, we easily realize from Eqs. (8) and (9) that the average b field still satisfies a free equation. As a consequence there is no source of interaction for b and the constraint (3) continues to hold in an average sense

$$\langle m^2 \partial_\mu A^\mu + \partial_\mu J_L^\mu \rangle = 0 . \tag{10}$$

We conclude that it is stable under renormalization. On the other hand, the field $\tilde\vartheta$ obviously couples to $A_\mu^{\perp}$, via the gauge anomaly

$$\mathcal{L}_{\tilde\vartheta, A^{\perp}} = \tilde\vartheta \partial^\mu J_{\mu L} = \frac{e^3}{48\pi^2} \tilde\vartheta F_{\mu\nu} \tilde F^{\mu\nu} . \tag{11}$$

This coupling generates divergences at higher orders in the loop expansion which require counterterms in the Lagrangian.

The most dangerous term contains the coupling of two vertices

$$\langle T[F_{\mu\nu}(x)\tilde F^{\mu\nu}(x), F_{\rho\sigma}(y)\tilde F^{\rho\sigma}(y)] \rangle = (c_1 m^2 \Box + c_2 \Box^2)\delta^4(x-y) + \text{finite terms} , \tag{12}$$

c_1 and c_2 being divergent dimensionless coefficients in the limit $\omega \to 2$, 2ω being the space-time dimensions. The required counterterms in this case would be

$$\mathcal{L}_{\rm ct} = (Z_1 - 1)\tfrac{1}{2} m^2 (\partial_\mu \tilde\vartheta)^2 + Z_2 (\Box\tilde\vartheta)^2 . \tag{13}$$

The presence of higher derivatives in the kinetic term entails the appearance of a ghost in the spectrum of $\tilde\vartheta$. This ghost is unavoidable and destroys the perturbative unitarity; it just corresponds to an anomalous unitarity breaking. We remark that this basic point has been overlooked in previous treatments in similar contexts.

At this point it is worthwhile to notice that the degrees-of-freedom content of the above model is, besides the physical fields $A_\mu^{\perp}$, ψ, $\bar\psi$, a fully decoupled ghost b and an interacting scalar $\tilde\vartheta$, which has good signature at the tree level [see Eq. (9)], but generates through quantum corrections higher-derivative ghost contributions of a particularly bad type. Our idea is to forbid the occurrence of those quantum corrections by means of the introduction of a suitable extra ghost scalar field η with the Lagrangian

$$\mathcal{L}_\eta = -\tfrac{1}{2} m^2 (\partial_\mu \eta)^2 + \eta\,\partial_\mu J_L^\mu . \tag{14}$$

We remark that it couples to the anomaly with the same coefficient as the longitudinal part of A_μ. Therefore, if we consider the Green's functions of the transverse vector field, in which $\tilde\vartheta$ and η may only appear in the intermediate states, we immediately realize that their contributions cancel since they couple with the same coefficient but their propagators have opposite signs.

One can also understand this point in terms of the variables $\lambda = \tilde\vartheta + \eta$ and $\varphi = \tilde\vartheta - \eta$; in fact, the relevant Lagrangian as a function of those variables becomes

$$\mathcal{L}_{\lambda,\varphi} = \tfrac{1}{2} m^2 \partial^\mu \lambda\, \partial_\mu \varphi + \lambda\, \partial_\mu J_L^\mu . \tag{15}$$

The path integration over φ and λ gives, irrespective of the order,

$$\left\langle \exp\left[i\int \mathcal{L}_{\lambda,\varphi} \right] \right\rangle_{\lambda,\varphi} = \langle \delta(\Box\lambda) \rangle_\lambda = \text{const} , \tag{16}$$

namely, the dependence on the anomaly has completely disappeared.

To prove the unitarity of the S matrix we follow the Becchi-Rouet-Stora-Tyutin (BRST) procedure.[8] We add to the Lagrangian a term involving a couple of Faddeev-Popov ghosts

$$\mathcal{L}_{\rm FP} = i\,\partial_\mu \bar c\, \partial^\mu c , \tag{17}$$

$c, \bar c$ being Hermitian Grassmann fields. A first symmetry of the total Lagrangian $\mathcal{L}_0 + \mathcal{L}_b + \mathcal{L}_\eta + \mathcal{L}_{\rm FP}$ is realized by the following transformation:

$$\delta_1 A_\mu = \partial_\mu c , \quad \delta_1 b = c , \quad \delta_1 \bar c = -i m^2 b , \tag{18}$$

with all the other variations vanishing. We remark that one might also vary the fermion fields, but must take the contribution of the measure into account.[9] The corresponding conserved BRST charge turns out to be

$$Q_1 = \int d^3\bar x\, m^2 : b \overleftrightarrow{\partial}_0 c : . \tag{19}$$

We remark that Q_1 is not nilpotent of order two, but of order three, at variance with the usual gauge case.

The second invariance of the BRST type we consider is provided by the transformation

$$\delta_2 b = c\,,\quad \delta_2\eta = -c\,,\quad \delta_2\bar{c} = im^2(\eta-\vartheta)\,, \tag{20}$$

with all the other variations being zero. Actually this variation leads to the invariance of the action and not of the Lagrangian. The corresponding current

$$j_2^\mu(x) = m^2(\partial^\mu\eta c - \partial^\mu c\,\eta + \partial_\mu c\,\vartheta - \partial_\mu\vartheta c) \tag{21}$$

is conserved and gives rise to the BRST charge

$$Q_2 = \int d^3\bar{x}\, m^2{:}(c\overleftrightarrow{\partial}_0\eta + \vartheta\overleftrightarrow{\partial}_0 c){:}\,, \tag{22}$$

which is also nilpotent of order three.

We remark that the charges Q_1 and Q_2 forbid the appearance of divergent extra terms in the effective action apart from arbitrary polynomials of the variable $\lambda = \tilde{\vartheta}+\eta$ times BRST-invariant combinations of canonical dimension four. Nevertheless, Eq. (16) survives the presence of those counterterms and guarantees that λ still behaves like a free field. As a consequence, only the restriction of the S matrix in a suitably defined physical subspace is expected to be renormalizable, as we discuss below.

Finally, we discuss the structure of the space of the physical states in connection with the unitarity of the S matrix. It is convenient to choose as independent fields $\tilde{\vartheta}$, b, and η, as they do not mix under propagation and, in particular, b is a free field within this choice.

Of course we also have the charge Q_G related to the dilatation of the Faddeev-Popov fields, whose eigenvalues are the "ghost numbers." Then in the subspace with zero ghost number, the condition

$$Q_1\,|\,\text{phys}\rangle = 0 \tag{23}$$

is equivalent to imposing

$$b^{(-)}\,|\,\text{phys}\rangle = 0\,. \tag{24}$$

As a second condition we require

$$Q_2\,|\,\text{phys}\rangle = 0\,, \tag{25}$$

which, together with Eq. (23), implies

$$(Q_1-Q_2)\,|\,\text{phys}\rangle = \int d^3\bar{x}\, m^2{:}(\tilde{\vartheta}+\eta)\overleftrightarrow{\partial}_0 c{:}\,|\,\text{phys}\rangle = 0\,. \tag{26}$$

If we now remember that $\tilde{\vartheta}+\eta$ is a free field [see Eq. (16)], in the sector with the ghost number equal to zero, Eq. (26) is equivalent to the condition

$$(\tilde{\vartheta}+\eta)^{(-)}\,|\,\text{phys}\rangle = 0\,. \tag{27}$$

We now assume, as usual, the completeness of asymptotic in-out states. Equations (24) and (27) obviously become

$$b_{\text{as}}^{(-)}\,|\,\text{phys}\rangle = 0\,,\qquad (\tilde{\vartheta}_{\text{as}}^{(-)}+\eta_{\text{as}}^{(-)})\,|\,\text{phys}\rangle = 0\,. \tag{28}$$

The nontrivial allowed physical states are therefore generated by the creation operators $A_{\mu,\text{as}}^{\perp(+)}$ and $\tilde{\vartheta}_{\text{as}}^{(+)}+\eta_{\text{as}}^{(+)}$, remembering that $\tilde{\vartheta}_{\text{as}}$ and η_{as} have opposite commutation relations.

All the states generated by $A_{\mu,\text{as}}^{\perp(+)}$ are of positive norm and correspond to the three physical polarizations of the massive vector field. The other combination $\tilde{\vartheta}_{\text{as}}^{(+)}+\eta_{\text{as}}^{(+)}$ creates zero-norm states.

Those zero-norm states, although $\tilde{\vartheta}$ and η are coupled to the left current and thereby with the transverse vector field, remain free due to the exact compensation mechanism described earlier and expressed by Eq. (16). In other words, if those states were present as initial states, they could not actually create final transversal vectors states.

We conclude that there exists a restriction of the S matrix in the subspace defined by conditions (23) and (25) which is unitary and block diagonal.

We remark that our Lagrangian (9) could also be obtained by performing on the Lagrangian (1) a gauge transformation and by interpreting the group parameter as a new degree of freedom to be functionally integrated over.[5] The gauge fixing (4) should be chosen in this case to be $-m^2\partial_\mu b A^\mu$. As already described, this procedure does not lead by itself to a consistent theory, but a new field η must be introduced whose Lagrangian (14) is dictated by the result of the group-integration method. It might be an open interesting question whether this extra term could be interpreted in a geometrical way.

We end by noticing that our treatment in the limit $m\to 0$ becomes meaningless as all the additional kinetic Lagrangians disappear. As a consequence, the quantization of a chiral gauge theory in four dimensions is still an open challenge.

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