CP Violation in Neutral-*B* Decays to *CP* Eigenstates

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CP asymmetries in neutral-*B* decays to *CP* eigenstates are studied in the standard model. Whereas usually one assumes a single decay amplitude which induces *CP* violation via B-$\bar{B}$ mixing, we investigate additional effects due to two interfering decay amplitudes. We estimate these effects in characteristic cases and suggest ways to experimentally distinguish between these two sources of asymmetries. The effects, which show up as time-integrated asymmetries at symmetric e^+e^- colliders operating at the $\Upsilon(4S)$, are quite small in Kobayashi-Maskawa- (KM-) allowed decays such as $B_d^0 \to K_S\psi$ and become large in KM-suppressed decays.

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In the standard model, *CP* violation in *B* decays may occur whenever there are at least two weak decay amplitudes with different Kobayashi-Maskawa (KM) factors which lead to a given final state. Of particular interest and simple to analyze theoretically are *CP* asymmetries in neutral-*B* decays to hadronic *CP* eigenstates. Under special circumstances these asymmetries may be approximately independent of uncertainties in hadronic matrix elements and hadronic final-state-interaction phases which usually plague *CP* asymmetries.[1] In this Letter we wish to clarify these circumstances and to study the general case with no such approximation.

Since there exists a high degree of B-$\bar{B}$ mixing,[2] an initial B^0 may decay to a final state f, $CP|f\rangle = \pm|f\rangle$, via two chains $B^0 \to B^0 \to f$, $B^0 \to \bar{B}^0 \to f$. In general, two weak amplitudes contribute to both $B^0 \to f$ and $\bar{B}^0 \to f$. This is illustrated in Fig. 1, which shows two quark diagrams for $\bar{B}_d^0 \to \pi^+\pi^-$. Figure 1(a) describes a KM-suppressed "spectator" tree-level diagram, possibly corrected for short-distance QCD effects. A *W*-exchange diagram has the same KM factors and will thus be included in the spectator amplitude. Figure 1(b) represents a "penguin" QCD-loop-induced diagram.[3] Another possibility is that there exist two interfering spectator amplitudes, such as in $B_d^0 \to D_{1,2}^0\pi^0$ ($b \to c\bar{u}d$, $b \to u\bar{c}d$). In both cases the second amplitude is expected to be rather small for KM-allowed decays (e.g., $B_d^0 \to K_S\psi, D_{1,2}^0\pi^0$) and to become significant ($B_d^0 \to D_{1,2}^0K_S$) or even dominant ($B_d^0 \to K_S\pi^0$) in KM-suppressed processes.

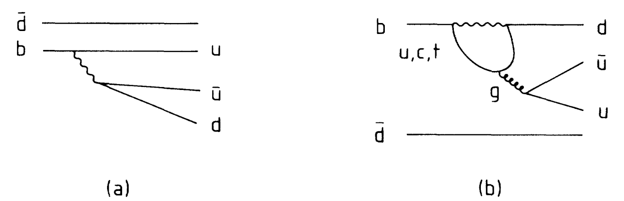

FIG. 1. (a) Spectator and (b) penguin diagrams for $\bar{B}_d^0 \to \pi^+\pi^-$.

If one neglects the second (penguin or spectator) amplitude, one may derive two important results:[1]

(a) The time-dependent asymmetry,

$$a(t) = \frac{\Gamma(B_{\rm phys}^0(t) \to f) - \Gamma(\bar{B}_{\rm phys}^0(t) \to f)}{\Gamma(B_{\rm phys}^0(t) \to f) + \Gamma(\bar{B}_{\rm phys}^0(t) \to f)} = \sin(2\phi)\sin(\Delta m t)\,, \quad (1)$$

[$\overset{(-)}{B}{}^0_{\rm phys}(t)$ is a state which was a pure $\overset{(-)}{B}{}^0$ at $t=0$] is given purely in terms of a combination of KM phases ϕ and the B-$\bar{B}$ mixing parameter $\Delta m/\Gamma$. This relation and its time-integrated form provide[1,4] future experimental tests of the standard model.

(b) On the $\Upsilon(4S)$ resonance a nonzero *CP* asymmetry measurement requires a determination of the time order in the two *B* decays. This was one of the arguments which led to considerations of the new concept of asymmetric e^+e^- colliders as factories of boosted $B\bar{B}$ pairs.[5,6]

These two features depend crucially on the assumption of a single amplitude. In the present Letter we are interested in the general case, where we keep the second amplitude and look for deviations from the features (a) and (b). We will suggest ways to experimentally observe these deviations and will estimate their size in characteristic cases.

We use the notations of Refs. 1 and 7 for the two physical neutral-*B* mesons

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \quad (m_L = m - \Delta m/2, \Gamma)\,,$$
$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle \quad (m_H = m + \Delta m/2, \Gamma)\,. \quad (2)$$

The corresponding masses and very approximately equal widths appear in the parentheses. $|q/p| = 1$ holds to a high degree of accuracy.[1] The time evolutions of initially pure B^0 and $\bar{B}^0$ states are[8]

$$|B_{\rm phys}^0(t)\rangle = g_+(t)|B^0\rangle + (q/p)g_-(t)|\bar{B}^0\rangle\,,$$
$$|\bar{B}_{\rm phys}^0(t)\rangle = (p/q)g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle\,, \quad (3a)$$

where

$$g_+(t) = \exp(-\tfrac{1}{2}\Gamma - im)\cos(\tfrac{1}{2}\Delta mt)\,,$$
$$g_-(t) = \exp(-\tfrac{1}{2}\Gamma - im)i\sin(\tfrac{1}{2}\Delta mt)\,. \tag{3b}$$

The decay amplitudes of B^0 and $\bar{B}^0$ to a CP eigenstate f are given by

$$A \equiv A(B^0 \to f) = A_1 e^{i\phi_1} + A_2 e^{i\phi_2}\,,$$
$$\bar{A} \equiv A(\bar{B}^0 \to f) = \pm A_1 e^{-i\phi_1} \pm A_2 e^{-i\phi_2}\,. \tag{4}$$

The two terms on the right-hand side corrrespond to two amplitudes as illustrated by the example in Fig. 1. ϕ_1 and ϕ_2 are the two corresponding different KM phases. The complex hadronic amplitudes A_j $(j=1,2)$ contain, besides KM mixing factors, also hadronic final-state-interaction phases α_j, $A_j = |A_j|\exp(i\alpha_j)$.

The time-dependent rates for initially pure B^0 and $\bar{B}^0$ states to decay to a final state f at time t are obtained, respectively,[7] as

$$\Gamma(B^0_{\rm phys}(t) \to f) = |A|^2 e^{-\Gamma t}[|\xi|^2 \sin^2(\tfrac{1}{2}\Delta mt) + \cos^2(\tfrac{1}{2}\Delta mt) - \mathrm{Im}\xi \sin(\Delta mt)]\,,$$
$$\Gamma(\bar{B}^0_{\rm phys}(t) \to f) = |A|^2 e^{-\Gamma t}[\sin^2(\tfrac{1}{2}\Delta mt) + |\xi|^2 \cos^2(\tfrac{1}{2}\Delta mt) + \mathrm{Im}\xi \sin(\Delta mt)]\,, \tag{5a}$$

where

$$\xi = \frac{q}{p}\frac{\bar{A}}{A}\,. \tag{5b}$$

The time-dependent CP asymmetry defined in Eq. (1) is

$$a(t) = \frac{(1-|\xi|^2)\cos(\Delta mt) - 2\,\mathrm{Im}\xi \sin(\Delta mt)}{1+|\xi|^2}\,. \tag{6}$$

The asymmetry has two contributions. The second term is the well-known asymmetry induced by the interference of A *and* $\bar{A}$ via mixing. The first term, which does not appear in Eq. (1), is due to CP violation in the decay $B^0 \to f$ itself. It follows from the different decay rates of B^0 and $\bar{B}^0$ to f that $|\xi| \neq 1$. This requires having two amplitudes A_1 and A_2 with different KM phases, $\phi_1 \neq \phi_2$, and different hadronic final-state-interaction phases, $\alpha_1 \neq \alpha_2$. It also contains $\exp(-\Gamma t)\cos(\Delta mt)$, the difference between the probabilities that a B^0 and a $\bar{B}^0$ oscillate to a B^0 state at time t. This "direct decay" asymmetry, diluted by B-$\bar{B}$ mixing, would exist also with no mixing ($\Delta m = 0$) as it does for charged B's.[3] Given by $\cos(\Delta mt)$, an even function of time, it does not vanish when integrated over positive and negative times, namely, when time order cannot be measured. This will be demonstrated below in the case of tagging B mesons at $\Upsilon(4S)$.

The time-integrated $(t>0)$ asymmetry obtained from Eq. (5a) is

$$\mathcal{A} = \frac{\int_0^\infty [\Gamma(B^0_{\rm phys}(t) \to f) - \Gamma(\bar{B}^0_{\rm phys}(t) \to f)]dt}{\int_0^\infty [\Gamma(B^0_{\rm phys}(t) \to f) + \Gamma(\bar{B}^0_{\rm phys}(t) \to f)]dt}$$
$$= \frac{1-|\xi|^2 - 2\,\mathrm{Im}\xi(\Delta m/\Gamma)}{(1+|\xi|^2)[1+(\Delta m/\Gamma)^2]}\,. \tag{7}$$

In the approximation of neglecting A_2, $\xi = \exp(-2i\phi)$, where ϕ is a convention-independent combination of KM phases. This leads to the more familiar and simpler form[1] of Eq. (5a)

$$\Gamma(B^0_{\rm phys}(t) \to f) = |A|^2 e^{-\Gamma t}[1 + \sin(2\phi)\sin(\Delta mt)]\,,$$
$$\Gamma(\bar{B}^0_{\rm phys}(t) \to f) = |A|^2 e^{-\Gamma t}[1 - \sin(2\phi)\sin(\Delta mt)]\,. \tag{8}$$

Subsequently the time-dependent asymmetry is given by Eq. (1) and the time-integrated $(t>0)$ asymmetry has the well-known expression

$$\mathcal{A} = \frac{\sin(2\phi)(\Delta m/\Gamma)}{1+(\Delta m/\Gamma)^2}\,. \tag{9}$$

To recapitulate, Eqs. (8), (1), and (9) are approximations obtained when the second amplitude in Eq. (4) is neglected. Equations (5), (6), and (7) are the corresponding equations with no such approximation.

To demonstrate the role of the $\cos(\Delta mt)$ term in the

TABLE I. Spectator and penguin couplings in $\bar{B}^0_d$ decays to CP eigenstates.

Quark process	Final state	Spectator coupling	Penguin coupling	$\lvert A_2/A_1\rvert$
$b \to c\bar{c}s$	$K_S\psi$	λ^2	$(\lambda^5 \text{ to } \lambda^4)\frac{\alpha_s}{12\pi}\ln\frac{m_t^2}{m_c^2}$	λ^6 to λ^3
$b \to c\bar{c}d$	D^+D^-	λ^3	$(\lambda^4 \text{ to } \lambda^3)\frac{\alpha_s}{12\pi}\ln\frac{m_t^2}{m_c^2}$	λ^4 to λ
$b \to u\bar{u}d$	$\pi^+\pi^-$	λ^4 to λ^3	$(\lambda^3 \text{ to } 2\lambda^3)\frac{\alpha_s}{12\pi}\ln\frac{m_t^2}{m_c^2}$	$2\lambda^3$ to 1
$b \to u\bar{u}s$	$K_S\pi^0$	λ^5 to λ^4	$\lambda^2\frac{\alpha_s}{12\pi}\ln\frac{m_t^2}{m_c^2}$	λ to λ^{-2}

asymmetry Eq. (6), let us consider its application in measuring CP violation at $\Upsilon(4S)$ with asymmetric e^+e^- beams.[6] Measuring the asymmetry requires tagging, namely, one must know whether the state at $t=0$ was a B^0 or a $\bar{B}^0$ meson. At the $\Upsilon(4S)$ the two neutral-B mesons are produced in a p wave and due to Bose statistics must be in mutually orthogonal flavor states. They remain in a coherent state of two orthogonal B mesons as long as neither of them has decayed. This implies that when one B is observed to decay to a flavor-specific mode (say as a B^0) the other is simultaneously identified as having the opposite flavor ($\bar{B}^0$). Alternatively, when one B decays to a CP eigenstate f, the other meson must be at this moment in an orthogonal state, which has a zero amplitude to decay to f. Hence, the rates for tagging one B and observing the other one to decay to a CP eigenstate are, in the two possible time orders,

$$\Gamma(t)\equiv\Gamma(B^0(t=0),f(t))=|A|^2|C|^2e^{-\Gamma t}\left[\frac{1+|\xi|^2}{2}-\frac{1-|\xi|^2}{2}\cos(\Delta m t)+\mathrm{Im}\xi\sin(\Delta m t)\right], \quad (10a)$$

$$\bar{\Gamma}(t)\equiv\Gamma(\bar{B}^0(t=0),f(t))=|A|^2|C|^2e^{-\Gamma t}\left[\frac{1+|\xi|^2}{2}+\frac{1-|\xi|^2}{2}\cos(\Delta m t)-\mathrm{Im}\xi\sin(\Delta m t)\right], \quad (10b)$$

$$\tilde{\Gamma}(t)\equiv\Gamma(f(t=0),B^0(t))=|A|^2|C|^2e^{-\Gamma t}\left[\frac{1+|\xi|^2}{2}-\frac{1-|\xi|^2}{2}\cos(\Delta m t)-\mathrm{Im}\xi\sin(\Delta m t)\right], \quad (10c)$$

$$\bar{\tilde{\Gamma}}(t)\equiv\Gamma(f(t=0),\bar{B}^0(t))=|A|^2|C|^2e^{-\Gamma t}\left[\frac{1+|\xi|^2}{2}+\frac{1-|\xi|^2}{2}\cos(\Delta m t)+\mathrm{Im}\xi\sin(\Delta m t)\right], \quad (10d)$$

where C is the B^0 decay amplitude to a flavor-specific mode.

In the customary discussion, in which one assumes a single amplitude for $B^0\to f$, ($|\xi|=1$), one has $\Gamma(t)=\bar{\tilde{\Gamma}}(t)$ and $\bar{\Gamma}(t)=\tilde{\Gamma}(t)$. One forms an asymmetry by subtracting the sum $\Gamma+\bar{\tilde{\Gamma}}$ from $\bar{\Gamma}+\tilde{\Gamma}$.[6] This requires measuring the time order.

In the general case studied here, all four time distributions of Eqs. (10) are different. This is due to the new $\cos(\Delta m t)$ terms. The relation $\Gamma+\bar{\Gamma}=\tilde{\Gamma}+\bar{\tilde{\Gamma}}$ still holds. In addition to the usual CP asymmetry [the subscript $(-)$ denotes an odd function of time]

$$a_-(t)=\frac{(\bar{\Gamma}+\tilde{\Gamma})-(\Gamma+\bar{\tilde{\Gamma}})}{(\bar{\Gamma}+\tilde{\Gamma})+(\Gamma+\bar{\tilde{\Gamma}})}=\frac{-2\,\mathrm{Im}\xi}{1+|\xi|^2}\sin(\Delta m t), \quad (11)$$

which requires measuring the time order, one may define a second asymmetry

$$a_+(t)=\frac{(\bar{\Gamma}+\bar{\tilde{\Gamma}})-(\Gamma+\tilde{\Gamma})}{(\bar{\Gamma}+\bar{\tilde{\Gamma}})+(\Gamma+\tilde{\Gamma})}=\frac{1-|\xi|^2}{1+|\xi|^2}\cos(\Delta m t). \quad (12)$$

This asymmetry can be measured without determining the time order of two B decays and, in general, is expected to be nonzero. In a symmetric e^+e^- collider (and, of course, in an asymmetric one) one may measure the time-integrated asymmetry

$$\mathcal{A}_+=\frac{\int_0^\infty[(\bar{\Gamma}+\bar{\tilde{\Gamma}})-(\Gamma+\tilde{\Gamma})]dt}{\int_0^\infty[(\bar{\Gamma}+\bar{\tilde{\Gamma}})+(\Gamma+\tilde{\Gamma})]dt}=\frac{1-|\xi|^2}{1+|\xi|^2}\left[1+\left(\frac{\Delta m}{\Gamma}\right)^2\right]^{-1}. \quad (13)$$

Both asymmetries a_+ and $\mathcal{A}_+$ measure CP violation in the decay $B^0\to f$ itself. Alternatively, one may look for this source of CP violation by searching for $\cos(\Delta m t)$ terms in the time distributions of Eqs. (10).

The effect of a second amplitude in Eq. (4) is to modify $\xi=\exp(-2i\phi)$. This depends on hadronic matrix elements and on KM and final-state-interaction phases. For $|A_2/A_1|\ll 1$ one finds, to first order in $|A_2/A_1|$,

$$\frac{2\,\mathrm{Im}\xi}{1+|\xi|^2}\simeq-\sin(2\phi)+2\left|\frac{A_2}{A_1}\right|\cos(2\phi)\sin(\phi_1-\phi_2)\cos(\alpha_1-\alpha_2),$$
$$\frac{1-|\xi|^2}{1+|\xi|^2}\simeq-2\left|\frac{A_2}{A_1}\right|\sin(\phi_1-\phi_2)\sin(\alpha_1-\alpha_2). \quad (14)$$

Because of the unknown final-state-interaction phases, $\sin(2\phi)$ cannot be determined unambiguously from measurements of the two constant coefficients describing the asymmetries $a_-(t)$ and $a_+(t)$. Note that unless $|A_2/A_1|$ is extremely tiny, small values of $\sin(2\phi)$ may be modified significantly when $\sin(\phi_1-\phi_2)$ is large.

As a rule, the effect of the penguin amplitude is expected to be small for KM-allowed decays (corresponding to $b\to c\bar{c}s$), to become significant for "once-KM-suppressed" decays ($b\to c\bar{c}d$, $b\to u\bar{u}d$), and to be very large for "twice-KM-suppressed" processes ($b\to u\bar{u}s$). Examples of these cases are shown in Table I, in which we give the couplings corresponding to the appropriate spectator (A_1) and penguin (A_2) amplitudes. Note that the two amplitudes must have different KM factors, $\phi_1-\phi_2\neq 0$. $\lambda=0.22$ is the parameter introduced[9] to parametrize the quark mixing matrix. We use $\lambda^2<|V_{ub}/V_{cb}|<\lambda$, where the m_t-dependent lower limit is derived from the measurement of B-$\bar{B}$ mixing and from

CP violation in $K \to 2\pi$. For allowed values of m_t one has

$$\frac{\alpha_s(m_b^2)}{12\pi}\ln\frac{m_t^2}{m_c^2} \sim \lambda^2 . \qquad (15)$$

The penguin-to-spectator coupling ratio for the KM-allowed transition $b \to c\bar{c}s$ is, at most, at a level of a fraction of a percent. This ratio may be as large as about 5%, 20% for $b \to c\bar{c}d$, $b \to u\bar{u}d$, respectively, and is expected to be of order 1 (1–4) for $b \to u\bar{u}s$-type processes. We did not include include in Table I examples of pure penguin processes, such as $B_d^0 \to K_S\phi$, which have no spectator diagrams and where two penguin amplitudes interfere.

A_2/A_1 is determined by a model-dependent ratio (P/S) of penguin-to-spectator-operator hadronic matrix elements. In some models[10] it is of order 1, and in other estimates,[11] penguin matrix elements are enhanced.[12] These various estimates may soon face experimental tests.[13] Conservative lower and upper limits are $\lambda < P/S < 1/\lambda$. This is used to obtain the limits of the last column of Table I, which cover large ranges. Our own rough estimate is $P/S \sim 1$ for the four final states in Table I. However, we would rather leave it to experiments to determine this ratio.

CP eigenstates which cannot be reached by a penguin operator have always two different tree amplitudes which interfere. Examples are $B_d^0 \to D_{1,2}^0\pi^0 (b \to c\bar{u}d, b \to u\bar{c}d)$ and the KM-suppressed decays $B_d^0 \to D_{1,2}^0 K_S$ $(b \to c\bar{u}s, b \to u\bar{c}s)$. The corresponding ratios of the two amplitudes which contribute to these transitions are at the level of (1–5)% and (20–100)%, respectively.

The purest cases are processes of the type $B_d^0 \to K_S\psi$ induced by $b \to c\bar{c}s$, in which $|A_2/A_1| < 1\%$. The effect of A_2 on determining $\sin 2\phi(K_S\psi)$, already known to be larger than 0.1,[1,4] is estimated from Eq. (14) to be at most at the level of 0.02. The rather small contribution of the penguin amplitude may be verified experimentally by measuring a vanishing asymmetry $\mathcal{A}_+$ defined in Eq. (13), $\mathcal{A}_+ < 2\%$, and by setting upper limits on the $\cos(\Delta m t)$ terms in the time distributions of Eqs. (10). This seems to be feasible in a proposed search for CP violation in $B_d^0 \to K_S\psi$ with asymmetric e^+e^- beams,[6] although the limits obtained may not be as strict as our theoretical estimates. Setting such limits on a direct decay asymmetry in $B_d^0 \to K_S\psi$ is useful for a reliable experimental measurement of the corresponding KM phase. Observing an asymmetry $\mathcal{A}_+$ larger than our estimate would indicate physics beyond the standard three-family model.

The direct decay asymmetry in another type of KM-allowed decay, $B_d^0 \to D_{1,2}^0\pi^0$, in which two spectator amplitudes interfere, may be an order of magnitude larger than in $B_d^0 \to K_S\psi$. Finally, from the preceding discussion one expects rather large direct decay asymmetries in certain KM-suppressed neutral-B decays. A careful experimental study of these asymmetries in the above-described manner may help in clarifying the uncertainties in the thereby-determined combinations of KM phases. This may be useful for future tests of the standard model, albeit some remaining uncertainties due to unknown final-state-interaction phases.

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