Shape Fluctuation as Responsible for Signature Inversion

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The problem of signature inversion is studied by using a model of particles coupled to a rotor. We have assumed an axially symmetric equilibrium deformation and taken into account the γ vibration around it. The calculated signature dependence is found inverted, in agreement with the experimental data if we adopt a set of moments of inertia which favor the rotation around the shortest axis, but not if we employ the set of the irrotational-flow model.

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The phenomenon called signature inversion has been observed in ^{155}Ho,[1] ^{157}Ho,[1,2] ^{157}Tm[3], ^{159}Tm,[4] ^{161}Lu,[5] ^{163}Lu,[6] and ^{165}Lu,[7] and has received much attention. Signature is a quantum number associated with the rotation of a deformed nucleus around a principal axis by 180° and defined as $\alpha_I = \frac{1}{2}(-1)^{I-1/2}$, where I is the total nuclear spin. A rotational band whose bandhead is determined by a quasiparticle moving in a unique-parity orbital specified by j, such as $h_{11/2}$ and $i_{13/2}$, consists of levels $I=j, j+1, j+2, \ldots,$ and is split into two sequences of $I=j$ (mod2) and $I=j+1$ (mod2) according to the signature. The former is shifted downwards in energy compared with the latter in almost all the experimentally known cases; hence the names adopted: the favored band for the former and the unfavored for the latter. This shift is well understood in terms of the Coriolis coupling. In the $h_{11/2}$ proton bands of these Ho, Tm, and Lu isotopes, however, the favored sequence is found to lie higher in energy than the unfavored one for $I > I_c$, I_c denoting the spin at which the first band crossing takes place. This situation is called a signature inversion. In the discussed unique-parity rotational bands an $h_{11/2}$ quasiproton is excited for $I < I_c$ and in addition two $i_{13/2}$ quasineutrons for $I > I_c$. This configuration change gives rise to a specific phenomenon called the rotational alignment: the quasiparticles align their angular momenta along the total angular momentum. This phenomenon is a consequence of the action of the Coriolis force. Obviously the number of quasiparticles changes at the band crossing. Despite the difference between the configurations below and above I_c, the Coriolis force invariably continues to favor the levels of $I-j=$ even over the ones of $I-j=$odd in the ordinary model of particles plus a symmetric rotor.

The signature dependence of an energy spectrum can be seen clearly when illustrated in the form $\Theta_I \equiv [E(I) - E(I-1)]/2I$, where $E(I)$ is the energy of the state of spin I. The energy spectrum of ^{157}Ho is illustrated in Fig. 1(a) in this way, and we see that the signature dependence is normal below the band crossing and that it is inverted above it for a rather wide range of spin.

Bengtsson *et al.*[8] found that a signature-inverted energy spectrum is obtained in the cranking model for limited regions of rotational frequency and Fermi energy when a triaxially deformed nucleus is cranked around the shortest axis ("positive-γ rotation" in the Lund convention of the triaxial deformation γ). It was suggested[9] that the spectrum of ^{157}Ho may be understood in the cranked shell model if γ is assumed to be negative below the band crossing, which is necessary to produce the ob-

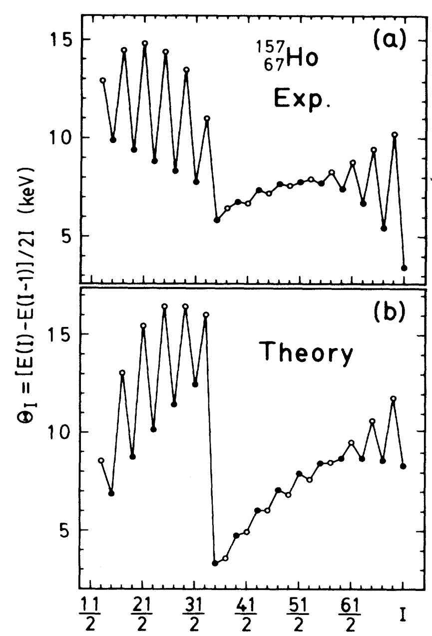

FIG. 1. (a) The experimental and (b) the theoretical energy spectrum of the unique-parity rotational band of ^{157}Ho in the form of Θ_I vs I. Solid and open circles represent Θ_I's of $I=j$ (mod2) and Θ_I's of $I=j+1$ (mod2), respectively. The experimental data are taken from Ref. 1.

served large normal signature splitting, and to be positive above it, the condition necessary for the signature inversion. One of the present authors (A.I.) and Åberg transplanted[10] the idea of positive-γ rotation to the particle plus triaxial rotor model and found that the signature inversion takes place for positive-γ rotation, in accordance with Bengtsson's prediction, but only for rather large values of γ, i.e., for $\gamma > 18°$. The study has also indicated that only very weak inversion is expected for the Fermi energies of the discussed Ho, Tm, and Lu isotopes. Matsuzaki *et al.*[11] performed detailed cranked-shell-model calculations, but obtained no signature inversion for any of those nuclei. They attributed the failure to the smallness of the calculated equilibrium values of γ, which are around 5° for ^{157}Ho and 9° for ^{161}Lu. To our knowledge no cranking calculation has ever reproduced the observed signature inversion quantitatively.

It was reported[12] that the quadrupole-type proton-neutron interaction between rotation-aligned quasiprotons and quasineutrons gives no signature inversion within a reasonable range of the interaction strength. Assuming a *p-n* interaction derived from the Schiffer-True interaction[13] for the aligned particles, we have carried out particle-rotor-model calculations.[14] Signature inversion is seen to take place for some combination of parameter values but it occurs only instantaneously for a very narrow range of spin, and cannot be identified with the experimentally observed signature inversion. Thus the effective *p-n* interaction alone does not explain the signature inversion either.

This lack of a satisfying explanation of the signature inversion has motivated us to look for a model that can reproduce this remarkable phenomenon under reasonable assumptions and to find out what degrees of freedom are involved in it. The triaxial shape fluctuation around an axially symmetric equilibrium deformation, which is called the γ vibration, is known to produce characteristic signature dependences in the $\Delta I = 1$ $E2$ and $M1$ transitions.[15] Such prominent effects of the γ vibration on the signature-related properties suggest that it may well influence the signature dependence of the energy spectra and may cause the signature inversion.

We have carried out the calculations in the model of particles plus a symmetric rotor, in which the configurations of $(\pi h_{11/2})^1$ and $(\pi h_{11/2})^1(\nu i_{13/2})^2$ and the excitation of the γ vibration have been explicitly taken into account. Such a model is a straightforward extension of the one of Ref. 15 to include three-quasiparticle configurations in addition to the one-quasiparticle one. We do not give all of the details here but outline only a few facts. The dynamical deviation of the rotor from axial symmetry, which is exactly what is meant by the γ vibration, affects the correlation between particles and rotor through the shape dependence of the mean field and through that of the moments of inertia. The latter can be written in the small-γ limit as

$$\frac{\hbar^2}{2}\gamma\left(\frac{d}{d\gamma}\mathcal{J}_1^{-1}(\gamma)\bigg|_{\gamma=0}(I_1-J_1)^2+\frac{d}{d\gamma}\mathcal{J}_2^{-1}(\gamma)\bigg|_{\gamma=0}(I_2-J_2)^2\right),$$

an expression which assumes the following form in the irrotational-flow model:

$$-\frac{1}{\sqrt{3}}\frac{\hbar^2}{2\mathcal{J}}\hat{\gamma}[(I_+^2+I_-^2)-2(I_+J_++I_-J_-)+(J_+^2+J_-^2)]\,.$$

We should note here the importance of the behavior of the moments of inertia around the 1 and 2 axes at small γ's. Quantities characterizing the γ vibration are its excitation energy and the matrix element of γ between the vacuum and the first excited vibrational state, which is related to the $E2$ transition strength as

$$b \equiv [B(E2{:}0_g^+ \rightarrow 2_{K=2}^+)/B(E2{:}0_g^+ \rightarrow 2_{K=0}^+)]^{1/2}\,.$$

In the present calculations we assume $b=(0.06)^{1/2}$, which is considered to be a reasonable value for the nuclei of interest. We use the quadrupole force as an effective *p-n* interaction and fix its strength according to Bohr and Mottelson.[16] Multiquasiparticle configurations of the BCS approximation contain a spurious state related to the violation of the particle-number conservation, and we have removed it from the model space.

First, we have assumed the irrotational moments of inertia and carried out a number of calculations, aiming to reproduce the characteristics of the unique-parity rotational band in ^{157}Ho. We have varied the Fermi energy for the protons, λ_p, around $\epsilon_{h_{11/2},\Omega=7/2}$ and λ_n around $\epsilon_{i_{13/2},\Omega=3/2}$. The other parameters are varied in the following ranges: [0.25,0.30] for β, [0.8,1.5] in MeV for Δ_p, [0.8,1.5] in MeV for Δ_n, and [20,35] in keV for $\hbar^2/2\mathcal{J}$. We have varied the strength of the quadrupole *p-n* interaction in the range of a factor of 3 and applied the surface delta interaction also. In spite of these efforts we have obtained only negative results. Signature inversion has been seen in some calculated spectra for a very narrow region of the spin which contains only a few spin values. Although we have not swept all the lattice points of the parameter space, we believe that the observed signature inversion cannot be described using the irrotational moments of inertia.

However, we should not disregard the possibility that the γ vibration may cause the signature inversion, because we have only carried out calculations in the model with the assumed specific set of moments of inertia and the model is not yet fully explored. The results could be quite different with a different set of moments of inertia. As a simple but quite a different choice, we employ the γ-reversed irrotational moments of inertia (which are obtained by reversing the sign of γ in the irrotational moments of inertia):[17]

$$\mathcal{J}_\kappa(\gamma)=\tfrac{4}{3}\mathcal{J}\sin^2(\gamma+2\pi\kappa/3)\,.$$

This effectively changes $\mathcal{J}_1$ and $\mathcal{J}_2$ and, thus, the largest one corresponds to the shortest axis. This change simply implies the need of changing the sign of the rotation-vibration interaction in the computer code.

In Fig. 1(b) a calculated result is presented. Here the Fermi energies are

$$\lambda_p=\epsilon_{h_{11/2},\Omega=5/2}+0.9[\epsilon_{h_{11/2},\Omega=7/2}-\epsilon_{h_{11/2},\Omega=5/2}]$$

and

$$\lambda_n=\epsilon_{i_{13/2},\Omega=13/2}\,.$$

We have assigned 0.89 MeV to E_γ, which is the observed excitation energy of the lowest $K=2$ state in ^{156}Dy. The binding energies and $B(E2)$'s of the neighboring even-even nuclei are used to estimate the gap energies as $\Delta_p=1.2$ MeV and $\Delta_n=1.3$ MeV, and the deformation as $\beta=0.27$. The moment of inertia is adjusted to reproduce the spectrum well, and the result is $\hbar^2/2\mathcal{J}=24$ keV. The most remarkable feature of Fig. 1(b) is that the signature inversion is reproduced at the right place, i.e., just above the band crossing. The calculated spectrum is satisfactory in other respects as well: The size of the inversion and the range of its extension in spin compares very well with the experimental data and the band crossing is reproduced at the right spin value. We should note that these satisfactory features are obtained with reasonable parameter values and that our model yields the signature inversion above the band crossing not only instantaneously for a specific combination of parameter values but rather steadily in a small but finite region of the parameter space. This feature is essential for the present model to be realistic, since the nuclei in which the signature inversion is observed differ in parameter values, especially in deformation and Fermi energies.

If one draws a line through the averages of adjacent open and solid circles in Fig. 1(b), which represent the effective moment of inertia corresponding to the theoretical spectrum, one finds that it grows too fast with spin both below and above the band crossing compared with the corresponding experimental line. This inaccuracy must be largely related with the rigid-rotor assumption of the model. The nuclei of interest and their even-even neighbors show large deviation from the rigid-rotor spectrum. This may be due to possible variation of β deformation and pairing correlation with spin. We hope to obtain better results if such degrees of freedom are incorporated in the model.

In order to see how the signature inversion is brought about, we have decomposed the total energy into the contributions from the various terms in the Hamiltonian. The rotational energy term, $H_{\rm rot}=\sum_i(I_i-J_i)^2\hbar^2/2\mathcal{J}_i(\gamma=0)$, is found only to make a large contribution to the normal signature dependence. On the other hand, H_γ, the sum of the rotation-vibration and particle-vibration couplings, works to invert the signature dependence. Their expectation values are shown in Fig. 2. An important point about these two contributions is that they increase with spin at almost the same pace. This makes it possible for H_γ to cope with the strong normal signature dependence produced by $H_{\rm rot}$ for a wide range of spin. Other terms such as the quasiparticle energy and the *p-n* interaction are found to work in favor of the signature inversion, but they do not grow with spin so rapidly as $H_{\rm rot}$. This fact explains the result of Refs. 12 and 14; i.e., the *p-n* interaction can hardly be the major cause of the signature inversion.

After this analysis, we can view the signature inversion as a consequence of the competition between the Coriolis coupling, which always works to produce the normal signature splitting, and the other terms which are found to work in the opposite direction when the γ-reversed irrotational moments of inertia are assumed. If we change the moments of inertia back to the irrotational ones, H_γ is found to work in favor of the normal signature dependence.

In conclusion, we have studied the problem of signature inversion in the framework of a model of particles plus a symmetric rotor with the γ vibrational degree of freedom taken into account. It is found that the observed signature inversion is hardly reproduced if the irrotational moments of inertia are employed. Introducing a different set of moments of inertia by changing the sign of γ appearing in the irrotational-flow model, we have reproduced the experimental signature inversion very well. Decomposing the total energy into various terms, we have found that the particle-vibration and rotation-vibration couplings give the major contribution to the signature inversion. Thus we have proposed a new theoretical model which gives a reasonable description of the phenomenon of the signature inversion. Since the key ingredient of this model is the dynamical fluctuation around an axially symmetric equilibrium deformation,

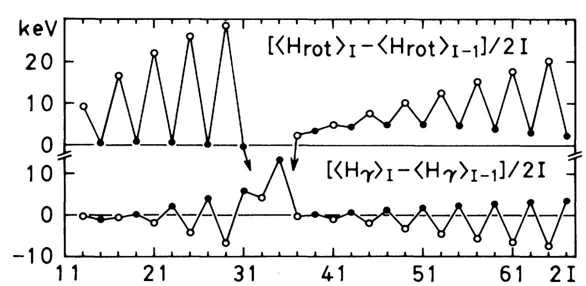

FIG. 2. The contributions to the total energy from $H_{\rm rot}$ and H_γ, which are defined in the text, as functions of I.

the present view of the phenomenon is in contrast to the view of Refs. 8 and 9 in which a nuclear system with a permanent triaxial deformation is cranked around the shortest axis. However, we cannot deny a possible connection between the two models because our set of moments of inertia had to be such that the one around the shortest axis was the largest. Moreover, such a set of moments of inertia is likely to lead to rotation around the shortest axis when the nuclear shape deviates from axial symmetry, which is in other words positive-γ rotation.

Calculations of the energy spectra for other nuclei where the signature inversion is observed and of the $E2$ and $M1$ transition strengths are in progress and will be published elsewhere.

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[1]G. B. Hagemann *et al.*, Nucl. Phys. **A424**, 365 (1984).

[2]G. B. Hagemann *et al.*, Phys. Rev. C **25**, 3224 (1982).

[3]M. A. Riley *et al.*, in *Proceedings of the Conference on High-Spin Nuclear Structure and Novel Nuclear Shapes, 13–15 April 1988* (Argonne National Laboratory Report No. ANL-PHY-88-2), p. 211; (to be published).

[4]A. J. Larabee and J. C. Waddington, Phys. Rev. C **24**, 2367 (1981).

[5]C.-H. Yu *et al.*, Nucl. Phys. **A489**, 477 (1988).

[6]K. Honkanen *et al.*, in *Nuclei Off the Line of Stability,* edited by R. Meyer and D. Brenner, American Chemical Society Symposium Series No. 324 (American Chemical Society, Washington, DC, 1986), p. 555.

[7]S. Jónsson *et al.*, Nucl. Phys. **A422**, 397 (1984).

[8]R. Bengtsson *et al.*, in Proceedings of the Twentieth International Winter Meeting on Nuclear Physics, Bormio, Italy, 1982 (unpublished), p.144.

[9]R. Bengtsson *et al.*, Nucl. Phys. **A415**, 189 (1984).

[10]A. Ikeda and S. Åberg, Nucl. Phys. **A480**, 85 (1988).

[11]M. Matsuzaki *et al.*, Prog. Theor. Phys. **79**, 836 (1988).

[12]I. Hamamoto, Phys. Lett. B **193**, 399 (1987).

[13]J. P. Schiffer and W. W. True, Rev. Mod. Phys. **48**, 191 (1976).

[14]T. Shimano and A. Ikeda (unpublished).

[15]A. Ikeda, Nucl. Phys. **A439**, 317 (1985).

[16]Aa. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, pp. 509 and 512.

[17]I. Hamamoto and B. R. Mottelson, Phys. Lett. **132B**, 7 (1983).