Current Drive by Plasma Waves and Helicity Conservation

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The connection between current drive by plasma waves and helicity conservation is discussed. The mean current drive depends on the difference between the viscous and resistive damping, whereas the helicity input depends on the sum of the damping rates. The discrepancy is due to the helicity lost through fluctuating currents. In a purely resistive plasma this loss is twice the helicity input, and the helicity used in the current drive is equal but opposite to the input.

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Recently, Ohkawa and others[1-3] have described plasma current drive by low-frequency Alfvén waves, and have interpreted this in terms of helicity balance. The argument runs as follows. Consider a circularly polarized transverse wave, in a uniform plasma and uniform magnetic field $(0,0,B_0)$, described by the vector potential

$$\mathbf{A}=(A,iA,0)\exp[i(kz-\omega t)]\,, \tag{1}$$

where ω is real and $k=k_0(\omega)+ik_1(\omega)$ (as for a wave from an antenna at $z=0$). The helicity density associated with the wave is

$$H_\omega=\langle\mathbf{A}\cdot\mathbf{B}\rangle=k_0A^2\exp(-2k_1z)\,. \tag{2}$$

(Note that there would be no helicity associated with a plane-polarized wave and that the usual difficulties in defining a gauge-invariant localized helicity[4] can be overcome because B_z is constant.) The current j_z driven by the wave is calculated from the helicity balance equation

$$2\eta j_zB_0=-v_A\,\partial H_\omega/\partial z=2k_1v_AH_\omega \tag{3}$$

(where v_A is the Alfvén wave speed and η the resistivity). Then, writing $A(z)=A\exp(-2k_1z)$,

$$j_z=j_0=k_1k_0v_AA^2(z)/\eta B_0\,. \tag{4}$$

However, this calculation assumes that all the helicity is available to drive the mean current and ignores the dissipation of helicity associated with fluctuating currents.

In this Letter I show that it is essential to include the helicity dissipated in the fluctuating currents in order to obtain the correct current drive from helicity balance. In fact, in a purely resistive fluid, the helicity dissipation associated with the fluctuating currents is actually twice that which drives the mean current and of opposite sign. For each unit of helicity input, *two* units are dissipated by the fluctuating currents and *minus one* unit drives the mean current. [Consequently if only the mean current is considered in the helicity balance, as in Eq. (3), one obtains the correct magnitude of the driven current, but with the wrong sign.]

Consider a simple viscous fluid with Ohm's law $\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{j}$. Then the wave satisfies the equations

$$-\omega\mathbf{B}=(kB_0)\mathbf{v}+i\eta k^2\mathbf{B}\,, \tag{5a}$$

$$-\omega\rho\mathbf{V}=(kB_0)\mathbf{B}+i\rho\mu k^2\mathbf{v}\,, \tag{5b}$$

and the dispersion equation is

$$(\omega+i\eta k^2)(\omega+i\mu k^2)=k^2v_A^2\,. \tag{6}$$

When η and μ are small, $k_0\simeq\omega/v_A$ and the wave damping is

$$k_1=(\eta+\mu)k_0^2/2v_A\,. \tag{7}$$

The mean current induced by the wave can be calculated directly from Ohm's law. One has

$$\eta\langle j_z\rangle=\langle\mathbf{v}\times\mathbf{B}\rangle \tag{8}$$

and if (5a) (which itself follows from Ohm's law) is used to express $\mathbf{v}$ in terms of $\mathbf{B}$, then

$$\eta\langle j_z\rangle=(\omega k_1-\eta|k|^2k_0)A^2(z)/B_0\,. \tag{9}$$

(The electrostatic field, or return current, which maintains $\nabla\cdot\mathbf{j}=0$ in the steady state can be ignored in the present discussion.) Note that in Eq. (9), any nonresistive damping appears only through k_1. Using the value of k_1 given by Eq. (7) the wave-driven current is

$$\eta j_\omega=\frac{k_0A^2(z)}{B_0}\frac{(\mu-\eta)k_0^2}{2}\,. \tag{10}$$

On the other hand, if we use the same damping k_1 in Eq. (4) (which represents the result of balancing wave helicity against dissipation by the mean current), we obtain

$$\eta j_0=\frac{k_0A^2(z)}{B_0}\frac{(\mu+\eta)}{2}k_0^2 \tag{11}$$

which does not agree with (10).

This discrepancy is due to the omission of the helicity dissipated in the fluctuating currents associated with the wave. When this is included, the correct helicity balance equation becomes

$$-v_A\,\partial H_\omega/\partial z = 2\eta\langle j_z\rangle B_0 + 2\eta\langle \mathbf{j}_\omega\cdot\mathbf{B}_\omega\rangle\,. \tag{12}$$

Here, the input of helicity by the wave is

$$-v_A\,\partial H_\omega/\partial z = 2k_1 v_A H_\omega = (\eta+\mu)k_0^2 H_\omega\,, \tag{13}$$

while the helicity dissipated by the fluctuating currents is

$$2\eta\langle \mathbf{j}_\omega\cdot\mathbf{B}_\omega\rangle = 2\eta k^2\langle \mathbf{A}\cdot\mathbf{B}\rangle = 2\eta k_0^2 H_\omega\,. \tag{14}$$

Consequently, according to Eq. (12) the mean current is given by

$$2\eta\langle j_z\rangle B_0 = (\mu-\eta)k^2 H_\omega \tag{15}$$

in full agreement with Eq. (10).

These results show that the current driven by wave absorption is not directly related to the helicity input. Other sources of helicity loss must be accounted for. In the present example, current drive depends on the difference between viscous and resistive damping, whereas helicity input depends on the sum. The balance is made up by the helicity lost in fluctuating currents. It is remarkable that, if only resistivity is important, the loss through fluctuations is exactly twice the input and the helicity dissipated in the current drive is equal in magnitude but opposite in sign to the helicity input.

One may conclude that in many cases direct calculation of current drive[5] may be more appropriate than consideration of helicity. However, the latter should be valuable when the damping involves nonlinear or turbulent processes similar to those involved in plasma relaxation.[4]

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