Correlation Length of the Three-State Potts Model in Three Dimensions

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A Monte Carlo study is carried out for the three-state Potts model in three dimensions. The correlation length is shown to remain finite and to be discontinuous at the transition point. It is also shown that the transition exhibits genuine first-order characteristics in all aspects.

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Considerable effort has been invested in the study of the behavior of the three-state Potts model in three dimensions, especially in the determination of the order of its phase transition. The Monte Carlo study of the internal energy of the order parameter indicated that the transition is weakly first order.[1-4] There is, however, some circumstantial evidence[3,5] indicating that the correlation length increases towards the critical point, which is usually taken as a characteristic of a second-order phase transition. No direct numerical attempts have so far been made[6] to measure the correlation length to settle this problem.

This model has recently attracted our great interest because of a claim made by the APE Collaboration that the finite-temperature phase transition of QCD may be second order.[7] This claim is based on their observation that the correlation length of the system appears to be divergent.[7] On the other hand, the Columbia University group found a small but finite discontinuity in the internal energy at the transition point.[8] This status looks quite similar to that for the three-state ($q=3$) Potts model in three dimensions. In fact this system is known to be an effective model for the Polyakov line for the QCD system at finite temperatures.[9] This motivates our study of the Potts model with the hope that it would provide a hint to resolve the confusing status of the QCD finite-temperature phase transition.

In this letter we report a precision Monte Carlo study of the phase transition of the $q=3$ Potts model on a simple cubic (sc) lattice in three dimensions, with a special emphasis on the behavior of the correlation length close to the critical point. We show that the correlation length remains finite and is discontinuous at the transition point. We also demonstrate that the correlation length appears to diverge, if a proper averaging procedure is not taken.

Very recently we have received a paper[10] in which a similar Monte Carlo analysis was made with a statistics almost comparable to ours. The authors of Ref. 10 have concluded that the transition is of first order and also shown that the apparent divergence of the correlation length extracted from a naive procedure does not represent the physical one. The procedure taken by them, however, does not give a correct physical correlation length in the vicinity of the transition as we discuss later. There also emerged another Monte Carlo analysis of the $q=3$ Potts model with the inclusion of antiferromagnetic next-to-nearest-neighbor interactions by the APE group,[11] which stresses the similarity between this system and their result for the finite-temperature QCD, i.e., possible divergence of the correlation length near the transition point. The correlation length measured by them, however, seems to correspond to the unphysical one.

We take the Hamiltonian

$$H = -\tfrac{3}{2}\sum_{\langle i,j\rangle}(\delta_{\sigma_i\sigma_j}-1)\,, \tag{1}$$

with $\langle i,j\rangle$ the sum over the nearest-neighbor spin pairs in the sc lattice, and the statistical system is defined by $Z=\exp(-\beta H)$. Equation (1) is also written as

$$H = -\sum_{\langle i,j\rangle}(\mathrm{Re}\,s_i s_j - 1) \tag{2}$$

using a Z_3-valued spin variable s_i. We employ a lattice with sizes $N_s^2\times N_t = 32^2\times 48$ and $48^2\times 64$ with the periodic boundary condition and make typically $(0.4-1)\times 10^6$ sweeps at each β with a heat-bath Monte Carlo updating algorithm. To examine the size dependence we also made several runs on $12^2\times 16$ and $24^2\times 32$ lattices. To extract the correlation length we use a zero-momentum projected correlation function,

$$C(t) = \langle O(t)O(0)\rangle - \langle O(t)\rangle\langle O(0)\rangle\,, \tag{3}$$

with $O(t)$ the average of N_s^2 spins on a plane located at t in the N_t direction.

On a $32^2\times 48$ lattice the system shows a clear flip-flop signature between the disordered (symmetric) and ordered (broken) phases from $\beta=0.3669$ to 0.3671. The typical tunneling time is several times 10^4 sweeps. This flip-flop behavior is exemplified in Fig. 1 with respect to the order parameter $\Phi(\beta)$ defined by Ref. 1, $\Phi(\beta) = \frac{3}{2}\langle n^*\rangle - \frac{1}{2}$, with n^* the maximum population of the

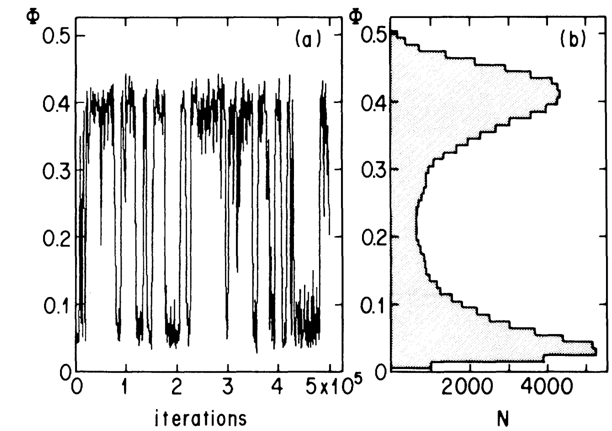

FIG. 1. (a) Time history of the order parameter Φ at $\beta=0.3670$ on a $32^2\times 48$ lattice. The first 5×10^5 sweeps of the run (a million sweeps total) are shown. (b) Histogram of Φ corresponding to (a). Here all of the data (1×10^6 sweeps) are used.

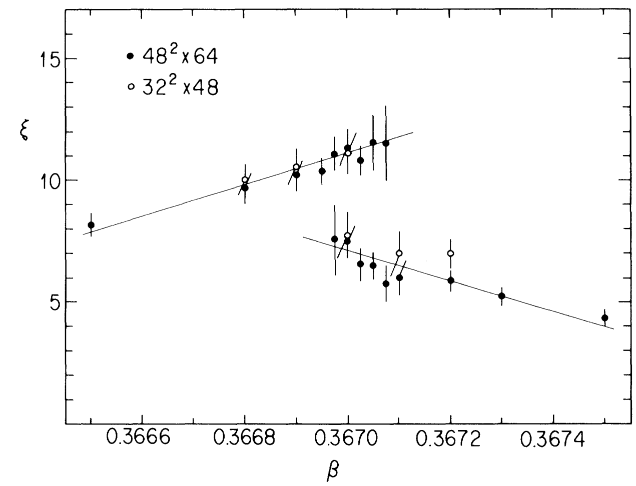

FIG. 2. The correlation length as a function of β on a $48^2\times 64$ and a $32^2\times 48$ lattice. Lines are drawn to guide the eye.

states projected onto the Z_3 axes.

On a $48^2\times 64$ lattice we observed a clear two-state signal for the range $0.366975 \leq \beta \leq 0.367075$. A typical time for staying in one state is several times 10^5 sweeps. Our estimate of β_c on this lattice size is $\beta_c=0.367025 \pm 0.000050$. This coexistence of the two states is counted as a clear signal for a first-order transition.

We estimate the distance-dependent correlation length defined by

$$\xi(t) = -\{\ln[C(t+1)/C(t)]\}^{-1}. \tag{4}$$

(In actual calculation, we take the effect of the periodic boundary condition into account.) When $\xi(t)$ stays at a constant value for a considerably wide range of t, we can extract the correlation length ξ from the data of $\xi(t)$. In the disordered phase the second term of (3) is negligibly small and is set equal to zero.

On a $48^2\times 64$ lattice two states are clearly separated in our data, and we computed the correlation length separately for two phases. On a $32^2\times 48$ lattice we have rapid tunneling between two phases. We then estimated the correlation length separately for disordered and ordered states by dividing the run into two sections. Here we used a criterion $\Phi > 0.23$ (< 0.23) to separate the ordered (disordered) phase [the number corresponds to the minimum position of the double peak in the histogram of Φ, see Fig. 1(b)].

We present in Fig. 2 the correlation length for both disordered and ordered phases. We estimated ξ from $\xi(t)$ for $t \simeq (0.5-2)\xi$ [$\xi(t)$ varies by about $\lesssim 10\%$ in this range], and the error shown stands for both statistical errors obtained by grouping the runs and systematic errors due to the variation of $\xi(t)$ in the t range that concerns us. We see that the correlation length changes little when the size of the lattice is increased from $32^2\times 48$ to $48^2\times 64$. We then conclude that ξ stays finite at $\beta=\beta_c$ as the volume $V\to\infty$ ($V=N_s^2\times N_t$) when one approaches from either side of β_c. Our estimate is

$$\xi(\beta\to\beta_{c-}) = 11\pm 1\,, \tag{5}$$

$$\xi(\beta\to\beta_{c+}) = 7\pm 1\,. \tag{6}$$

Let us now attempt to calculate the correlation length without separating the two phases on a lattice with size $32^2\times 48$ or smaller. Here we do not subtract the second term of (3). The result presented in Fig. 3 shows that the correlation length ξ' thus defined takes a value much larger than ξ and it is as large as $\gtrsim N_t/2$. We also found that ξ' increases as $\sim N_t$ near the critical point. From such a measurement one might conclude that the correlation length is divergent at $\beta=\beta_c$.

It is easy to see that this superficial divergence is caused by the fact that $\langle O(t)O(0)\rangle$ does not exhibit an exponential decay but approaches a constant due to an admixture of the ordered phase by the tunneling. A similar phenomenon is observed in the ordered phase for the Ising model in two and four dimensions.[12,13] The divergent correlation length is interpreted as merely representing the mass gap between the two degenerate minima in the symmetry-broken states. It is pointed out that the true correlation length is to be extracted from the second harmonics of the exponential decay for this case.[13] In our $q=3$ Potts model, however, the tunneling occurs between the four degenerate minima (one in the symmetric state and three in the symmetry-broken states), and these degeneracies will be lifted. After the removal of this degeneracy, these four states have different symmetries and hence have different physical correlation lengths. Therefore, one needs higher harmonics to analyze the correlation function, if one applies a similar analysis to the present case. The authors of Ref. 10 assumed, however, that the correlation function

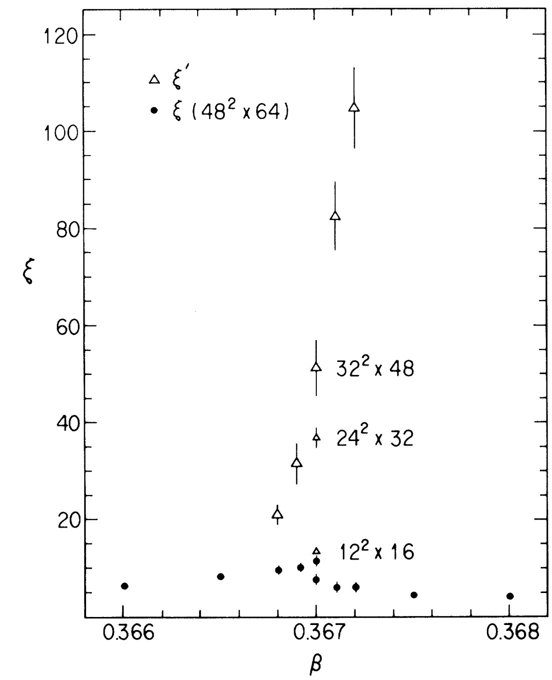

FIG. 3. The "correlation length" ξ' obtained from an exponential-decay fit to the correlation function without separating the ordered and disordered phases of the run ($32^2\times 48$ lattice). Data for other sizes are also shown at $\beta=0.3670$ to exhibit the lattice-size dependence of ξ'. Solid points are the true correlation length ξ as given in Fig. 2 ($\xi'=\xi$ for $\beta<0.3668$).

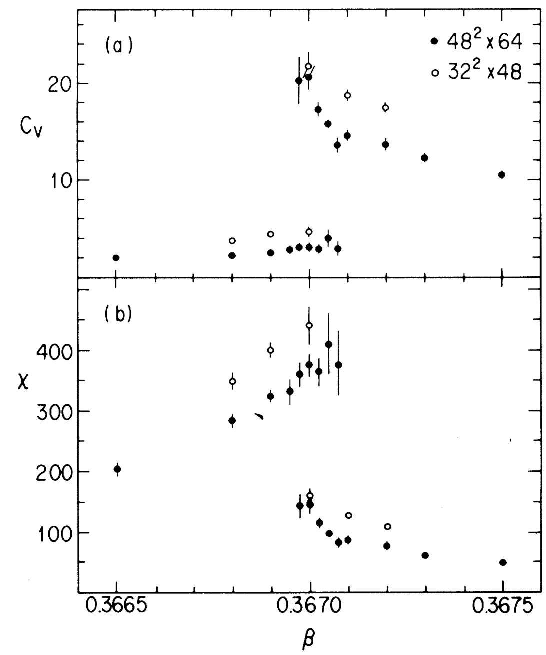

FIG. 4. (a) The specific heat and (b) the susceptibility as functions of β on a $48^2\times 64$ and a $32^2\times 48$ lattice.

is given by a sum of two exponential terms and extracted the physical correlation length from the second harmonics also in the case of the $q=3$ Potts model in three dimensions. Therefore this analysis failed to reveal the discontinuity of the correlation length at the transition point, while they noticed the difference between the tunneling correlation length and the physical correlation length.

In our analysis we have also measured the specific heat $[C_v=V^{-1}(\langle H^2\rangle-\langle H\rangle^2)/9]$ and the susceptibility $[\chi=V^{-1}(\langle(\sum s_i)^2\rangle-\langle\sum s_i\rangle^2)]$. We found (see Fig. 4) that all quantities stay finite as $\beta\to\beta_{c}\pm$. This contrasts with the observation in Ref. 3 where it was found that they obey a power law satisfying hyperscaling. We note that the measurement of Ref. 3 was made far from the critical point in view of the present calculation; our analysis shows that a power-law behavior is lost near the critical point.

In conclusion, we have shown that the three-state Potts model exhibits a behavior typical of a genuine first-order transition in the vicinity of the critical point in all aspects: A clear two-state signal is seen; the correlation length is finite for both $\beta\to\beta_c\pm$ with $\xi(\beta_c+)\neq\xi(\beta_c-)$; and the specific heat and the susceptibility are also finite. Our analysis suggests that the superficial power-law behavior claimed in the literature[3,5] arose from the fact that physical quantities were calculated for the region quite far away from the transition point, where the second-order fixed point in the metastable region[2] may control the behavior.

Our final comment is that if the correlation length is measured without separating the two phases, the leading exponential behavior of the correlation function leads to a superficially divergent correlation length, which is not taken to be the physical correlation length. We may expect that a similar situation emerges also for the finite-temperature QCD system.[7]

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