Defining the Configuration Space of String Field Theory

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We propose a space for bosonic and heterotic string field theories in which all classical vacua are included naturally. We also illustrate how space-time emerges. The existence of a new string symmetry beyond the usual Becchi-Rouet-Stora-Tyutin symmetry is pointed out.

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String theory, in particular, the heterotic string,[1] offers the potential of being the ultimate theory of all forces and matter in nature.[2] In recent years, many string models based on the heterotic string theory have been constructed. Some of them look surprisingly realistic, in terms of the gauge symmetry and chiral fermions. However, the number of models is numerous and there is no known reason why a particular model is favored over any of the rest. In terms of string field theory, these models correspond to the classical vacua of the heterotic string field theory. Dynamics of the string field will hopefully select out the vacuum that describes the world we live in. Hence a proper formulation of string field theories is of great importance in unraveling the underlying string symmetry as well as dynamics.

Unfortunately, string field theory as formulated up to now[3,4] is woefully short of this eventual goal. This statement needs some clarification. It is well known that the covariant closed-string field-theory formulation has encountered difficulties. In particular, the covariant heterotic string field theory is missing. However, even if the covariant closed-string theory were written down, it would still be useless in answering the dynamical questions such as why the theory chooses any particular vacuum. This follows from the choice of space upon which the string field is defined. For simplicity, consider the closed bosonic string field, $\Phi(X_\mu(z,\bar{z}),b(z),c(z),\bar{b}(\bar{z}),\bar{c}(\bar{z}))$, where X_μ ($\mu=0,1,2,\ldots,25$) is the string variable while c ($\bar{c}$) and b ($\bar{b}$) are the ghost and the antighost for left (right) movers. To define the string field, the variables X_μ are chosen to be the coordinates. However, this fixes the classical string vacuum to the 26-dimensional string or one of its toroidal compactifications. Starting from such a choice, it seems impossible to explore all the other classical vacua (e.g., left-right asymmetric models) that we know exist. A proper formulation of a string field theory, in general, entails an action. To properly define the action, it is necessary to specify the space in which the strong fields live, i.e., the coordinates of the string field. It is clearly important, as a necessary first step, to define the string field in a space such that all classical vacua emerge naturally; in fact, these vacua should be on an equal footing before dynamics (i.e., interactions) start to play a role.

In this paper, we propose a configuration space upon which string fields should be defined. We shall explicitly construct this space for the closed bosonic string and the heterotic string field. We shall illustrate, in general, how the classical vacua emerge. In this formulation, space-time is a derived concept. Our proposal clearly shows the existence of a very deep underlying string symmetry. To start, (super)conformal symmetry is taken to be essential to the formulation of string field theory. It is known that a large class of conformal fields can be bosonized,[5-8] e.g., (super)conformal ghosts, all (super) conformal models in the minimal series as well as W algebras, which includes parafermions. Our key assumption is that all two-dimensional (super)conformal fields can be bosonized (actually, the assumption needed is somewhat weaker; we require only the bosonization of conformal fields that appear in string theories). We propose that these bosons are the fundamental coordinates of string fields. To make the discussion concrete, we start with a working hypothesis. We shall comment on it later.

Let us concentrate for the moment on the closed bosonic string. We introduce 51 chiral bosons[5,6] (the reason for the number 51 will be explained later):

$$T(z)=\sum_{ij}\eta^{ij}(-\tfrac{1}{2}\partial\phi_{Li}\partial\phi_{Lj}+i\alpha_{Li}\partial^2\phi_{Lj}), \tag{1}$$

where the diagonal metric is Minkowskian, $\eta=(-1,1,1,\ldots,1)$. The central charge of ϕ_{Lj} is $c_{Lj}=1-12\eta_{jj}\alpha_{Lj}^2$, where $j=0,1,2,\ldots,50$. The total central charge of this set must be 26,

$$\sum_j c_{Lj}=\sum_j(1-12\eta_{jj}\alpha_{Lj}^2)=26, \tag{2}$$

or

$$\alpha_L^2=\eta^{ij}\alpha_{Li}\alpha_{Lj}=\tfrac{25}{12}, \tag{3}$$

where α_{Lj} are real. The constraint Eq. (3) assures that $c_{L0}\geq 1$ and the rest $c_{Li}\leq 1$. Here $\langle\phi_i(z)\phi_j(w)\rangle=-\eta_{ij}\times\ln(z-w)$. Let us introduce a similar set of 51 right-moving chiral bosons $\phi_{Rj}(\bar{z})$. Denote $\boldsymbol{\alpha}=(\boldsymbol{\alpha}_L,\boldsymbol{\alpha}_R)=(\alpha_{L0},\alpha_{L1},\ldots,\alpha_{R0},\alpha_{R1},\ldots)$. Now we can define the string field in terms of these chiral bosons and the bc ghosts, $\Phi(\phi_{Li},\phi_{Ri},b,c,\bar{b},\bar{c})$. Of course, the bc ghosts can be bosonized in the same way, if desired. These bosons can be expanded into modes,

$$a_{Ljn} = i\oint \frac{dz}{2\pi i} z^n \partial\phi_{Lj}(z)\,, \tag{4}$$

where

$$[a_{Lin}, a_{Ljm}] = \eta\delta_{n,-m}\eta_{ij}\,,$$

$$b_n = \oint \frac{dz}{2\pi i} z^{n+1} b(z), \quad c_n = \oint \frac{dz}{2\pi i} z^{n-2} c(z)\,.$$

Defining the SL(2,C)-invariant world-sheet vacuum $|0\rangle$, we have

$$\Phi = [s(\mathbf{p}) + g_{ij}(\mathbf{p}) a^{Li}_{-1} a^{Rj}_{-1} + \ldots] c_1 \bar{c}_1 |\mathbf{p}\rangle\,, \tag{5}$$

where $|\mathbf{p}\rangle = e^{i\mathbf{p}\cdot\boldsymbol{\phi}(0)}|0\rangle$. Here $s(\mathbf{p})$ is the tachyon state in the momentum space and $g_{ij}(\mathbf{p})$ contains the "would-be" graviton. It is clear that the space we introduced is much bigger than the Fock space of the physical states in the closed bosonic string. The important point is that by varying the charge vector $\boldsymbol{\alpha}$ [satisfying the constraint Eq. (3) for left and right movers separately], the space is big enough to include the Fock spaces corresponding to all the classical vacua of the closed bosonic string. In fact, it is even bigger than that. For any specific choice of the central charges, there are numerous unphysical states. For example, let $c_i = 1$ for $i = 0, 1, \ldots, 25$, and $c_i = 0$ for the rest for both the left and the right movers; this corresponds to the usual bosonic string except that the 25 ($c_i = 0$ nonunitary) conformal fields introduce numerous unphysical states. In the string field theory, we must project them out. Let us introduce a projection operator $\mathcal{P}(\boldsymbol{\alpha}, \boldsymbol{\phi})$ to remove these unphysical states when we go on shell. Although the explicit form of this operator $\mathcal{P}(\boldsymbol{\alpha}, \boldsymbol{\phi})$ is not known, operationally we have a pretty good idea of how it functions. For specific choices of $\boldsymbol{\alpha}$, we can project out all the nonunitary Virasoro representations, leaving behind a unitary conformal family.[9] We shall elaborate on this projection in a moment.

The string field action can then be (symbolically) written down as

$$S = \int \mathcal{D}\phi_a \mathcal{D}b\, \mathcal{D}c [\tfrac{1}{2}\langle \Phi \mathcal{K} \mathcal{P} \Phi \rangle + \tfrac{1}{3}\langle \Phi\Phi\Phi \rangle]\,. \tag{6}$$

Here, the action is essentially the usual string field action,[3,4,10,11] except for the inclusion of $\mathcal{P}$ and $\boldsymbol{\alpha}$; the range of $\boldsymbol{\alpha}$ is constrained by Eq. (3). On shell, the projection operator $\mathcal{P}(\boldsymbol{\alpha}, \boldsymbol{\phi})$ keeps only unitary conformal families. The kinetic operator $\mathcal{K}$ includes Becchi-Rouet-Stora-Tyutin (BRST) operators; on shell, the BRST symmetry removes the unphysical timelike modes. Note that $\mathcal{P}$ is required to commute with $\mathcal{K}$. On shell, the kinetic operator $\mathcal{K}$ guarantees left-right level matching and provides the kinetic operator for each physical field. The explicit form of the kinetic operator $\mathcal{K}$ is not settled yet. In the absence of the interaction term, we can choose[10,11] either $\mathcal{K} = (\bar{c}_0 Q + c_0 \bar{Q})\delta(L_0 - \bar{L}_0)$ or $\mathcal{K} = 2\pi Q\bar{Q}/\sin[\pi(L_0 + \bar{L}_0)]$ where the left (right) BRST charge operator Q ($\bar{Q}$) obeys $Q^2 = \bar{Q}^2 = 0$, and L_0 ($\bar{L}_0$) is the corresponding Virasoro generator. Actually, the difficulty associated with the covariant formulation of closed-string field theory is not essential to our construction. We could easily have applied the idea to the light-cone string field theory where the string field action is known.[4] In that case, we only introduce 48 chiral bosons, with $c = 24$ for the transverse modes. All classical vacua that can be recovered from the light-cone formulation will have at least one space and one time dimension. This is not such a big loss if we can write down the action explicitly.

At this point, it is useful to compare this formulation with the free gauge-field theory:

$$S = \tfrac{1}{2}\int dx\, A_\mu K P^{\mu\nu} A_\nu$$

$$= \tfrac{1}{2}\int dx\, A_\mu \partial^2 (\delta^{\mu\nu} - \partial^\mu \partial^\nu / \partial^2) A_\nu\,. \tag{7}$$

The gauge symmetry $\delta A_\mu = \partial_\mu \lambda$ follows simply because $P^{\mu\nu}\partial_\nu\lambda = 0$. In the free covariant string field-theory case, $\mathcal{P}(\boldsymbol{\alpha}, \boldsymbol{\phi})$ clearly allows a very large symmetry, i.e., all $\delta\Phi$ that satisfy $\mathcal{P}\delta\Phi = 0$. On shell, the theory is unitary by construction. All negative-norm states from the 50 chiral bosons with Euclidean metric are removed by the projection operator $\mathcal{P}$ while the negative-norm states from the timelike boson ϕ_0 are removed by the BRST operators in $\mathcal{K}$. Off shell, negative-norm states beyond those from the timelike modes are present in huge numbers, in general. We believe this new symmetry, resulting from the projection $\mathcal{P}$, is the true string symmetry. In comparison, the usual BRST symmetry in string field theory is simply a compact way of organizing the gauge symmetries and kinematics that are already present in quantum field theory for massless and massive particles.

This approach is very different from the approach implicit in many recent works, e.g., derivation of the equations of motion via the vanishing of the β functions of nonlinear σ models,[12] and the renormalization-group flow of two-dimensional nonlinear σ models towards conformal fixed points.[13] In Euclidean space, these approaches demand unitarity off shell and (super)conformal symmetry is required only on shell. Our approach demands (super)conformal symmetry both on and off shell, while unitarity is required only on shell. This embraces the same philosophy of gauge theories.

The operational meaning of the projection operator $\mathcal{P}(\boldsymbol{\alpha}, \boldsymbol{\phi})$ needs some clarification. Consider any boson with $c < 1$. For any given p in a set of discrete momenta (discrete because chiral boson with anomaly takes value on a circle), the Fock space F^c_p corresponding to any $e^{ip\phi}$ is *a priori* very large,

$$F^c_p = \{ |n, p\rangle = a^{n_1}_{-1} a^{n_2}_{-2} \cdots |p\rangle \}\,. \tag{8}$$

Furthermore, the Fock space has a natural Virasoro structure from Eq. (1). Let us consider any model in the

$c<1$ unitary series, which can be bosonized. In general, F_p^c as Virasoro module contains negative-norm states. For specific choices of the highest weight $h=p(p-2\alpha)/2$ permitted in the unitary series, the Fock space F_p^c is still much bigger than the irreducible representation of Virasoro algebra V_h, and, in general, contains negative-norm states. Feigin and Fuchs[5] (FF) developed a way to project out all states except those in the Virasoro representation: $P_{\rm FF}:F_p^c\rightarrow V_h$ where h is an allowed highest weight.

Going through all choices of $\mathbf{p}$, we have $\mathcal{P}(\boldsymbol{\alpha},\boldsymbol{\phi})$: $\{F_p^c\}\rightarrow\oplus_i V_{h_i}$. For $c<1$, the final result is well known and the set of representations $\oplus V_{h_i}$ is automatically guaranteed to form a unitary conformal family C_α; i.e., the operator-product expansion of any two fields in C_α closes in C_α. For $c=26$, we further require the closure property on the set of irreducible representations. Otherwise it is projected out. For a given choice of $\boldsymbol{\alpha}$, there should be a unique unitary conformal family C_α (up to discrete symmetries) and $\mathcal{P}(\boldsymbol{\alpha},\boldsymbol{\phi})$ projects out everything else. For some choices of $\boldsymbol{\alpha}$, C_α can be trivial; i.e., it contains only the identity element. For a fixed $\boldsymbol{\alpha}$, the equation of motion for the free-string field, $\mathcal{K}\mathcal{P}\Phi=0$, may have more than one solution. Given the classical solution, we may still have to check if it is consistent with quantum mechanics, i.e., check for anomalies as in the case in quantum field theory. This means checking the consistency of the one-loop diagrams. In particular, modular invariance at the one-loop level must be maintained.

Let us illustrate the above points with the case of two bosons (c_1,c_2) whose total central charge is $c_1+c_2=1$. We discuss how the projection $\mathcal{P}$ operates on the Fock space as we move from the $(\frac{1}{2},\frac{1}{2})$ case to the (1,0) case. This example also illustrates how a space dimension can arise. Of course we can connect the two cases by going along the Z_2 orbifold and then along the torus by changing radius, so that the theory remains unitary throughout;[14] but instead we shall vary *away* from unitary conformal theory by following $c_1=\frac{1}{2}+y$ and $c_2=\frac{1}{2}-y$ where y varies from 0 to $\frac{1}{2}$.

At the point $(\frac{1}{2},\frac{1}{2})$ the projection operator $\mathcal{P}$ essentially eliminates all the Fock space except the three unitary representations of the Ising model, i.e.,

$$\mathcal{P}(\boldsymbol{\alpha},\boldsymbol{\phi}):F_p^{1/2}\otimes F_p^{1/2}\rightarrow(V_0\oplus V_{1/2}\oplus V_{1/16})\otimes(V_0\oplus V_{1/2}\oplus V_{1/16})\,. \tag{9}$$

Therefore, the theory describes *either* two copies of Ising models *or* a complex Dirac fermion. For $y\neq 0,\frac{1}{2}$, the theory is not unitary and hence is projected out. At the end point (1,0), $\mathcal{P}(\boldsymbol{\alpha},\boldsymbol{\phi})$ acts on the $c=0$ boson alone. Since the only unitary representation of the $c=0$ Virasoro algebra is the identity representation, i.e., $\mathcal{P}(\boldsymbol{\alpha},\boldsymbol{\phi}):\oplus_p F_p^0\rightarrow I$; the projection essentially eliminates this boson. For the $c=1$ boson, the closure condition for unitary conformal family (coming from interaction vertex) implies the single valuedness on the world sheet. Consider

$$\exp[ip_L\phi(z)+ip_R\phi(\bar z)]\exp[iq_L\phi(w)+iq_R\phi(\bar w)]$$
$$=(z-w)^{-p_Lq_L}(\bar z-\bar w)^{-p_Rq_R}\exp[i(p_L+q_L)\phi(w)+i(p_R+q_R)\phi(\bar w)]+\cdots\,. \tag{10}$$

The single-valuedness condition is satisfied if $p_Lq_L-p_Rq_R\in Z$. The level-matching condition from $\mathcal{K}$ implies $p_L^2/2+N=p_R^2/2+\bar N$. Therefore, the momenta are

$$p_L=\left(nR+\frac{m}{2R}\right),\quad p_R=\left(nR-\frac{m}{2R}\right),\quad n,m\in Z\,, \tag{11}$$

where the radius R is arbitrary. In the limit $R\rightarrow\infty$, the momentum $p_L=p_R$ takes continuous values, and a_{L0} can be identified with a_{R0}. If we choose 26 bosons, including ϕ_0, to have $c=1$ and the rest to have $c=0$, we can repeat the above procedure for each $c=1$ boson. The $g_{ij}(\mathbf{p})$ in Eq. (5) become $g_{\mu\nu}(\mathbf{p})$ as functions of continuous momenta $\mathbf{p}$, where $\mu,\nu=0,1,\ldots,25$. The space-time then emerges from the Fourier transformation of $\mathbf{p}$. Clearly this is merely one particular solution out of many possibilities.

Note that the requirement that $\mathcal{P}$ and $\mathcal{K}$ commute imposes a strong constraint on $\mathcal{P}$. In particular, $\mathcal{P}$ can only decouple $c=0$ conformal fields. Suppose, for fixed $\boldsymbol{\alpha}$, that there are central charges not in the unitary series, say some $c_i<0$; then *either* $\mathcal{P}$ projects out everything leaving behind a trivial solution, *or* the corresponding $c_i<0$ fields combine with some $c>0$ fields to yield nontrivial unitary conformal families. That is to say, $\mathcal{P}$ cannot completely project out any boson with nonzero central charge; otherwise, the effective central charge in the BRST operator will not be 26, i.e., $\mathcal{P}$ would not commute with $\mathcal{K}$.

Now we are ready to explain our working hypothesis of choosing the 51-dimensional space for the bosonic string. The smallest central charge for any unitary conformal field is $c=\frac{1}{2}$, so we introduce 50 chiral bosons to replace the 25-space dimensions. We can have only one time dimension, otherwise the BRST symmetry could not remove all the timelike modes. This gives the 51 dimensions. Of course, we can consider higher-dimensional space if desired. The crucial point is that for fixed $c=26$, the number of dimensions conceivably needed to describe all possible conformal fields needed in string field theory is finite.

Applying the same hypothesis to the heterotic string, we introduce 51 holomorphic bosons plus one for the ghost for left movers and 29 antiholomorphic bosons (with two time components) plus three for the (super)ghosts for the right movers.

In summary, we introduce a configuration space upon which string fields are defined and demand (super)conformal symmetry both on and off shell while unitarity only on shell. The enlarged space includes all classical vacua and hopefully the true vacuum of nature as well. We point out that the projection on an enlarged space introduced in this paper also suggests a way to classify conformal field theories.

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