Gravitational Radiation from Primordial Solitons and Soliton-Star Binaries

Marcelo Gleiser
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
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The possibility that both the formation of nontopological solitons in a primordial second-order phase transition and binary systems of soliton stars could generate a stochastic gravitational-wave background is examined. The present contribution of gravitational radiation to the energy density of the Universe from these processes is estimated for a number of different models. The detectability of such contributions from the timing measurements of the millisecond pulsar and spaceborne laser interferometry is briefly discussed and compared to other cosmological and local sources of background gravitational waves.

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Theories that conserve an additive quantum number N carried either by a complex scalar field or by a spin-$\frac{1}{2}$ field have been shown to exhibit localized, stable field configurations known as nontopological solitons (NTS's).[1] Within the context of renormalizable interactions, the charge-carrying field ϕ interacts with a real scalar field σ in such a way as to have different masses at the distinct vacua of the theory. Frieman, Gelmini, Gleiser, and Kolb (hereafter FGGK) studied NTS's with a potential given by[2]

$$V(|\phi|,\sigma)=\tfrac{1}{8}\lambda_1(\sigma^2-\sigma_0^2)^2+h|\phi|^2(\sigma-\sigma_0)^2 + \tfrac{1}{3}\lambda_2(\sigma-\sigma_0)^3\sigma_0+g|\phi|^4+\Lambda\,, \qquad (1)$$

the constant Λ being adjusted to give $V=0$ at the absolute minimum of the potential.

From (1) it is easy to see that for $\sigma=\sigma_0$, the local minimum of V, the field ϕ is massless and for $\sigma=\sigma_-$, the global minimum of V (not exactly at $-\sigma_0$), ϕ acquires a mass $m_\phi^2=h(\sigma_- -\sigma_0)^2$. Thus, massless ϕ particles are trapped inside a configuration with σ at its local minimum which is separated from the absolute vacuum by a domain wall of thickness of order σ_0^{-1}. The mass and radius of the configurations that minimize the energy are found to be, in the limit of small coupling g, $M=(4\pi/3)\sqrt{2}\Lambda^{1/4}N^{3/4}$ and $R=(N/4\Lambda)^{1/4}$, where N is the global conserved charge coming from the invariance of V under $\phi\rightarrow e^{i\theta}\phi$.[2] Note that the mass grows as $N^{3/4}$. Thus, for N bigger than a minimum charge $N_{\min}$, the NTS is classically stable. These solitons are hereafter called type I. If the coupling g is not negligible, or if the charge-carrying field has spin $\frac{1}{2}$, the NTS will behave as ordinary matter with its mass growing linearly with the charge N.[3] These solitons are hereafter called type II. In this way, NTS's are reminiscent of the bag models of the 1970's.[4]

Properties of the NTS's such as their mass and radius will be given in terms of their charge N and the relevant vacuum energy m. (In the example above, $m\sim\Lambda^{1/4}\sim\lambda_1^{1/4}\sigma_0$. In general, m will be of the order of the symmetry-breaking scale of the model.) Thus, type-I NTS's have $M=mN^{3/4}$ and $R=m^{-1}N^{1/4}$, while type-II NTS's have $M=mN$ and $R=m^{-1}N^{1/3}$. (Only NTS solutions with nondegenerate vacua will be considered here.) As the charge N is increased, gravitational corrections to the total energy of the configurations become progressively more important. As remarked by Lee,[5] it is easy to estimate the charge N for which this occurs by obtaining the Schwarzschild radius of the configuration. For type-I NTS's one finds $N_c\sim(M_{\rm Pl}/m)^4$, $M_c\sim M_{\rm Pl}^3/m^2$, and $R_c\sim M_{\rm Pl}/m^2$, while for type II one finds $N_c\sim(M_{\rm Pl}/m)^3$, $M_c\sim M_{\rm Pl}^3/m^2$, and $R_c\sim M_{\rm Pl}/m^2$, where $M_{\rm Pl}$ is Planck's mass. It is clear from these results that for a mass scale $m\simeq1$ GeV, "soliton stars" have masses and radii comparable to neutron stars.[6] It is worth mentioning that *purely gravitational* bound states of charge-carrying fields have been first discussed in the literature independently on NTS's. For complex scalar fields, many different kinds of "boson stars" have been found; a free or nonminimally coupled massive field gives rise to configurations ("minisoliton stars") with critical mass $M_c\simeq0.6M_{\rm Pl}^2/m$ and radius $R_c\simeq3m^{-1}$,[7] while fields with $V(|\phi|)=\lambda|\phi|^4$ give rise to configurations with $M_c\simeq0.22\alpha^{1/2}M_{\rm Pl}^2/m$, in the limit of a large α, with $\alpha\equiv\lambda M_{\rm Pl}^2/4\pi m^2$.[8]

In this Letter I would like to suggest a few mechanisms by which NTS's could generate a stochastic gravitational-radiation background (GRB), both from their formation in a primordial phase transition and by assuming that soliton (or boson) stars contribute significantly to the dark matter in the Universe and that, in analogy to ordinary stars and compact objects, they form binary and multiple systems. It should be pointed out that other primordial sources of GRB have been proposed in the literature, such as in the transition from a de Sitter inflationary phase to a radiation-dominated era,[9] cosmic strings,[10] primordial black-hole formation,[11] and from a first-order QCD transition by Witten[12] and Hogan.[13] A plane gravitational wave with amplitude h has an energy density (see Refs. 6 and 13) $\rho_g=(16\pi G)^{-1}\langle\dot h^2\rangle$. A stochastic background is a superposition of such waves with random Fourier amplitudes for each mode and polarization. The volume-averaged density of waves is, in units of $\rho_c \equiv 3H_0^2/8\pi G$, with $H_0 \equiv 100 h_{100}$ km(s Mpc)$^{-1}$ the Hubble constant,

$$\Omega_g(\nu) \equiv \frac{\nu}{\rho_c}\frac{d\langle\rho_g(\nu)\rangle}{d\nu} \simeq \langle h_\nu^2\rangle \left[\frac{\nu}{H_0}\right]^2, \qquad (2)$$

where $\langle h_\nu^2\rangle$ is the mean-square amplitude in a broad band of frequency $\delta\nu \simeq \nu$.

The primordial formation of NTS's has been discussed recently by FGGK. They have shown that it is possible to form NTS's in a primordial second-order phase transition in which the soliton "lives" in the false vacuum of the potential given in Eq. (1); there are two possible formation mechanisms, depending on whether one or both vacua percolate. In the first case (below percolation), the Universe will consist of a true vacuum "sea" with isolated bags of false vacuum which will be rendered stable by the kinetic pressure coming from the confined massless ϕ particles. In the second case (above percolation), an infinite domain wall with complicated topology will permeate the Universe. The dynamics of the wall will be a result of the interplay between the tension force on the wall and the false vacuum pressure that will tend to squeeze away the false vacuum regions.[14] As a result, regions of false vacuum will be pinched off trapping a charge N inside, giving rise to NTS's. The distribution in sizes of the NTS's formed in the below-percolation case is given by percolation studies in three-dimensional lattices.[15] The number density of bags with charge N at the formation temperature ($\equiv T_G$, the Ginzburg temperature, given in terms of the formation time t_G, in a radiation-dominated Universe, by $t_G \simeq 0.3 g_*^{-1/2} M_{\rm Pl} T_G^{-2}$, where g_* is the number of relativistic degrees of freedom at T_G) can be written as

$$n_G(N) = (N/N_0)^{-3/2}\exp(-N/N_0)/V_\xi \equiv \alpha(N)/V_\xi\,, \qquad (3)$$

where $N_0 \equiv \eta n_\phi V_\xi$ is the charge in a correlation volume, defined here as $V_\xi \equiv (2\xi)^3 \simeq 8m^{-3}$, η is the excess ratio of particles over antiparticles, and n_ϕ is the number density of ϕ particles at T_G, $n_\phi \simeq \zeta(3)T_G^3/\pi^2$. ($T_G$ is properly defined as the temperature when thermal fluctuations freeze out due to the expansion of the Universe. Here $T_G \sim m$ is assumed, although it could be lower depending on the nature of the phase transition.)

At formation, the false vacuum bags are nonspherical, and will pulsate away the excess mass representing the departure from sphericity by means of gravitational radiation with frequency $\nu \sim \langle R\rangle^{-1}$, where $\langle R\rangle$ is the average radius of the configuration. Because of the complicated interplay between the pulsations, the motion of the walls, and the kinetic pressure from the confined particles, the NTS's will attain spherical symmetry at a time t_2 and will stop radiating. (The possibility of bag fragmentation is neglected here.) In what follows, a simple-minded calculation of the GRB generated from type-I NTS's at formation will be presented. The result for type-II solitons is given at the end. Assume that the dominant energy-loss mechanism is by gravitational radiation. (The NTS's will also "evaporate" similarly to strange matter lumps.[16] More will be said about this later.) Assume further that the NTS's radiate their energy mostly in the quadrupole mode with constant frequency $\nu \sim \langle R\rangle^{-1} = mN^{-1/4}$; the standard quadrupole formula gives[6]

$$\dot M_g \sim -GM^2R^4\nu^6 \sim -(m^2/M_{\rm Pl})^2 N\,. \qquad (4)$$

By using the mass and radius of the spherical configurations, it is assumed that the bulk matter in quadrupole motion has, on average, the equilibrium mass and that the motion is, on average, over the equilibrium radius. If there were no other damping forces, the bags would radiate all their mass in a time $\tau \sim M/|\dot M_g| \sim M_{\rm Pl}^2 m^{-3} N^{-1/4}$. However, as pointed out earlier, this will not be the case. The mass radiated away (ΔM) can be expressed in terms of the efficiency parameter $\epsilon \equiv 1-\Delta M/M$. In this way, using the rate given in Eq. (4), the time t_2 at which the surviving mass is ϵM is roughly $t_2 \sim t_G[1+3.3 g_*^{1/2}(1-\epsilon)N^{-1/4}M_{\rm Pl}/m]$. An estimate for ϵ (and thus t_2) could be obtained with better knowledge of the bag's dynamics, which should be studied in more detail.

In an adiabatic expansion, the ratio $n_G(N)/s$, where $n_G(N)$ is given in Eq. (3) and $s = 2\pi^2 g_* T_G^3/45$ is the entropy density of radiation at T_G, is kept constant. Thus, the number density of bags with charge N between t_G and $t_G + dt_G$ is

$$dn_G \sim -\frac{3\times 10^{-2}}{g_*^{3/4}}\frac{\alpha(N)}{t_G^{5/2}} M_{\rm Pl}^{3/2} dt_G\,, \qquad (5)$$

while at a time $t_G < t' < t_2$,

$$dn' \sim -3\times 10^{-2} M_{\rm Pl}^{3/2} g_*^{-3/4}\alpha(N) t_G^{-1} t'^{-3.2} dt_G\,.$$

The energy density radiated by bags of charge N in the time interval dt' is $d\rho_g' \sim -(m^4 M_{\rm Pl}^{-2} N) dn' dt'$. It is convenient to refer all quantities to the end of the radiation-dominated era, $t_{\rm eq} \simeq (4\times 10^{10}\,{\rm s})(\Omega h_{100}^2)^{-2}[T_0/(2.7\ {\rm K})]$, where T_0 is the present cosmic background temperature. The red-shifted energy corresponding to $t_{\rm eq}$ is $d\rho_g^{\rm eq} \sim (t'/t_{\rm eq})^2 d\rho'$. The frequency and frequency range at $t_{\rm eq}$ are, respectively, $\nu_{\rm eq} \sim (t'/t_{\rm eq})^{1/2} mN^{-1/4}$ and $d\nu_{\rm eq} \sim (t' t_{\rm eq})^{-1/2} mN^{-1/4} dt'$. Thus, the expression for $d\rho_g/d\nu$ at $t_{\rm eq}$ can be written in terms of t_G as

$$\frac{d\rho_g^{\rm eq}}{d\nu_{\rm eq}} \sim 1.7\times 10^{-2}\nu_{\rm eq}^2 g_*^{-1}(t_{\rm eq}t_G^3)^{-1/2} N^{7/4}\alpha(N) dt_G\,. \qquad (6)$$

The total contribution from NTS's of charge N can be obtained by integrating over t_G, the limits being determined by the time interval in which the bags can radiate, $t_G < t < t_2$. Expressing t_G in terms of $\nu_{\rm eq}$ as t_G

$=v_{eq}^2 t_{eq} m^{-2} N^{1/2}$, the integration gives

$$d\rho_g^{eq}/dv_{eq} \sim 3.4\times 10^{-2} v_{eq} (g_* t_{eq})^{-1} \alpha(N) m N^{3/2}.$$

Neglecting t_2 is a good approximation whenever $m \ll M_{Pl}$, even for poor efficiency ($\Delta M \ll M$), since the factor $N^{-1/4} M_{Pl}/m$ in the expression for t_2 is roughly always between $(M_{Pl}/m)^{1/4}$ and M_{Pl}/m. Thus, contrary to cosmic strings, most of the radiation from NTS's comes shortly after they are formed, consistently with the naive model assumed here. In terms of the radiation energy density at t_{eq}, $\rho_\gamma^{eq} \simeq 0.1 M_{Pl}^2/t_{eq}^2$, $d\rho_g^{eq}/dv_{eq}$ is

$$\frac{d\rho_g^{eq}}{dv_{eq}} \sim 0.1\, v_{eq} g_*^{-1} (\rho_\gamma^{eq})^{1/2} \left(\frac{m}{M_{Pl}}\right) \alpha(N) N^{3/2}. \qquad (7)$$

Since the quantity $(d\rho_g/dv)(v\rho_\gamma^{1/2})^{-1}$ is a constant of the expansion, today's spectrum is obtained from Eq. (7) by dropping the subscripts and superscripts "eq." Using $t_0 = 3\times 10^{17}$ s for the present age of the Universe, $t_{eq} = 4\times 10^{10}$ s, and that the red-shifted dominant frequency is $v_0 = v_{eq}(t_{eq}/t_0)^{2/3} \sim 2.8\times 10^{11} (N_{g_*})^{-1/4}$ Hz, the contribution in gravitational radiation from NTS's with charge N is, in units of the critical density ρ_c,

$$\Omega_g \sim 2.1\times 10^{-9} (m/M_{Pl}) \alpha(N) N, \qquad (8)$$

where $g_* = 100$ and $h_{100} = 1$ were used.

As there is an absolute upper bound on Ω_g from nucleosynthesis ($\Omega_g \lesssim 10^{-5}$, Ref. 11), interesting values for the GRB from NTS's obtain whenever the product $(m/M_{Pl})\alpha(N)N$ is at least bigger than unity. Of course, the result in Eq. (8) is only for bags of charge N; the total Ω_g is properly obtained by summing over bags with all possible values of N, given, in principle, in the interval $N_{min} \lesssim N \lesssim N_c \sim \eta (M_{Pl}/m)^3$. ($N_c$ is the maximum charge inside the horizon at formation.) However, as mentioned before, NTS's can evaporate with only the largest bags surviving. In fact, following Ref. 16, it is easy to see that the evaporation rate for type-I NTS's is $\dot{N}_{evap} \sim m_\phi T^2 \exp(-I_b/T) f_\phi m^{-2} N^{1/2}$, where m_ϕ is the mass of the ϕ particles outside, $I_b \sim m_\phi$ (approximation good for large N) is the binding energy, and f_ϕ is the absorption efficiency. Thus, for a NTS to survive until today, its charge has to be, roughly, $N_{evap} \gtrsim (M_{Pl}/m)^2$. (Unless the formation temperature is much less than m.[16]) As the ratio between mass loss by evaporation ($\dot{M}_{evap} = \frac{3}{4} m N^{-1/4} \dot{N}_{evap}$) and by gravitational radiation is bigger than unity for $N \lesssim (M_{Pl}/m)^{8/3}$, $\dot{M}_{evap}/\dot{M}_g \sim M_{Pl}^2 m^{-2} N^{-3/4}$, it is reasonable to assume that the most gravitational radiation will be emitted by the bags that survived evaporation, with the spectrum dominated by the bags with $N \sim N_{evap}$. Thus, Eq. (8) can be rewritten as $\Omega_g \sim 10^{-9} \alpha(N_{evap}) M_{Pl}/m$. The maximum GRB from NTS's compatible with nucleosynthesis is obtained for $\alpha(N_{evap}) \sim 10^4 m M_{Pl}^{-1}$. Unfortunately, the exponential suppression is too strong for such a value of α to be obtained in most models, unless the couplings are very weak, for reasonable mass scales. For example, the model of Eq. (1) has

$$\alpha(N_{evap}) \sim 10^{-14} \lambda_1^{-3/2} (m/M_{Pl})^3 \exp[-10^9 \lambda_1^3 (M_{Pl}/m)^2]$$

with $\eta = 10^{-9}$. The amplitude for the waves corresponding to Ω_g of Eq. (8) can be obtained from Eq. (2), $h_0 \sim 10^{-34} (m/M_{Pl})^{1/2} [\alpha(N)]^{1/2} N^{3/4}$ or, in terms of N_{evap}, $h_0 \sim 10^{-34} (M_{Pl}/m)^2 [\alpha(N_{evap})]^{1/2}$. In order for a gravitational wave to be detected as noise in the timing of the millisecond pulsar, its amplitude has to be, for a timing stability of $\Delta t \sim 10^{-7}$, $h_d \gtrsim (\Delta t)/P_0 \sim 5\times 10^3 (N g_*)^{-1/4}$, where $P_0 = 2\pi/v_0$ is the present period. Thus, from the amplitude calculated above, a GRB can be detected whenever the inequality $(m/M_{Pl})^{1/2} [\alpha(N)]^{1/2} N \gtrsim 10^{37}$ is satisfied. For type-II NTS's the same calculation can be performed, giving, $\Omega_g \sim 10^{-9} (n/M_{Pl}) \alpha(N) N^{4/3}$ and $h_0 \sim 10^{-34} (n/M_{Pl})^{1/2} [\alpha(N)]^{1/2} N$. Note that for type-II NTS's $N_{evap} \sim (M_{Pl}/m)^3$.

Before leaving primordial NTS's I would like to mention another possible mechanism for gravitational wave emission in the above-percolation case. In the QCD phase transition, Witten suggested that gravitational radiation would be produced due to the collision of true vacuum bubbles in the percolation process.[12] The point is that the pinching mechanism is also a collision process and will generate a GRB. A gravitational wave generated at t_G would have today a wavelength and amplitude given by $\lambda_0 \sim 4\times 10^{15} [m/(1\ \text{TeV})] \lambda_G$ and $h_0 \sim 2\times 10^{-16} \times [(1\ \text{TeV})/m] h_G$. What are λ_G and h_G? A body with mass M and radius R has a gravitational field of order $\sim GM/R$. If two such bodies collide head on to a stop or are scattered through large angles, the gravitational waves generated have amplitude $h \sim \beta GM/R$, where β is the efficiency parameter for the emission of gravitational waves. Thus, assuming that the mass involved in such collisions is given roughly by the NTS mass of average radius $R \sim \lambda_G$, one obtains, for type-I NTS's, $h_G \sim \beta (m/M_{Pl})^2 N^{1/2}$. Thus, today, $\lambda_0 \sim 8 N^{1/4}$ cm and $h_0 \sim 2.3 \times 10^{-48} [m/(1\ \text{TeV})] \beta N^{1/2}$. For type-II solitons the wavelength and amplitude are now $\lambda_0 \sim 8 N^{1/3}$ cm and $h_0 \sim 2.3\times 10^{-48} \beta [m/(1\ \text{TeV})] N^{2/3}$. Putting $N = (0.1 M_{Pl}/m)^3$, $m = 100$ MeV, and $\beta \sim 1$, the results of Ref. 12 are reproduced, which are within the detectable range $h_d \gtrsim 2.3\times 10^{-15}$, for a timing stability of $\Delta t \sim 10^{-7}$. Assuming the same charge and efficiency for type-I NTS's, the amplitude is $h_0 \sim 2.3\times 10^{-24} [(1\ \text{TeV})/m]^{1/2}$. Thus, for reasonable mass scales the signal is quite weak and could perhaps be detected by laser interferometry in space; with an integration time of $\tau \sim 10^6$ s, standard coincidence techniques would allow a detection of amplitudes as weak as $h \sim 10^{-22} [\tau/(10^6\ \text{s})] (v_0 \tau)^{1/4}$. More details can be found in Ref. 13 and references therein.

I now wish to discuss a quite different mechanism for the generation of a GRB, coming from soliton- (or boson-) star binaries. Apart from the assumption that such objects exist, there is also the assumption that they would mostly appear in binary or multiple systems, which in turn implies that in their formation process a prototype region dense in scalar matter would fragment into smaller regions that would condense and orbit around each other, emitting gravitational radiation in the process. If such objects play any role in the missing-matter problem, it is not so far fetched to imagine such systems to be located in galactic halos. Of course, ordinary binary systems also contribute to the GRB. From standard results for binary systems,[6] the fraction of rest mass radiated in a Hubble time is $\Delta M \sim (H_0 P)^{-1}(P/R_S)^{-7/3}$, where R_S is the Schwarzschild radius related to the reduced mass of the system $\mu \equiv M_1 M_2/(M_1+M_2)$ and P is the orbital period.[13] Defining $\Omega_{\rm BS}(P)$ as the density in binaries with period P in units of the critical density, one obtains

$$\Omega_g^{\rm BS} \sim 10^{-9}\Omega_{\rm BS}(P)(\mu/M_\odot)^{7/3}[P/(10^4\,{\rm s})]^{-10/3}.$$

Taking $\Omega_{\rm BS}(P)=10^{-4}$, the typical contribution from binaries is roughly $\Omega_g^{\rm BS} \sim 10^{-13}$. Similarly, for the related amplitude h one finds, from Eq. (2), $h \sim 10^{-22}$ for frequencies of 10^{-3} Hz. For binary systems, the spiraling time is $\tau_0 = \frac{5}{256}(c^5/G^3)a_{\rm now}^4/M^2\mu$, where $a_{\rm now}$ is the present distance between the stars and $M = M_1+M_2$. Thus, in order for the system to have survived until today, $a_{\rm now} \gtrsim 2\times 10^{11}(M^2\mu/M_\odot^3)^{1/4}$ cm. Using the fact that the critical mass for soliton (or boson) stars can be written, in general, as $M = \alpha M_{\rm Pl}^3/m^2$, a binary system of identical soliton stars would exist today if $a_{\rm now} \gtrsim 4.1 \times 10^6 \alpha^{3/4}[(1\ {\rm TeV})/m]^{3/2}$ cm. Thus, a lower bound for the period is $P \gtrsim (a_{\rm now}^3/MG)^{1/2} \simeq 0.45\alpha^{5/8}[(1\ {\rm TeV})/m]^{5/4}$ s. The contribution to the GRB from soliton binaries is then

$$\Omega_g^{\rm SS} \lesssim 10^{-10}\Omega_{\rm SS}(P)\alpha^{1/4}[(1\ {\rm TeV})/m]^{1/2}. \qquad (9)$$

For $\alpha \sim 1$, and taking $\Omega_{\rm SS}(P) \simeq 1$, soliton binaries dominate over ordinary binaries so long as $m \lesssim 10^6$ TeV. The related amplitude is, for a frequency $\nu = 10\alpha^{-5/8} \times [m/(1\ {\rm TeV})]^{5/4}$ Hz, $h \sim 3\times 10^{-25}[(1\ {\rm TeV})/m)]^{3/2}$. Repeating the same calculation for minisoliton stars with $M = 0.6 M_{\rm Pl}^2/m$, one obtains $\Omega_g^{\rm MSS} \lesssim 10^{-14}\Omega_{\rm MSS}(P)[(1\ {\rm TeV})/m]^{1/4}$ and $h \lesssim 10^{-37}\Omega_{\rm MSS}(P)^{1/2}[(1\ {\rm TeV})/m]^{3/4}$. If m is the axion mass $m \sim 10^{-5}$ eV, minisoliton stars would have a mass comparable to the Earth's mass and minisoliton binaries would generate a GRB with typical amplitude $h \sim 10^{-24}$ and frequency $\nu \sim 4.5$ Hz, for $\Omega_{\rm MSS} \approx 1$.

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