Classification of Beam Breakup Instabilities in Linear Accelerators

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With the use of a standard model, beam breakup growths in all types of linacs are shown to be classified by only two dimensionless parameters which depend on the beam current, beam energy, pulse length, machine length, breakup mode frequency, focal strength, and transverse shunt impedance. This classification suggests that rf cure plays a more important role in the control of beam breakup than focusing in a long pulse beam. The converse holds for a short pulse beam.

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Beam breakup (BBU) instabilities,[1-3] after some twenty years of study, have received considerable renewed interest.[4-10] Their manifestation lead to, at best, a degradation of the beam brightness and at worst, to total beam disruption. Although the control of BBU has always evolved around focusing and manipulation of the structure mode,[11] the great variety of linacs would lead one to suspect that the effective means of stabilization of BBU would depend very much on the accelerator type under consideration.

In fact, different scalings of BBU growth have been found for different types of linacs.[1-3] The modern theory of beam breakup began with Panofsky and Bander[1] who, in explaining the pulse shortening phenomena observed during the first operation of the SLAC Two Mile Accelerator, established BBU growth scalings for linacs with weak focal strength, low current, and long pulse length. Somewhat later, Neil, Hall, and Cooper[2] used an entirely different approach and found a different scaling for linacs with a strong focusing field, high beam current, and moderate pulse length. This scaling has been used extensively in the Advanced Test Accelerator (ATA) experiments at Livermore. Yet another different scaling was obtained by Chao, Richter, and Yao[3] when they assessed the beam's dipole deflection for the next generation linear collider, whose beam is characterized by moderate focal strength, short pulse length, moderate current, and a long propagation distance. For convenience of reference, these three types of scalings will be designated as A, B, and C, respectively. They all are of the cumulative type.[1]

In this paper, we present two dimensionless parameters, s_1 and s_2, which completely characterize cumulative BBU of all types, whether it be A, B, C, or a new type D, as shown in Fig. 1. These two dimensionless parameters depend on the beam current, beam energy, pulse length, machine length, focal strength, breakup mode frequency, and transverse shunt impedance. That such a classification is possible is a result of the fact that BBU of various types are governed by the same equations. In the evaluation of the Green's function to these equations, the above-mentioned dimensionless parameters emerge, the magnitude of which alone would determine the nature of BBU growth. This Green's function will also be examined via a mode coupling approach. This approach, being standard in microwave electronics,[12,13] gives a clear indication as to why BBU of type C, for example, might be more readily stabilized with focusing (e.g., betatron frequency spreads) whereas control of type A would rely more on rf cure (e.g., lowering of the quality factor Q, stagger tune); control of BBU of type B would require both rf cure and external focusing. Much of these, of course, are consistent with the years of experience in the operation of various types of linacs. Some unity of perspective is sought in this paper by a single analysis of a standard model.

For simplicity, we consider a continuum model of a

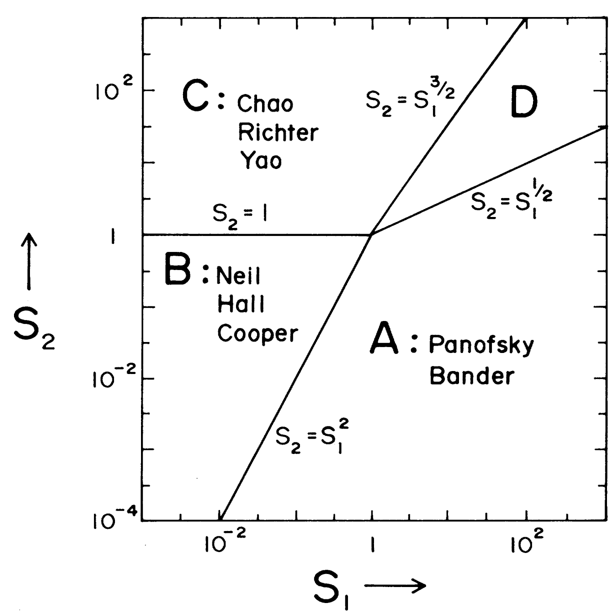

FIG. 1. Classification of cumulative BBU in linacs according to $s_1 \equiv 2\epsilon/\Omega^2$ and $s_2 \equiv 2\epsilon Z/\Omega(T-Z)$.

beam with current I, relativistic mass factor γ, and velocity $v \simeq c$ in a focusing system of betatron frequency ω_c inside a series of identical accelerating units of separation L. Let $\xi(z,t)$ be the transverse displacement of the beam, from its center axis, at position z from the injector at time t, $a(z,t)$ be the transverse Lorentz force per unit rest mass produced by a deflecting dipole mode. This mode is characterized by frequency ω_0, quality factor Q, and transverse shunt impedance $Z_\perp(\Omega)$ in units of Ω. The governing equations for ξ and a are[1,2]

$$\left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right]\left[\gamma\left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right]\xi\right] + \gamma\omega_c^2\xi = a(z,t)\,, \tag{1}$$

$$\frac{\partial^2 a}{\partial t^2}+\frac{\omega_0}{Q}\frac{\partial a}{\partial t}+\omega_0^2 a = 2\gamma\omega_0^4\epsilon\xi(z,t)\,. \tag{2}$$

Equation (1) describes the deflection of the beam by the mode and Eq. (2) describes the excitation of the mode by the beam's transverse displacement ξ. In Eq. (2), $\epsilon=(v/\omega_0 L)[Z_\perp(\Omega)/30Q](I/17\gamma\ \mathrm{kA})$ is the dimensionless coupling constant which determines the strength of BBU interaction.

It is not difficult to show that Eqs. (1) and (2) may be cast into an equivalent integro-differential equation[3] for $x(s,\zeta)\equiv\xi(z,t)$:

$$\frac{\partial}{\partial s}\left[\gamma\frac{\partial x}{\partial s}\right]+\gamma k_\beta^2 x = \frac{e^2}{m_0 c^2}\int_0^\zeta d\zeta' f(\zeta')W(\zeta-\zeta')x(s,\zeta')\,, \tag{3}$$

in which the independent variables (s,ζ) are related to (z,t) by $s=z$, $\zeta=t-z/v$. In Eq. (3), $k_\beta=\omega_c/v$, f is the number of electrons per unit length, W is the "wake function" defined by

$$W(\zeta)=W_0 e^{-\omega_0\zeta/2Q}\left[\frac{\sin\omega_1\zeta}{\omega_1}\right], \tag{4}$$

where $\omega_1\equiv\omega_0(1-1/4Q^2)^{1/2}\simeq\omega_0$. For Eq. (3) to be equivalent to Eqs. (1) and (2), W_0 is related to the coupling constant ϵ by

$$(e^2/m_0)W_0 f = 2\gamma\omega_0^4\epsilon\,, \tag{5}$$

where e is the electron charge and m_0 is the electron rest mass. Equation (3) has often been used.[3,4,7,8,10] For short bunch, $\omega_1\zeta \ll 1$, the approximation $W(\zeta)\simeq W_0\zeta$ leads to the scalings of type C.[3] Hereafter, we shall focus mainly on Eqs. (1) and (2); as these equations would allow more ready interpretation of the results in terms of mode coupling than Eq. (3), even if Eq. (3) is entirely equivalent to the system of Eqs. (1) and (2), once the identification (5) is made.

To proceed further, we now consider a coasting beam with constant γ, ω_c, and ϵ. (The extension to an accelerated beam may be accomplished with the transformation given in Ref. 3.) Assuming a wavelike solution of the form $\exp(i\omega t - ikz)$, Eqs. (1) and (2) yield the following dispersion relationship,

$$D(\omega,k)\equiv[(\omega-kv)^2-\omega_c^2](\omega^2-i\omega\omega_0/Q-\omega_0^2) - 2\omega_0^4\epsilon=0\,, \tag{6}$$

which describes the coupling[14] between the beam mode $(\omega-kv\simeq\pm\omega_c)$ and cavity mode $(\omega\simeq\omega_0+i\omega_0/2Q)$. The coupling strength is ϵ. The construction of a dispersion relation for Eqs. (1) and (2) allows[13] the Green's function to these equations to be written as

$$G(z,t)\sim\int d\omega\, e^{i\omega t - ik(\omega)z}\,, \tag{7}$$

which gives the response at (z,t) to an impulse excitation at $z=0$, $t=0$. The propagation number $k(\omega)$ in (7) is to be solved from the dispersion relation (6).

A saddle-point calculation, to be published elsewhere, yields the following dominant contribution to the Green's function (7):

$$|G(z,t)|\sim e^{-(T-Z)/2Q}e^{\Gamma(T-Z)}\,, \tag{8}$$

where

$$\Gamma=\Gamma(s_1,s_2)=\mathrm{Im}\left\{\sqrt{\psi}\left[1+\frac{s_2^2/s_1}{(\psi-1)^2}\right]\right\}. \tag{9}$$

In Eqs. (8) and (9), all quantities are dimensionless: $T\equiv\omega_0 t$, $Z\equiv\omega_0 z/v$, $T-Z>0$ by causality,[13] and with $\Omega\equiv\omega_c/\omega_0$,

$$s_1\equiv 2\epsilon/\Omega^2\,; \tag{10a}$$

$$s_2\equiv 2\epsilon Z/\Omega(T-Z)\,; \tag{10b}$$

$\psi=\psi(s_1,s_2)$ is the meaningful solution [i.e., one which yields highest growth] to the fourth-degree polynomial

$$(\psi-1)^4+s_1(\psi-1)^3-s_2^2\psi=0\,. \tag{11}$$

The first factor $\exp[-(T-Z)/2Q]$ in Eq. (8) represents the natural decay of the deflecting mode and is independent of the accelerator type. The second factor $\exp[\Gamma(T-Z)]$ gives the BBU growth and reflects the accelerator type *only* through $\Gamma=\Gamma(s_1,s_2)$. Note that the machine length (Z) and the pulse length $(T-Z)$ enter only in s_2 but not in s_1. ($T-Z$ may be regarded as the normalized pulse length since assigning a value to $vt-z$ is equivalent to labeling a particular beam slice from the beam head.)

An examination of Eq. (11) shows that ψ assumes a different dependence on s_1 and s_2 according to the values of s_1 and s_2 in the various domains (Fig. 1): (A) $s_2\ll s_1^{1/2}$ and $s_2\ll s_1^2$, (B) $s_1^2\ll s_2\ll 1$, (C) $s_2\gg 1$ and $s_2\gg s_1^{3/2}$, and (D) $s_1^{3/2}\gg s_2\gg s_1^{1/2}$. It may further be shown from Eqs. (9) and (11) that the exponentiation factor $\Gamma(T-Z)$ in Eq. (8) assumes the following dependences within these various domains:

$$\Gamma_A(T-Z) \simeq \frac{3\sqrt{3}}{4}\left(\frac{s_2^2}{s_1}\right)^{1/3}(T-Z)$$
$$=1.64[\epsilon Z^2(T-Z)]^{1/3}, \tag{12a}$$

$$\Gamma_B(T-Z) \simeq s_2^{1/2}(T-Z)$$
$$=\left[\frac{2\epsilon Z}{\Omega}(T-Z)\right]^{1/2}, \tag{12b}$$

$$\Gamma_C(T-Z) \simeq \frac{3\sqrt{3}}{4}s_2^{1/3}(T-Z)$$
$$=\left(\frac{3^{3/2}}{4}\right)\left[\frac{2\epsilon Z}{\Omega}(T-Z)^2\right]^{1/3}, \tag{12c}$$

$$\Gamma_D(T-Z) \simeq 2\left(\frac{s_2^2}{s_1}\right)^{1/4}(T-Z)$$
$$=2^{5/4}[\epsilon Z^2(T-Z)^2]^{1/4}. \tag{12d}$$

Expression (12a) is the exponentiation factor originally established by Panofsky and Bander for type-A BBU growth [cf. Eq. (33) of Ref. 1]. Equation (12b) was first obtained by Neil, Hall, and Cooper for type B [cf. the exponent in Eq. (5.13) of Ref. 2]. Equation (12c) was reported by Chao, Richter, and Yao for type C [cf. the exponent in Eq. (16) of Ref. 3]. Equation (12d) does not seem to have been reported previously. [The zero focusing limit corresponds to type A, since $s_1 \to \infty$ faster than s_2 as $\Omega \to 0$ according to Eq. (10).]

The exponentiation rates given in Eqs. (12) are just the dominant contribution from the saddle-point calculation of the Green's function (7). The next-order contribution has been shown to yield the milder amplitude variations which were also given in Refs. 1–3 for types A–C, respectively. The boundaries separating the various types shown in Fig. 1 are not sharp. A uniformly valid solution crossing the boundary between type A and type B, for example, was given in Refs. 9 and 14.

The above unified analysis allows us to draw some general conclusions regarding the control of BBU by rf cure and by external focusing in various types of linacs. Since the growth rate Γ is independent of Q and the effect of finite Q enters separately in Eq. (8), let us set $Q=\infty$ for convenience of exposition. Then the dispersion relation (6) gives

$$k(\omega)=\frac{\omega}{v}\pm\frac{1}{v}\left(\omega_c^2+\frac{2\epsilon\omega_0^4}{\omega^2-\omega_0^2}\right)^{1/2}. \tag{13}$$

Substitution of (13) into (7) shows that, without performing the integral, at fixed z, the asymptotic behavior ($t \to \infty$) of the Green's function is dictated by the singularity $\omega=\omega_0$ in Eq. (13). Note that $t \to \infty$ at fixed z corresponds to $s_2 \to 0$ from Eq. (10b) which, in turn, corresponds to type A according to Fig. 1. A modification in ω_0 leads to a strong modification in this singularity in Eq. (13) and consequently to the reduction of BBU growth of type A. Physically, modifications of ω_0 may come from stagger tuning, spreads in the breakup mode frequency; and the lowering of Q may also be interpreted as an effective change of ω_0. On the other hand, modification of ω_c does not change the singular behavior of $k(\omega)$ in Eq. (13). Thus, introduction of spreads in betatron frequencies would not be as effective as rf cure for type A. Some of these aspects were already noted.[9,11,15]

On the other hand, we may introduce the variable $k'=k-\omega/v$ and rewrite the Green's function (7) as

$$G \sim \int dk' e^{-ik'z+i\omega(k')(t-z/v)}, \tag{14}$$

where $\omega(k')$ is also obtained from Eq. (6):

$$\omega(k')=\left(\omega_0^2+\frac{2\epsilon\omega_0^4/v^2}{k'^2-\omega_c^2/v^2}\right)^{1/2}. \tag{15}$$

Now, at fixed $t-z/v$, the behavior of G in Eq. (14) as $z \to \infty$ is dictated by the singularity $k'=\omega_c/v$ in Eq. (15). Note from Eq. (10b) that fixing $t-z/v$ and letting z become large corresponds to $s_2 \gg 1$, i.e., type C in Fig. 1. Thus, Eq. (15) shows that a modification of the betatron frequency (such as betatron frequency spread) would lead to a strong modification of BBU growth of type C. This observation corroborates the recent finding by Chernin and Mondelli[8] who showed that a spread in the betatron frequency may result in a strong stabilization of BBU in the parameter regime of a 500-GeV collider. As a change in ω_0 does not alter the singular behavior in Eq. (15), rf cure would not be as effective in the control of BBU of type C. [This is actually obvious from the solution (8) which shows clearly that damping due to finite Q is unimportant for short pulse beams (small $T-Z$), regardless of the propagation distance Z.]

Since type B lies between type A and type C in the (s_1,s_2) plane (Fig. 1), suppression of BBU of type B may require *both* rf cure and external focusing. It is perhaps not a coincidence that control of BBU in ATA has required a Q as low as 4 *and* betatron frequency spreads which are provided by an ion channel generated along the beam path with a laser.[16]

In summary, a classification of the cumulative BBU is given for all types of linacs based on just two dimensionless parameters. This classification is based on the standard equations (1) and (2) [or, equivalently, Eq. (3)]. A simple argument is provided on the roles of rf cure and of external focusing for BBU control in various types of linacs.

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