Relativistic Solitons and Shocks in Magnetized e^--e^+-p^+ Fluids

Tzihong Chiueh [a]
Department of Astrophysical, Planetary and Atmospheric Sciences, University of Colorado at Boulder, Boulder, Colorado 80309
(Received 5 December 1988)

A new type of relativistic magnetosonic soliton, which is electrically charged with a gigavolt potential, is found to exist in a magnetized electron-positron-proton plasma. Relativistic collisionless shocks resulting from such solitons can carry an even larger electric potential at the shock front. GeV electrons and positrons in some active astrophysical sources may be produced due to acceleration by these electric fields.

PACS numbers: 95.30.Qd, 52.35.Sb, 52.35.Tc

It is commonly believed that in some active astrophysical sources the plasmas consist predominantly of electron-positron pairs and the plasma motion is relativistic. For example, flows emanating from pulsars are relativistic and pair dominated, and may deliver a sizable fraction of the pulsar rotational energy into the surrounding nebulae.[1,2] Energetic jets also appear to be commonly emerging from compact objects. High-resolution very-long-baseline–interferometer radio maps suggest that relativistic outflows exist within several light years from the cores of the active galactic nuclei (AGN), and some jets can even remain relativistic out to several thousand light years.[3] On the other hand, a cascade of e^--e^+ pairs has become an attractive scenario for explaining the fairly universal radiation spectra of AGN's.[4] If that is true then a large population of pairs in the AGN's will be inevitable, and shocked pair plasmas due to relativistic accretion and ejection will be commonplace.[5]

Radio emission from these sources is probably the result of synchrotron radiation. For a typical magnetic field strength, of order microgauss to milligauss, a Lorentz factor of order 100 to 1000 for electrons is required to account for the radio frequency. It has been a puzzle as to how a large amount of GeV electrons should commonly emerge.[2,6] Plasmas in these astrophysical settings are collisionless on the time scale of interest, and thus collisionless shocks or nonlinear waves may well be responsible for the electron acceleration. The classical picture for the development of collisionless weak shocks is through the formation of solitons, where the nonlinear steepening is balanced by the plasma dispersion.[7] When the soliton amplitude approaches some critical value, a strong field gradient develops, which then couples to dissipation and a shock can form. A detailed study of relativistic nonlinear waves in a pure e^--e^+ plasma was conducted by Kennel and Pellat,[8] in which various waves were analyzed and the dispersion relations derived. Magnetosonic solitons and shocks in a pure e^--e^+ plasma were found to be charge neutral.[8-10] Although pairs are likely to be the dominant constituents of the plasmas in strong sources, minority protons must also exist. In this work we confine ourselves to the regime where protons dominate the mass density, but not the number density, of the plasma. We discover a new type of soliton that is charged with a gigavolt electric potential and may be responsible for the omnipresent radio emission.

Charged soliton.— Cross-field charge separation in a strong flow occurs because the electrons are tied to the field lines but the momentum-carrying protons slip across the field lines on the length scale comparable to the proton inertial length. In a nonrelativistic flow, electrons usually can compensate for their inability to move across the field by sliding along the field to catch up with the protons and shield out the electric field, if the flow is not exactly perpendicular to the field lines and is also slow enough, i.e., a subluminal (along the field) flow. However, in a relativistic flow, protons move too fast for electrons to follow; therefore a charge-separation electric field can be set up. In the following analysis we shall assume a cold-fluid theory. We let all species have the same flow speed far upstream, where the flow $U_1\hat{\mathbf{x}}$ and the magnetic field $B_1\hat{\mathbf{z}}$ are uniform. Finally, we consider only the situation in which the magnetic field is always in the z direction and no variation in the z and y directions is allowed.

In the soliton frame, the soliton equation sought can be derived from the continuity and the momentum equations of each species, i.e., $n_\alpha U_\alpha = N_1 U_1$, $m_\alpha U_\alpha dU_\alpha/dx = q_\alpha[\gamma_\alpha E_x + (V_\alpha/c)B_z]$, $m_\alpha U_\alpha dV_\alpha/dx = q_\alpha[\gamma_\alpha E_y - (U_\alpha/c)B_z]$, together with Maxwell's equations, i.e., $dB_z/dx = -(4\pi/c)\sum_\alpha q_\alpha n_\alpha V_\alpha$, $E_y = E_1 = \text{const}$, $dE_x/dx = 4\pi \times \sum_\alpha \gamma_\alpha q_\alpha n_\alpha$. Here α denotes the index for each species, U and V are the x and y components of the flow four-velocity, n is the proper density, and N_1 and U_1 are constants, the proper density and the flow four-velocity far upstream. Although a finite constant W_α, the z component of the velocity, in principle exists, it can be trivially incorporated and we shall leave it out in this analysis for the sake of convenience.

To simplify the notation, let $n_p = fN_p$, $n_+ = (1-f) \times N_+$, and $n_- = N_-$, where f is the number density fraction of protons compared to the number density of electrons. Far upstream, we demand that $N_p = N_+ = N_- = N_1$. Before making any approximation, we observe that this set of equations has the following three known integrals:

$$\{fM_p\gamma_p + m_e[(1-f)\gamma_+ + \gamma_-]\}N_1U_1c^2 + c\frac{E_1B_z}{4\pi} = T^{01}, \tag{1}$$

$$\{fM_pU_p + m_e[(1-f)U_+ + U_-]\}N_1U_1 + \frac{1}{8\pi}(B_z^2 - E_x^2) = T^{11}, \tag{2}$$

$$\{fM_pV_p + m_e[(1-f)V_+ + V_-]\}N_1U_1 - \frac{E_1E_x}{4\pi} = T^{21}. \tag{3}$$

They are, respectively, conservation of the energy flux, the longitudinal momentum flux, and the transverse momentum flux. In the presence of a charge-separation electric field, we can use the electric potential ϕ to obtain a new integral. Multiplying the x component of the momentum equations by $m_aq_an_aU_a/|q_a|$ and summing up the resultant equations for all species, we obtain

$$\frac{1}{2}\frac{d}{dx}\{f(M_pU_p)^2 + m_e^2[(1-f)U_+^2 - U_-^2]\} = \{fM_p\gamma_p + m_e[\gamma_- + (1-f)\gamma_+]\}eE_x + \{fM_pV_p + m_e[V_- + (1-f)V_+]\}\frac{eB_z}{c}$$

$$= -\frac{eT^{01}}{N_1U_1c^2}\frac{d\phi}{dx}, \tag{4}$$

where we have used the boundary condition, $T^{21} = 0$, for the second equality. This equation can be integrated once.

Now, we shall assume that the proton inertia dominates ($1 > f \gg 10^{-3}$) and ignore the electron terms on the left-hand side of these equations. The four algebraic equations, the relation $E_x = -d\phi/dx$, and the ion continuity equation can be solved straightforwardly for the six unknowns U_p, N_p, V_p, B_z, E_x, and ϕ. We first use Eqs. (2) and (4) to obtain $B_z^2 - E_x^2$ as a function of ϕ. Next, we use Eq. (1) to obtain γ_p; together with the definition of the Lorentz factor γ_p, we finally have

$$e_x^2 = \frac{1}{\sigma(1+\sigma)}\left(\frac{d\Phi}{d\xi}\right)^2 = -Q(\Phi) \equiv b^2 - \Theta^2, \tag{5}$$

where

$$e_x \equiv \frac{E_x}{B_1},$$

$$b \equiv \frac{B_z}{B_1} = \frac{\sigma}{2(1+\sigma)}\left[\left(\frac{1+\sigma}{\sigma}\right)^2 + \Theta^2 - \frac{1/\gamma_1^2 + \Phi_0 - \Phi}{\sigma^2}\right],$$

$$\Theta^2 \equiv 1 + \frac{2\beta_1[\beta_1 - (\Phi_0 - \Phi)^{1/2}]}{\sigma},$$

$$\sigma \equiv \frac{B_1^2}{4\pi f M_p N_1 c^2 \gamma_1^2} \equiv \frac{U_A^2}{c^2} \equiv \frac{\gamma_1^2\beta_1^2}{M_A^2}, \quad \Phi_0 \equiv \beta_1^2,$$

$$\Phi \equiv \frac{2(1+\sigma)\beta_1\phi}{r_cB_1}, \quad \xi \equiv \frac{2x(1+\sigma)^{1/2}\beta_1}{r_c\sqrt{\sigma}}, \quad \beta_1 \equiv \frac{E_1}{B_1},$$

$r_c \equiv U_1M_pc/eB_1$ is the proton gyroradius, and M_A and U_A are the Mach number in the soliton frame and the proper magnetosonic speed. Two points are worth noting. First, the length of the relativistic soliton scales with the proton gyroradius r_c for large σ; in the nonrelativistic limit, $\sigma \to 0$, the length scale is the ion inertial length. Second, the maximum electric potential energy $e\phi$ in a soliton is approximately the proton rest mass energy; in the nonrelativistic limit, $e\phi$ becomes the upstream proton kinetic energy.

Note that $Q(\Phi)$ is the Sagdeev potential, and the solution Φ can be understood by using the analogy of the trajectory of a particle rolling down a potential well.[7] It can be shown that $\Phi = 0$ is a stationary point where $d\Phi/d\xi = 0$, $b = 1$, $V_p = 0$, and $U_p = U_1$. When $M_A > 1$, the Sagdeev potential admits bound solutions (Fig. 1), and when $M_A < 1$, the Sagdeev potential is convex at $\Phi = 0$, and thus admits an oscillating solution. The small-amplitude oscillating solution can be shown to satisfy the following dispersion relation: $\beta_1^2 = S[1 + k_x^2r_c^2S]$, where $S \equiv \sigma/(1+\sigma)$. In the nonrelativistic limit, it is reduced

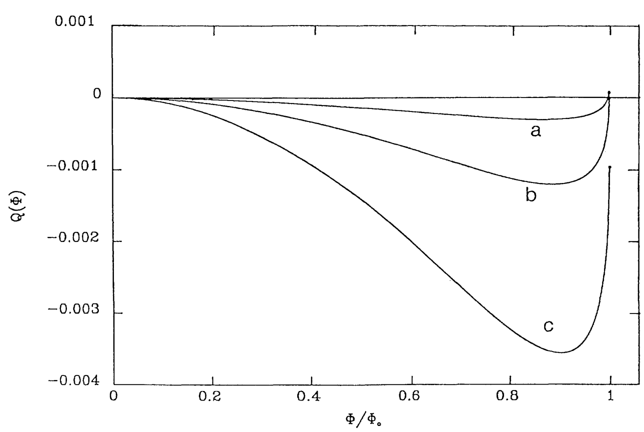

FIG. 1. The Sagdeev potential of Eq. (5) for $\gamma_1 = 10$ and $M_A/M_{Ac} = 0.8$ (curve a), 1 (b), and 1.2 (c). In its analogy to the particle orbit in a constrained well under gravity, Φ, $d\Phi/d\xi$, and ξ can be regarded as the displacement, the velocity, and the time, respectively. At the far end of the potential, there exists a singularity at which the "force" is infinitely large.

to the zero-m_e limit of the lower hybrid dispersion relation: $\omega^2/k_x^2 = U_A^2/(1+k_x^2 U_A^2/\omega_{pi}^2)$, where ω_{pi} is the proton plasma frequency. The soliton has parity symmetry: U_p, N_p, Φ, and b are symmetric, and V_p and e_x are antisymmetric. Except for the relativistic effects and the fact that its thickness is determined by the proton, rather than the electron, inertial length scale, this soliton looks like the classical soliton of Adlam and Allen.[7]

There exists a critical M_{Ac} at which the proton density diverges. In Fig. 1, the end of each curve represents the point at which the Sagdeev potential becomes singular ($\Phi = \Phi_0$). The physical solution can realize this singularity when the end point has a negative value of $Q(\Phi)$. By setting $d\Phi/d\xi = 0$, and $\Phi = \Phi_0$ we can determine M_{Ac} and the critical-field strengths. It follows that $\sigma_c = (\gamma_1 - 1)/2$, $b_c = (1+2\beta_1^2/\sigma_c)^{1/2} = 1+2/\gamma_1$, and finally $e\phi_c = \beta_1 M_p U_1 c/2(1+\gamma_1)$. It is instructive to compare these results with those in a pure e^--e^+ plasma[8] and those in a nonrelativistic ion-electron plasma.[7] For the former, $M_{Ac}^2 = 2(1+\gamma_1)$ and $b_c = 1+2/\gamma_1$; for the latter, $M_{Ac}^2 = 4$ and $b_c = 3$. They all agree in the nonrelativistic limit; particularly surprising is the agreement between the critical values of the two relativistic solitons, since they have very different natures.

Beyond the critical Mach number, $d\Phi/d\xi$ can never return to zero before encountering the singularity, a fact clearly demonstrated by Fig. 1. One can further show that the spatial structure of the singularity behaves as $b - b_s \sim |x|^{1/2}$, $e_x - e_{xs} \sim |x|^{1/2}$, and $U_p \sim |x|^{1/2}$, where b_s and $e_{xs} \equiv E_x/B_1$, are the values of b and e_x at the singularity. The electric potential Φ and the electric field $[-Q(\Phi)]^{1/2}$ increase with the Mach number (Fig. 1); b also increases accordingly from Eq. (2), in contrast to the charge-neutral solution for which $b-1$ remains small,[8] of order $1/\gamma_1$. This distinction becomes relevant when one is concerned with the immediate upstream configuration of a shock.

Charged shock.—Above the critical Mach number collisionless shocks can arise, and the fluid solution may be valid only in part of the upstream region. We now wish to examine to what extent the fluid solution becomes contaminated by the kinetic effects of the shock, in an attempt to estimate the lower bound for the electric potential predicted by the fluid theory. For a pure e^--e^+ plasma, Alsop and Arons[9] proposed that a reflected flow, a kinetic effect caused primarily by the magnetic force,[10] could avoid the fluid singularity. In our case, the electric force can also contribute to reflection of the incoming protons, similar to what has been observed in the Earth bowshock[11] and studied theoretically.[12] The rationale for our estimation of the validity of the fluid theory is based on the understanding that no information should propagate faster than the shock in a fluid; hence particle reflection, a kinetic effect, is the only means to inform the upstream field of a change. The extent to which this can occur is roughly one proton gyroradius upstream of the fluid singularity. When this length is much smaller than the length scale of the fluid soliton, the valid region of the fluid solution can extend up to the immediate neighborhood of the singularity. However, when the opposite is true or even when the two lengths are comparable, the fluid solution will fail entirely. Therefore, the criterion that these two lengths be equal defines a second critical Mach number M_{Ac2}, beyond which the electric potential drop predicted by the fluid theory is not a reliable estimate.

The existence of M_{Ac2} is a relativistic phenomenon,[13] in that the soliton length can exceed the proton gyroradius only in the case of sufficiently large γ_1, as already suggested by the definition of ξ after Eq. (5). For large γ_1, the electric potential is far too weak to stop the incoming flow and the particle reflection relies primarily on the magnetic force. The gyroradius must be evaluated using the *downstream,* rather than the upstream, proper magnetic field, an important distinction pointed out previously by Alsop and Arons.[9] Equating the proton gyroradius so determined with a numerically determined soliton length, defined as $\phi_{\max}/E_{x\,\max}$, we can fix the second critical Mach number. A summary of the results is listed in Table I. As is clear in Table I, the electric potential energy can be up to tens of GeV but is still too small compared with the incoming proton energy to contribute to proton reflection. Note that the listed electric potential is simply an estimate of the lower bound; it is not clear to what extent the electric potential can continue to increase beyond the second critical Mach number.

Astrophysical implications.—In this section, we will confine ourselves to discussing relativistic flows. An interesting point raised earlier is the possibility of electron acceleration by the charge-separation electric field of the soliton. Not every soliton can survive to develop into a large-scale shock, and hence it is likely that the small-scale solitons and shocklets, riding on the large-scale background flow, outnumber large-scale shocks to produce "diffusive heating" in active sources. We find that each relativistic soliton can be electrically charged up to a potential drop of approximately 1 GV across a width of order of the proton gyroradius ($U_1 M_p c/eB_1$).

We have so far analyzed only the perpendicularly propagating soliton. If the magnetic field is oblique to

TABLE I. Second critical Mach numbers and field strengths for various Lorentz factors.

γ	M_{Ac2}/M_{Ac}	$b_{c2}-1$	ϕ_{c2} (GV)
3000	6.4	0.025	35
1000	4.9	0.05	20
300	3.7	0.08	10
100	2.8	0.13	6.5
30	2.1	0.23	3.5
10	1.6	0.45	2
2.1	1	0.9	0.7

the direction of propagation, i.e., $B_x \neq 0$, this type of soliton can survive only when the phase velocity along the magnetic field exceeds the speed of light. In such situations B_x can be Lorentz transformed away, and in this frame the analysis given above remains unchanged except that the z components of the velocities are not zero, yielding a much larger Lorentz factor. By contrast, if the phase velocity along the magnetic field is subluminal, then E_y can in turn be transformed away so that the upstream flow is along the magnetic field; since e^- and e^+ are now able to catch the wave along the magnetic field, the electric potential drop can no longer be large across the soliton (or wave). That is, $e\phi$ changes from the GeV scale to an MeV (or $\gamma_1 m_e c^2$) scale as the propagation changes from superluminal to subliminal along **B**.

Precisely owing to the superluminal nature of this type of soliton, no Landau damping should occur and the soliton must propagate with little dissipation. The soliton may, however, become unstable and evaporate. This may occur when the soliton propagates into a region of decreasing Alfvén speed where it steepens and forms a shocklet, or when the soliton propagates into an environment that changes the direction of propagation from a superluminal one to a subluminal one, during which process the GeV electric potential energy must be released. Instabilities associated with the development of a shocklet may be caused by the relative drifts between electrons and protons. They may lead to electromagnetic fluctuations inside the soliton, such that a sizable random potential drop along the field lines occurs, thereby irreversibly accelerating particles. To calculate the available energy, the energy flux per soliton in the laboratory frame ($E_y = 0$) can be shown to be

$$(T^{01})' = \gamma_1^2[(1+\beta_1^2)T^{01} - \beta_1(T^{00}+T^{11})]$$
$$= c\beta_1^3\gamma_1^2 B_1^2(b^2-1)/4\pi \sim c\gamma_1 B_1^2/\pi , \qquad (6)$$

independent of f, so long as $f \gg 10^{-3}$. If solitons occupy a volume filling fraction, f_v, of the entire plasma, and if a substantial fraction of the soliton energy can be deposited with electrons, then an average energy flux of order $f_v(T^{01})'$, carried by GeV-scale electrons, emerges. Note that energy density of energetic electrons $[\sim f_v(T^{01})'/c]$ is a factor $8f_v\gamma_1^3$ of the magnetic energy density U_B $(\equiv B_1^2/8\pi\gamma_1^2)$. That is, for a mildly relativistic flow, say $\gamma_1 = 5$, and a reasonable volume filling factor $f_v = 10^{-3}$, the estimated electron energy density can be in equipartition with U_B. (Energy equipartition is commonly assumed to estimate the electron energy density in synchrotron sources.[2]) Furthermore, if the synchrotron emission accounts for most of the radiation flux given out by these electrons, then the emission power per volume[14] is $2\times10^4 f_v\gamma_1^3\sigma_T U_B^2/m_e c$, with a critical frequency a factor $(M_p/m_e)^2$ of the nonrelativistic cyclotron frequency, where σ_T is the Thomson scattering cross section.

For large-scale relativistic shocks, electron acceleration at the shock front has a profound implication for first-order Fermi acceleration. Traditionally, electrons are regarded as not capable of being shock accelerated because of their long mean free paths and rapid radiation cooling. Therefore, to have electrons undergo acceleration as effectively as protons, electrons may need to have a GeV energy to begin with. We suggest that the > GeV/particle free energy stored at the turbulent shock front and sustained by the impingement of an inflow may well be a natural battery for injecting high-energy seed electrons.

I would like to thank F. V. Coroniti for insightful suggestions and E. G. Zweibel for support and encouragement. Beneficial conversation with C. F. Kennel is acknowledged. I also thank M. C. Begelman for his careful reading of the manuscript. This work was supported by the NSF Grant No. ATM8506632 and the NASA Grant No. NAGW-91.

[a]Present address: Institute of Physics and Astronomy, National Central University, Chung-Li, Taiwan.

[1]J. Arons, Space Sci. Rev. **24**, 437 (1979); F. C. Michel, Rev. Mod. Phys. **54**, 1 (1982).

[2]C. F. Kennel and F. V. Coroniti, Astrophys. J. **283**, 694 (1984); **283**, 710 (1984).

[3]A. H. Bridle and R. A. Perley, Annu. Rev. Astron. Astrophys. **22**, 319 (1984); R. C. Walker, M. A. Walker, and J. M. Beson, Astrophys. J. **335**, 668 (1988).

[4]P. W. Guilbert, A. C. Fabian, and M. J. Rees, Mon. Not. Roy. Astron. Soc. **205**, 593 (1983); A. P. Lightman, A. Zdziarski, and M. J. Rees, Astrophys. J. Lett. **315**, L113 (1987).

[5]D. Kazanas and D. Ellison, Astrophys. J. **304**, 178 (1986); J. Blondin and A. Konigl, Astrophys. J. **323**, 451 (1987).

[6]F. V. Coroniti (private communication).

[7]D. A. Tidman and N. A. Krall, *Shock Waves in Collisionless Plasmas* (Wiley-Interscience, New York, 1971); R. Z. Sagdeev, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4; J. Adlam and J. Allen, Philos. Mag. **3**, 448 (1958).

[8]C. F. Kennel and R. Pellat, J. Plasma Phys. **15**, 335 (1976).

[9]D. Alsop and J. Arons, Phys. Fluids **31**, 839 (1988).

[10]A. B. Langdon, J. Arons, and C. E. Max, Phys. Rev. Lett. **61**, 779 (1988).

[11]N. Sckopke, G. Paschmann, S. J. Bame, J. T. Gosling, and C. T. Russell, J. Geophys. Res. **88**, 6121 (1983); W. A. Livesey, C. T. Russell, and C. F. Kennel, Geophys. Res. Lett. **9**, 1037 (1982).

[12]M. M. Leroy, Phys. Fluids **26**, 2742 (1983).

[13]The nonrelativistic soliton has a length scale of c/ω_{pi}, which is a factor $1/M_A$ smaller than the proton gyroradius, and hence no second critical Mach number of our sense should exist.

[14]G. B. Rybicki and A. P. Lightman, *Radiative Processes in Astrophysics* (Wiley-Interscience, New York, 1979).