Semenoff Replies: In his recent criticism[1] of my work on exotic statistics,[2] Hagen addresses two points. First is the use of the temporal $A_0=0$ gauge. Hagen asserts that, in a model with nondynamical matter once considered by him, it is incompatible with the solution of the equations of motion for A_μ. There is similar confusion elsewhere in the literature as to whether simultaneous use of the temporal and radiation-gauge conditions $A_0 \approx 0$, $\nabla\cdot\mathbf{A}\approx 0$ is allowed in electrodynamics. Indeed, it is not generally possible to find a gauge transformation which sets both conditions. What must be recognized is that canonical transformations are also allowed. This is why Dirac's[3] method of eliminating the gauge degrees of freedom associated with first-class constraints works. Of course the solutions of the equations of motion for gauge-variant fields look different in different gauges and with different canonical variables.

Second is my use of the multivalued function $\Theta(\mathbf{x}-\mathbf{y})$ to solve the constraints

$$\nabla\times\mathbf{A}(\mathbf{x})=\frac{2\pi}{\alpha}j^0(\mathbf{x}),\quad \nabla\cdot\mathbf{A}=0\,. \tag{1}$$

Here Θ is a multivalued function on the punctured plane R^2-{0} whose gradient is given by the single-valued vector field $\nabla_i\Theta(\mathbf{x})=-\epsilon_{0ij}x^j/\mathbf{x}^2$. This implies that Θ cannot be extended to a differentiable function at the origin, in fact $\nabla\times\nabla\Theta(\mathbf{x})=2\pi\delta(\mathbf{x})$ and satisfies $\nabla^2\Theta=0$. This is all we need to show that

$$\mathbf{A}(\mathbf{x})=-\frac{1}{\alpha}\int d^2y\,\nabla\Theta(\mathbf{x}-\mathbf{y})j^0(\mathbf{y}) \tag{2}$$

solves (1) and is single valued. We are further required to extract the gradient operator from the integration in (2). The resulting quantity $I(\mathbf{x})=(1/2\pi)\int d^2x\,\Theta(\mathbf{x}-\mathbf{y})\times j^0(\mathbf{y})$ should solve $\nabla\times\nabla I(\mathbf{x})=j^0(\mathbf{x})$ and $\nabla^2 I(\mathbf{x})=0$. The first of these can be expressed in integral form $\oint_C dl\cdot\nabla I(\mathbf{x})=\int\int_D d^2x\,j^0(\mathbf{x})$, where D is a disk with a boundary, the closed curve C. If the charge density is concentrated at points the result is immediate. For distributed charge density the Riemann integral $I(\mathbf{x})$ is not analytic where $\mathbf{x}$ is interior to the charge distribution and must be defined by the continuum limit of the Riemann sum: "This is natural in a lattice regularization of the gauge theory;"

$$I(\mathbf{x})=\frac{1}{2\pi}\lim_{\epsilon\to 0}\sum_i \epsilon^2\Theta(\mathbf{x}-\mathbf{y}_i)j^0(\mathbf{y}_i)\,.$$

This is obviously a multivalued function of $\mathbf{x}$ with the desired properties. It is also obvious that when $\mathbf{x}$ is internal to the charge distribution $I(\mathbf{x})$ cannot be represented by an elementary function. In Hagen's example the charge distribution $j^0(\mathbf{x})=0$, $|\mathbf{x}|>a$, and $j^0(\mathbf{x})=\rho$, $|\mathbf{x}|\le a$, and

$$I(\mathbf{x})=\frac{1}{2\pi}\lim_{\epsilon\to 0}\sum_{|\mathbf{y}|\le a}\epsilon^2\Theta(\mathbf{x}-\mathbf{y}_i)\rho\,.$$

The limit is not an analytic function where $|\mathbf{x}|\le a$. However, its derivatives have a well defined limit and it has the requisite holomony. It is essential that the multivalued angle function is used. If, as Hagen has done, one uses the single-valued angle with a branch singularity, one obtains the wrong answer. Furthermore, construction of the angle θ as a single-valued function on the universal covering space of the punctured plane is an elementary exercise and demonstrates that the "multivalued" quantum field used in Ref. 2 is indeed well defined as a single-valued operator there.

Hagen has also suggested that the apparent triviality of the free anyonic theory is incompatible with the known renormalization of the parameters of Chern-Simons electrodynamics. On the contrary, the anyon theory is far from trivial. The nonlinear interacting gauge field theory with single-valued fields and relatively simple boundary conditions has been transformed to a theory of anyons where the field operators are multivalued, equations of motion are simple but the boundary conditions are complicated. The two dynamical problems are equivalent—demonstrating that was the point of Ref. 2—and an idea which has since been substantiated by many other authors.

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