Comment on "Canonical Quantum Field Theory with Exotic Statistics"

In a recent work[1] Semenoff claims to give an example of a three-dimensional field theory with exotic statistics and fractional spin. It is the purpose of this Comment to point out that there exists a crucial error in that work which negates the principal conclusions of the paper.

It may be useful to begin by pointing out that the model of Ref. 1 is a special case of the more general photonless gauge theory developed by the author.[2] It is described by the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}\phi^\mu\epsilon_{\mu\nu\alpha}\partial^\alpha\phi^\nu+g\phi^\mu j_\mu\,, \qquad (1)$$

where j^μ is an unspecified conserved current. It was shown in Ref. 2 that the (radiation-gauge) potentials ϕ^μ are

$$\phi_i(x)=-g\epsilon_{ij}\nabla_j\int d^2x'\,\mathcal{D}(x-x')j^0(x') \qquad (2)$$

and

$$\phi^0(x)=g\int d^2x'\,j(x')\times\nabla'\mathcal{D}(x-x')\,,$$

where the function $\mathcal{D}(x)$ is given by

$$\mathcal{D}(x)=\frac{1}{4\pi}\ln x^2+\text{const.}$$

Although Semenoff incorrectly states that ϕ^0 (his A^0) can be chosen to be zero [in contradiction of the field equations implied by (1)], this is not the "crucial error" referred to above.

The latter consists of the claim that the equation

$$\nabla\times\phi=-gj^0 \qquad (3)$$

is solved not only by (2) but equally well by

$$\phi_i(x)=-g\frac{1}{2\pi}\nabla_i\int\arctan(\mathbf{x}-\mathbf{x}')j^0(x')d^2x'\,. \qquad (4)$$

This equating of a gradient to a dual of a gradient must be viewed with some suspicion, a suspicion which is confirmed by a simple calculation.

Consider, in particular, the case of a classical charge density given by

$$j^0(x)=1/\pi R^2\,, \qquad (5)$$

for $r<R$ and zero otherwise. This will easily be recognized as equivalent to the problem of computing the magnetic field of an infinite straight wire in which the current is uniformly distributed over the circular cross section of the wire. Upon using the prescription (2) one obtains

$$\boldsymbol{\phi}=-\frac{g}{2\pi}\boldsymbol{\epsilon}_\phi\times\begin{cases}1/r, & r>R,\\ r/R^2, & r<R,\end{cases} \qquad (6)$$

where $\boldsymbol{\epsilon}_\phi$ is the unit vector orthogonal to the radial direction. This result is, of course, equivalent to the well-known result for the field of a long straight wire carrying current $-g$.

The prescription (4) is similarly evaluated, yielding

$$\boldsymbol{\phi}=\frac{g}{2\pi}\times\begin{cases}\boldsymbol{\epsilon}_\phi/r, & r>R,\\ \boldsymbol{\epsilon}_\phi r/R^2+(2r/R^2)\phi\boldsymbol{\epsilon}_r, & r<R,\end{cases} \qquad (7)$$

which agrees with (6) for $r>R$ but is characterized by the appearance of an unusual multivalued contribution in the radial direction for $r<R$.

It remains to take the curl of (6) and (7) in order to determine whether each solves Eq. (3). Although the curl of $\boldsymbol{\phi}$ as obtained from (6) is in agreement with (3) for j^0 given by (5) one finds that the "solution" (7) is everywhere curlfree. This confirms the expectation previously expressed concerning the inequivalence of a gradient and a dual of a gradient. However, it also illustrates convincingly that even the use of a multivalued kernel such as that employed by Semenoff does not enable one to avoid a contradiction.

This fact that $\boldsymbol{\phi}$ cannot be written as a gradient eliminates any possibility of extracting the effects of the interaction into an exponential as in Ref. 1. Indeed, the result of Semenoff that the (gauge-invariant) single-particle mass does not even get renormalized is at variance with perturbation theory. It is therefore only the failure of the exponentiation process itself that makes possible the elimination of that contradiction.

In sum there is no manifestation of exotic statistics in this theory.[3]

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C. R. Hagen
Department of Physics and Astronomy
University of Rochester
Rochester, New York 14627

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[1]G. W. Semenoff, Phys. Rev. Lett. **61**, 517 (1988).
[2]C. R. Hagen, Ann. Phys. (N.Y.) **157**, 342 (1984).

[3]It may be well to remark here that the anomalous term in the angular momentum found by the author in Ref. 2 has been shown *not* to require anomalous statistics [C. R. Hagen, Phys. Rev. D **31**, 2135 (1985)].