Test of Newton's Inverse-Square Law in the Greenland Ice Cap

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An Airy-type geophysical experiment was conducted in a 2-km-deep hole in the Greenland ice cap at depths between 213 and 1673 m to test for possible violations of Newton's inverse-square law. An anomalous gravity gradient was observed. We cannot unambiguously attribute it to a breakdown of Newtonian gravity because we have shown that it might be due to unexpected geological features in the rock below the ice.

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Some unified field theories[1] raise the possibility that forces exist in nature with ranges on the order of 10^2–10^5 m and coupling strengths close to that of gravity. If they exist, these new forces would be apparent as violations of Newton's inverse-square law. Recent geophysical measurements in a mine[2] and on a tall television antenna[3] have reported small deviations from the classical law. This paper describes a geophysical experiment to search for possible finite-scale, non-Newtonian gravity over a vertical distance of 213–1673 m in the glacial ice of the Greenland ice cap. The principal reason for the choice of experimental site is that the uniformity of the ice eliminates one of the major sources of uncertainty arising in the first of the earlier studies,[2] namely, the heterogeneity of the rocks through which the mine shaft passes. Our observations were made at Dye 3, Greenland, in a 2033-m-deep borehole, which reached the basement rock. The site is 60 km south of the Arctic Circle, 125 km inland from Greenland's east coast, and at a 2530-m elevation.

The Newtonian prediction of the gravity profile in the borehole, based on a density model of the ice and the topographic relief of the bedrock developed from geophysical measurements, was compared with measured values. Differences in gravity g were measured at several depths z and modeled by

$$g_m(z) \approx \gamma z - 4\pi G \rho_i z + g_r(z)\,, \tag{1}$$

where γ is the theoretical free-air gravity gradient, G is the Newtonian gravitational constant as determined in laboratory experiments, ρ_i is the ice density, and g_r is a correction to the gravity differences based on the attraction of the subice terrain. (The effect of the ice-surface topography is negligible.) Although Eq. (1) is adequate within the uncertainties of our experiment, a more exact expression[4] which accounts for $\rho_i \approx \rho_i(z)$, $\gamma \approx \gamma(z)$, and the Earth's ellipticity was used in the calculations. The gravity anomaly Δg is defined as the difference between the modeled gravity g_m and the observed gravity in the borehole $g_{\rm obs}$,

$$\Delta g = g_{\rm obs}(z) - g_m(z)\,. \tag{2}$$

Now we describe the steps taken to obtain the experimental observations and model calculations given in Table I. The uncertainties in this table include contributions from the measurements themselves and from imperfect knowledge of the ice density and the terrain, with the latter effect dominating. They do not reflect our ignorance of the density inhomogeneities in the underlying rock. This issue, which in the end has the least controlled systematic uncertainty, will be discussed below.

Before the measurements were made in Greenland, the borehole gravity meter was calibrated in Canada and Alaska over the range of gravity values expected in the Dye 3 borehole by comparison with the readings of an absolute gravity meter.[5] Uncertainties in calibration were 0.05 mGal in the borehole measurements at Dye 3.

Accurate location of the borehole gravity meter vertically is essential because of the large vertical gradient in g. This required knowledge of the borehole's trajectory

TABLE I. The absolute observation depths, z; the depths relative to the shallowest observation point, Δz; the theoretical free-air term, $\gamma \Delta z$; the effect due to the ice [approximately the second term in Eq. (1)], g_{ice}; the attraction of the subice terrain, g_r; the theoretical gravity differences, $g_m = \gamma \Delta z + g_{\text{ice}} + g_r$; the observed gravity differences, g_{obs}; the anomalies, $g_{\text{obs}} - g_m$; and the uncertainties, σ_g. The modeled and observed values are offset to make them both zero at $z = 213.22$ m, which is permitted since all the gravity observations are relative. All distances are in m and all gravity values are in mGal (1 mGal $= 10^{-3}$ cm/s^2).

z	Δz	$\gamma \Delta z$	g_{ice}	g_r	g_m	g_{obs}	$g_{\text{obs}} - g_m$	σ_g
213.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26
396.12	182.90	56.37	−14.06	−0.25	42.06	42.39	0.33	0.25
579.00	365.78	112.74	−28.13	−0.52	84.06	84.72	0.63	0.23
761.83	547.61	169.09	−42.21	−0.82	126.06	127.13	1.07	0.20
944.63	731.41	225.45	−56.31	−1.14	168.00	169.48	1.48	0.19
1309.40	1096.18	337.91	−84.44	−1.92	251.55	254.10	2.55	0.15
1491.18	1277.96	393.97	−98.47	−2.40	293.10	296.32	3.22	0.13
1673.23	1460.01	450.11	−112.52	−2.95	334.64	338.51	3.87	0.25

(which we obtained from downhole inclinometers) and a precise length calibration of the wireline from which the instrument was suspended. The wireline-length-measuring system was calibrated under load in a 1520-m mine shaft at the Consolidated Silver mine in Idaho by comparison with a laser geodimeter. This was done before and after the Greenland experiment: The measured depths are accurate to about 1 part in 10^4.[5]

Approximately 100 gravity observations were made in the borehole distributed over eight stations placed at 183-m intervals. The average gravity value at each depth has an estimated accuracy of 0.05 mGal.

In addition to the borehole observations, a surface-gravity survey was performed with LaCoste-Romberg relative-gravity meters. The region covered was 32 km in diameter and consisted of 25 sites on three rings and at the center, each site was repeatedly occupied with four gravity meters. Elevations for the sites were obtained with a combination of first-order optical leveling and satellite observations with the Global Positioning System. After corrections (to be described below) the gravity values are used to provide further constraints on possible subice density variations.

The properties of ice have been extensively studied and samples from Dye 3 have been analyzed in detail.[6] The three important factors affecting an ice-density model are pressure, temperature, and air content, all others being minor by comparison. Only moderately accurate knowledge of these parameters was needed to calculate density accurate to 7 parts in 10^4 over the depth range of our experiment.[5]

Additional gravity gradients are created by the undulations of the basement rock; it is therefore important to map the ice thickness for a considerable distance around the site of the experiment. Ice-penetrating radar surveys are traditionally used for this purpose. Airborne and ground radar surveys had previously been made in the vicinity of the site, providing moderately accurate coverage to a distance of more than 60 km.[7] We performed a more detailed surface radar survey within a 5-km distance from the Dye site using the Scott Polar Research Institute radar system.[5] The bedrock topography map was constructed from the recorded radar travel times by a three-dimensional migration algorithm along 124 radial lines. The resultant map has horizontal resolution of about 125 m and vertical uncertainty of about 5 m over 70% of the area; in a few exceptional places far from the borehole where reflections were very weak, the uncertainty rose to 50 m. The ice thickness ranges from approximately 1800 to 2100 m and is in general agreement with previous regional surveys. At the same time, the ice surface topography was also mapped using an electronic distance meter and theodolite, to an accuracy of about 1 m.

The gravitational effect of bedrock topography (the terrain correction) was computed at each gravity observation point, both on the ice surface and in the borehole, using two different techniques. Both methods depend upon an estimate of the bedrock density of 2.70 g/cm^3 given by Jezek, Roeloffs, and Greischer.[8] The first method uses vertical prisms; the second relies on spectral analysis of the topography around Dye 3. The two methods agreed.[5] Imperfect knowledge of the terrain was, in fact, the leading source of uncertainty in the gravity corrections.

After all these conventional adjustments are applied, there remains an unexplained gravity difference of 3.87 ± 0.36 mGal between the gravity value at a depth of 213 m and the one at a depth of 1673 m.

At this point the data could be taken as being indicative of an attractive, Yukawa force having a range on the order of 1 km and a strength of a few percent of G. However, before one can accept this exotic explanation, one must explore one further effect that is not as readily computed as the other corrections. So far the rock beneath the ice has been treated as a completely uniform solid. But, density variations in the rock generally produce vertical gravity gradients. Two lines of evidence point to the existence of such heterogeneities. First, geological studies of the coastal regions of Greenland show that mafic intrusions with densities from 2.8 to 3.0 g/cm^3 are found within the metamorphic basement. These high-density intrusives occupy a few percent of the exposed basement rock.[9] Second, our own surface-gravity map reveals anomalies within our network. They are typically 3 mGal, but in one case the anomaly reaches 10 mGal.

To demonstrate unambiguously the inadequacy of a purely Newtonian explanation one must show that no reasonable density variation in the basement can produce the observed anomalous borehole gravity profile without conflicting with the surface-gravity survey. It is well known that even complete gravity information outside a body cannot uniquely determine the internal density structure. However, the theory of ideal bodies[10] leads to a rigorous calculation of the smallest possible density contrast mathematically consistent with a finite number of gravity observations. If such a calculation showed that a geologically unacceptable density contrast was required to produce the measured gravity values, the case for a modification of Newton's law of gravity would be made.

In reality the situation is not quite so straightforward because the collected gravity values are imprecise; we have extended the standard theory so that the smallest density contrast is found subject to a specified weighted sum of the squared misfit to the observations. We solve the following optimization problem for a three-dimensional density distribution:

$$\min_{0 \le \Delta\rho \le \rho_0} \sum_{j=1}^{N} \frac{1}{\sigma_j^2} \{\Delta g_j - F_j[\Delta\rho]\}^2 = \chi^2[\Delta\rho] , \qquad (3)$$

where $F_j[\Delta\rho]$ is the integral for the gravitational attraction at the jth observation point due to variation of density $\Delta\rho$ (a function of position) beneath the ice; here Δg_j is the jth observed gravity value and σ_j its estimated uncertainty; all 25 surface values and 8 borehole values were used. The problem is solved approximately with a quadratic programming algorithm, which constructs a block model of the density distribution. It is easily shown that the value of ρ_0 causing χ^2 to achieve a specified value in Eq. (3) is the smallest possible density contrast consistent with that misfit. Thus by varying ρ_0 in Eq. (3) we can find the least density contrast within the basement necessary to reproduce the observations as measured by the size of the misfit functional.

There is one further complication: All the gravity measurements are relative, so that in principle the same arbitrary constant may be added to each one. In our treatment the constant is the contribution to gravity due to the anomalous density distribution that would be measured at our network's base station. In fact, to avoid the

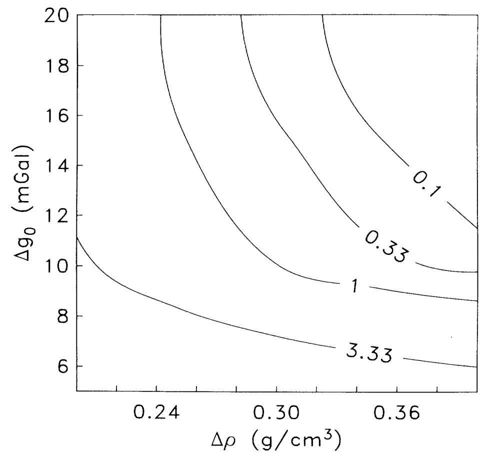

FIG. 1. A contour map of $\chi^2(33)$ obtained by fitting the 33 gravity station measurements with an ideal body of density contrast $\Delta\rho$ located at and below the rock-ice interface, in a field with regional gravity offset Δg_0. The specific case of $\Delta\rho = 0.30$ g/cm^3 and $\Delta g_0 = 10$ mGal is exhibited in Fig. 2.

necessity of huge anomalous masses whose presence would be inconsistent with geological considerations, we allow an offset as a parameter in our fit. Since our gravity survey reveals variations in the corrected surface gravity of up to 10 mGal, values on this order for the arbitrary constant are acceptable, but values several times larger would be difficult to justify.

We systematically varied ρ_0, the bound on the density contrast, and Δg_0, the constant gravity offset, and calculated the smallest possible misfit to the observations. The squared misfits normalized by the number of observations are shown in Fig. 1. The unit contour line defines a good fit to the data. Note that a good fit can be obtained for large but plausible density contrasts and acceptable offset values. For example, the mass distribution depicted in Fig. 2, generated by the ideal-body code, has a density constrast of 0.3 gm/cm^3 (larger than this is geologically unreasonable) and a regional offset of 10 mGal and simultaneously fits the surface and borehole gravity observations. Figure 2 (out to 22 km) implies that the high-density intrusives are $\ge 23\%$ of the area beneath the well, while $\approx 2\%$ of the exposed coastal rocks are of this high density.[9] On the other hand, nearly identical ideal bodies fit the surface data alone. The distribution found by any solution of Eq. (3) is not designed to find the actual one in the Earth. We have merely shown that such solutions are mathematically possible. The fundamental nonuniqueness of the inverse gravimetric problem makes it impossible to identify the actual density distributions responsible for the gravity signal.

The above indicates that any complete analysis of geophysical experiments searching for non-Newtonian gravi-

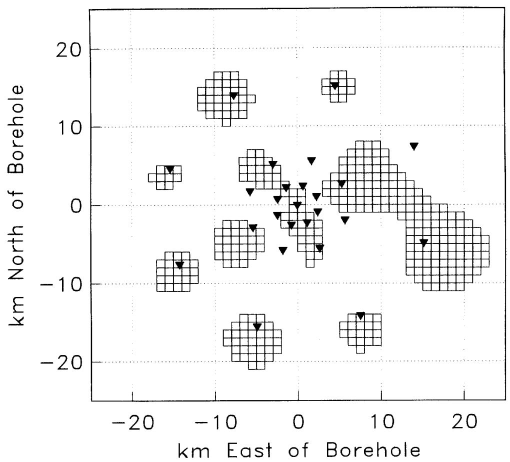

FIG. 2. The plan view, at the rock-ice interface, of the ideal body with density contrast $\Delta\rho = 0.30$ g/cm^3, in a regional offset field of $\Delta g_0 = 10$ mGal, which can model the gravity observations. The vertical dimension of each piece of the ideal body is roughly the same as its horizontal dimension. The inverted triangles denote the locations of the gravity measurements on the surface.

ty should include a careful estimate of the ability of small density contrasts to generate substantial vertical gravity gradients. This issue has been emphasized by Stacey and collaborators.[2]

In conclusion, we have found an anomalous gravity gradient that could be taken as evidence for non-Newtonian gravity. We have quantified this possibility by using regional gravity offset and density contrast as parameters to fit in an ideal-body analysis. This analysis finds that the data can be mathematically fitted with Newtonian gravity if one allows surprisingly large mafic intrusions in the bedrock. These findings could be further tested by performing an aeromagnetic survey of the region; this is currently being considered.

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