Generation of Dark Solitons in Optical Fibers

In the recent interesting Letter,[1] Krökel *et al.* have reported measurements of 0.3-psec dark pulses propagating through a 10-m single-mode optical fiber. It is well known that the counterbalancing between dispersion and nonlinearity that gives rise to optical bright envelope solitons requires a pulse propagating in the anomalous dispersion regime. Conversely, in the normal dispersion regime the same mechanism allows for the undistorted propagation of a hole in a cw background, or a fundamental dark soliton. Indeed, in the normal dispersion region Krökel *et al.*[1] observed two well-defined dark solitons which were created by a driving dark pulse.

In this Comment we aim to attract attention to the problem of dark-soliton generation in optical fibers. The problem is very important for an explanation of some results by Krökel *et al.*,[1] and also for the potential use of dark solitons in optical communication systems. We demonstrate that dark solitons may be created as pairs by *an arbitrary dark pulse without a threshold.* This result qualitatively differentiates dark solitons from bright ones (the latter may only be generated at some threshold of an input power; see, e.g., Ref. 2).

The propagation of short optical pulses in single-mode optical fibers is described by the normalized nonlinear Schrödinger equation

$$iu_x+\sigma u_{tt}+2|u|^2u=0. \tag{1}$$

In the case $\sigma=-1$, there are no bright solitons; instead the pulses undergo enhanced broadening and chirping. Other solutions of the nonlinear Schrödinger equation (1) for $\sigma=-1$ are dark solitons with the boundary conditions $|u|\to u_0=\text{const}$, as $t\to\pm\infty$. Let us consider for these boundary conditions the generation of dark solitons by a small-intensity hole created by a driving pulse at the edge of a fiber ($x=0$) (similar to the experiment of Ref. 1),

$$u(t,0)=u_0e^{i\alpha}+u_1(t),\quad |u_1(t)|\to 0 \text{ for } t\to\pm\infty.$$

According to the inverse scattering technique, to find which type of initial function generates solitons, one has to investigate the eigenvalue Zakharov-Shabat problem[3]:

$$\begin{aligned}(\Psi_1)_t&=i\lambda\Psi_1-iu(t,0)\Psi_2,\\(\Psi_2)_t&=-i\lambda\Psi_2+iu^*(t,0)\Psi_1.\end{aligned} \tag{2}$$

Each real discrete eigenvalue $|\lambda|<u_0$, $\lambda^2\equiv u_0^2-w^2$, corresponds to a dark soliton with amplitude w moving with velocity 2λ. The investigation of the eigenvalue problem (2) for a small $u_1(t)$ leads to the following results. For an *arbitrary* function $u_1(t)$ (which falls off fast enough at $t\to\pm\infty$), $|u_1|\ll u_0$, and negative

$$\delta=\text{Re}\left[e^{-i\alpha}\int_{-\infty}^{\infty}u_1(t)dt\right]<0, \tag{3}$$

there always exist two eigenvalues of the discrete spectrum

$$\lambda_{1,2}=\pm\lambda_0\equiv\pm u_0(1-\delta^2/2), \tag{4}$$

corresponding to a pair of dark solitons with equal amplitudes $u_0|\delta|$ and opposite velocities $\pm 2\lambda_0$. It means that for $\delta<0$ the dark-pulse solitons may be created without a threshold, i.e., by an infinitely small driving pulse. This analytical result explains the experimental conclusions of Krökel *et al.*[1] who, in particular, did not observe any threshold power for dark-soliton creation.

It is interesting to note that our results (3) and (4) for the eigenproblem (2) have an analogy with the famous Peierls problem in quantum mechanics: A one-dimensional well always contains a discrete level.[4]

The absence of a threshold for dark-soliton generation leads to important conclusions. One can easily create dark solitons in optical fibers by a small driving pulse, but, on the other hand, small (random or systematic) fluctuations acting on dark solitons will create additional secondary dark pulses (with the probability $p>\frac{1}{2}$). The latter, probably, will make impossible the effective use of dark solitons in optical communication systems.

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