Zeeman Effect on Magnetoresistance in High-Temperature Superconductors

A. G. Aronov, S. Hikami, and A. I. Larkin

Leningrad Nuclear Physics Institute, Gatchina, Leningrad, 188350, U.S.S.R.
Department of Pure and Applied Sciences, University of Tokyo, Meguro-ku, Komaba, Tokyo 153, Japan
L. D. Landau Institute for Theoretical Physics, Ul Kosygina 2, 117940 GSP-1 Moscow, U.S.S.R.

(Received 5 October 1988)

The Zeeman effect on the magnetoresistance in high-temperature superconductors is investigated above the transition temperature. It is found that this effect is dominant for the positive magnetoresistance in a parallel magnetic field and the phase relaxation time τ_φ can be estimated by a comparison of this term with experimental data for single-crystal films. This value is consistent with an estimate from the orbital effect of the Maki-Thompson term in a perpendicular magnetic field, and a temperature dependence of τ_φ proportional to $1/T$ is obtained from the experimental data.

PACS numbers: 74.40.+k, 74.75.+t

High-temperature superconductors in layered materials like the Y-Ba-Cu oxides show anisotropic superconducting fluctuations. Recently, theoretical formulas for the magnetoresistance of such high-temperature superconductors have been studied for perpendicular magnetic field $H_{\perp ab}$ (H is perpendicular to the a,b axes or to the CuO_2 plane).[1] These formulas are based upon the Aslamazov-Larkin (AL) term[2] and the Maki-Thompson (MT) term.[3,4] By comparison with the experiments of sintered samples of Y-Ba-Cu-O, the coherence lengths ξ_c and ξ_{ab} are estimated to be 2 and 16 Å, respectively.[5] These small values are consistent with the values of Oh *et al.*[6] which are obtained from the Aslamazov-Larkin term without a magnetic field.

The value of the phase relaxation time τ_φ has not been determined precisely since the applicable Maki-Thompson region is small for sintered samples and τ_φ has been estimated to be approximately of the order of 3×10^{-14} s.[5] Recently, the magnetoresistance of a single-crystal film[7] of Y-Ba-Cu-O has been measured in the $H_{\perp ab}$ and $H_{\parallel ab}$ cases and different behaviors were observed. In Fig. 1 we plot the magnetoresistance of single-crystal films as observed by Matsuda *et al.*[8]

In this Letter, we study a new effect in magnetoresistance, due to the Zeeman effect on the Maki-Thompson and Aslamazov-Larkin terms. Usually a large magnetoresistance is observed due to the effects of weak localization phenomena or the quantum interference effect.[9,10] Near the superconducting transition point, the localization effect is modified by the superconducting fluctuations. The Maki-Thompson term has been studied for this magnetoresistance.[11] The effect of the Maki-Thompson term in the magnetoresistance is due to an orbital effect and becomes larger than the AL term far from T_c as discussed in Ref. 1 for the $H_{\perp ab}$ case. However, for the $H_{\parallel ab}$ case, this orbital effect is absent in the two-dimensional limit. In such a case, the spin-splitting Zeeman effect on the Maki-Thompson and Aslamazov-Larkin terms becomes important. This effect has not been considered before in the literature[12] and we will discuss it in this Letter. Our formulas give reliable new estimates for the phase relaxation time τ_φ, which is an important quantity regarding the pair breaking of superconductors and the coherence lengths ξ_{ab} and ξ_c. Our analysis is also of interest with respect to the interplay between Anderson localization and superconductivity, since high-temperature superconductors are ideal quasi-two-dimensional systems due to their small $\xi_c \simeq 1$ Å.

The magnetic field breaks the singlet spin pair as a result of the paramagnetic effect. It suppresses the Maki-Thompson superconductor correction and it leads to the positive magnetoresistance. For the Zeeman effect, without the orbital effect, the correction of the conduc-

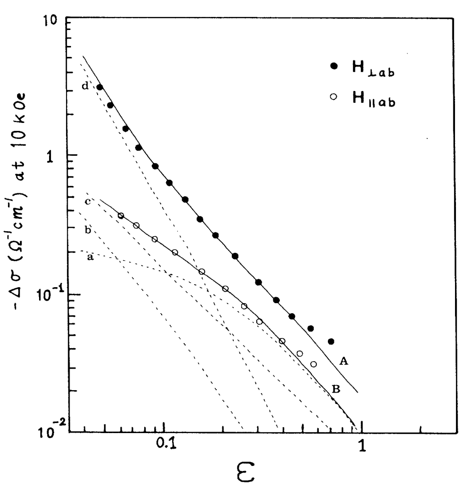

FIG. 1. The magnetoconductivity of a single-crystal film. The solid line A is the summation of the following contributions: a, MT-Zeeman; b, AL-Zeeman; c MT-orbital; and d, AL-orbital. The solid line B is the summation of contributions a and b. The experimental data of Matsuda *et al.* are plotted as circles. For theoretical estimation, the parameters $\xi_c = 1.5$ Å, $\xi_{ab} = 11.5$ Å, and $\tau_\varphi = 10^{-13}$ s at $T = 100$ K are used.

tivity in the Maki-Thompson term[1] is written as

$$\sigma'_{\mathrm{MTZ}}=\frac{2e^2}{\pi\hbar}D\int_0^\infty \frac{q\,dq}{2\pi}\int_0^{2\pi/d}\frac{dk_\parallel}{2\pi}\mathrm{Re}C(q,k_\parallel)\beta(q,k_\parallel)\,, \tag{1}$$

where

$$C(q,k_\parallel)=\frac{\xi_{ab}^2}{D(q^2\xi_{ab}^2+2\xi_c^2[1-\cos(k_\parallel d)]/d^2+\xi_{ab}^2/\tilde{L}_\varphi^2)}\,, \tag{2}$$

$$\beta(q,k_\parallel)=\frac{\pi^2}{4}\frac{1}{\epsilon+q^2\xi_{ab}^2+2\xi_c^2[1-\cos(k_\parallel d)]/d^2+\xi_{ab}^2/\tilde{L}_\varphi^2}\,, \tag{3}$$

D is a diffusion constant, $\xi_{ab}^2(0)=\pi\hbar D/8kT$, and d is the distance between conduction layers. ϵ is $(T-T_c)/T_c$. The quantity $\tilde{L}_\varphi$ is given by

$$\frac{\hbar}{\tilde{L}_\varphi^2}=\frac{\hbar}{D\tau_\varphi}+\frac{i\omega_s}{D}\,, \tag{4}$$

where $\omega_s=g\mu_B H$, μ_B is the Bohr magneton. We use $g=2$ in this Letter for the analysis. The notation Re in (1) means the real part of the quantities that follow. We consider the quasi-two-dimensional system as weakly coupled to the c-axis direction ($\xi_c \ll \xi_{ab}$). Near T_c, $|T-T_c| \le \hbar/\tau_\varphi k$, it is necessary to investigate the magnetic field dependence of β.[1] If ω_s is vanishing, the above formulas reduce to the usual Maki-Thompson term in the anisotropic case. Since the external magnetic field is parallel to the a,b axes, the momentum q is not quantized. The integrations can be done exactly. Since the magnetoresistance is proportional to the square of the magnetic field, except very near T_c, with the usual laboratory magnets, we consider only a small ω_s^2 term ($\omega_s\tau_\varphi/\hbar \ll 1$).

The magnetoconductivity in a small magnetic field becomes

$$\Delta\sigma'_{\mathrm{MTZ}}=\sigma'_{\mathrm{MTZ}}(H)-\sigma'_{\mathrm{MTZ}}(0)=-\frac{e^2}{16\epsilon d\hbar}\left(\frac{\omega_s\tau_\varphi}{\hbar}\right)^2\left[\frac{1+\delta}{(1+2\delta)^{3/2}}-\frac{1+\delta+\delta/\alpha}{[(1+\delta/\alpha)(1+2\delta+\delta/\alpha)]^{3/2}}\right]\,, \tag{5}$$

$$\alpha=2\xi_c^2(0)/d^2\epsilon\,, \tag{6}$$

$$\delta=16\xi_c^2(0)kT\tau_\varphi/\pi d^2\hbar\,. \tag{7}$$

This term $\Delta\sigma'_{\mathrm{MTZ}}$ is plotted in Fig. 1 for the value $\tau_\varphi=1\times10^{-13}$ s at $T=100$ K. We used $\xi_c=1.5$ Å and $d=12$ Å. The transition temperature of the sample in Fig. 1 is 85.5 K. As observed in the weak localization effect, the phase relaxation time τ_φ is different from the energy relaxation time τ_ϵ since there appear other relaxation times due to spin-orbit and spin-spin interactions, which are usually temperature independent. In our case, τ_φ is considered to be τ_ϵ since the estimated value of τ_φ is small compared to other possible relaxation times due to spin-orbit and spin-spin interactions. Therefore, the temperature dependence of τ_φ is suggested. We find from fitting Eq. (5) to the experimental values that the experiment in a parallel magnetic field is explained by $kT\tau_\varphi/\hbar \simeq 1.3$ in the considered temperature region. This $1/T$ dependence of τ_φ is similar to the $1/T$ dependence of the conductivity. This obtained value is consistent with the estimate from the magnetoresistance in $H_{\perp ab}$ using the orbital MT term $\Delta\sigma'_{\mathrm{MTO}}$ and the orbital AL term $\Delta\sigma'_{\mathrm{ALO}}$, which have the following magnetoconductivities[1]:

$$\Delta\sigma'_{\mathrm{MTO}}=-\frac{e^2}{8\epsilon\hbar d(1-\alpha/\delta)}\left(\frac{h^2}{6\epsilon^2}\right)\times\left[-\frac{1+\alpha}{(1+2\alpha)^{2/3}}+\frac{\delta^2}{\alpha^2}\frac{(1+\delta)}{(1+2\delta)^{3/2}}\right]\,, \tag{8}$$

$$\Delta\sigma'_{\mathrm{ALO}}=-\frac{e^2h^2}{64\hbar d\epsilon^3}\frac{(2+4\alpha+3\alpha^2)}{(1+2\alpha)^{5/2}}\,, \tag{9}$$

where $h=2e\xi_{ab}^2(0)H/\hbar c$. From the experimental data of Fig. 1 for $H_{\perp ab}$, we obtain $\tau_\varphi \approx 10^{-13}$ s by using the above equations of the orbital effect in addition to the Zeeman contribution of Eq. (5). The short pair breaking time τ_φ leads to a shift of the transition temperature ΔT_c. Our estimation of τ_φ gives $\Delta T_c/T_c \approx \pi\hbar/8kT_c\tau_\varphi \approx 0.3$, and the Ginzburg-Landau approximation may be adequate.

There is also a Zeeman effect on the Aslamazov-Larkin term due to the shift of the transition temperature. The shift of the transition temperature in a magnetic field is described by[13]

$$\ln\frac{T_c(H)}{T_c(0)}=-\mathrm{Re}\left[\psi\left(\frac{1}{2}+\frac{i\omega_s}{4\pi kT_c}\right)-\psi\left(\frac{1}{2}\right)\right] \simeq\frac{1}{2}\left(\frac{\omega_s}{4\pi kT_c}\right)^2\psi''\left(\frac{1}{2}\right)\,, \tag{10}$$

where ψ is the digamma function and $\psi''(\frac{1}{2})=-16.8$. The conductivity enhancement due to the Aslamazov-Larkin term is[1]

$$\sigma'_{\mathrm{AL}}(0)=\frac{e^2}{16\hbar d}\frac{1}{\epsilon'(1+2\alpha')^{1/2}}\,, \tag{11}$$

where $\epsilon'=\epsilon-\frac{1}{2}\psi''(\frac{1}{2})(\omega_s/4\pi kT_c)^2$, $\alpha'=2\xi_c^2(0)/d^2\epsilon'$, and $\epsilon'=[T-T_c(H)]/T_c(H)$. Therefore, we obtain the magnetoconductivity of the Aslamazov-Larkin Zeeman term as

$$\Delta\sigma'_{\mathrm{ALZ}} = -0.526\frac{e^2}{\hbar d\epsilon^2}\frac{1+\alpha}{(1+2\alpha)^{3/2}}\left(\frac{\omega_s}{4\pi kT_c}\right)^2 . \quad (12)$$

This term is small far from T_c, but it becomes larger than MT Zeeman term near T_c.

The observed magnetoresistance in a parallel magnetic field is well explained by the Zeeman effect on the MT and AL terms. In Fig. 1, we make a comparison with the experimental results in the $H_{\parallel ab}$ case. We have also made a best fit to the data of Fig. 1 for the $H_{\perp ab}$ case, with Eqs. (5), (8), (9), and (12). This leads to coherence-length values of $\xi_c = 1.5$ Å and $\xi_{ab} = 11.5$ Å. Thus it is possible to determine precisely the coherence lengths and the pair breaking time. We notice that the rather large difference of the magnitude of magnetoresistance between the experimental value and the theoretical formula was interpreted before as a c factor,[6] and indeed a c factor exists in sintered samples as $c \simeq 6$.[5] For the single-crystal film of our analysis, this c factor is almost 1 and the resistivity at the onset temperature is 60 $\mu\Omega$ cm.

We emphasize that the phase relaxation time τ_φ is more precisely determined from the data of $H_{\parallel ab}$, since δ is small and δ/α is independent of the value of $\xi_c(0)$. Our obtained value of τ_φ provides the reasonable value $kT\tau_\varphi/\hbar \simeq 1.3$. The consistency of the estimated value of τ_φ for both $H_{\perp ab}$ and $H_{\parallel ab}$ may suggest that the usual microscopic singlet pairing, like the BCS type, exists, and confirms that there is the ordinary Maki-Thompson fluctuation term in the high-temperature superconductors.

The ratio between the orbital and Zeeman effects in the Maki-Thompson term for $H_{\perp ab}$ and $H_{\parallel ab}$ is discussed by the following quantities:

$$\left(\frac{\omega_s}{a_\perp}\right)^2 = \left(\frac{\hbar}{4m_e D_\perp}\right)^2 , \quad (13)$$

$$\left(\frac{\omega_s}{a_\parallel}\right)^2 = \left[\frac{\hbar}{4m_e (D_\parallel D_\perp)^{1/2}}\right]^2 , \quad (14)$$

where $a_\parallel = 4(D_\parallel D_\perp)^{1/2} eH$, $a_\perp = 4D_\perp eH$, and $D_{\parallel,\perp}$ are the diffusion constants in the directions parallel to the c axis and to the a-b plane, respectively. If we take $\xi_{ab} \simeq 12$ Å, $\xi_c \simeq 1.5$ Å, we obtain $D_\perp \simeq 0.4\ \mathrm{cm^2\,s^{-1}}$ and $D_\parallel/D_\perp \simeq \frac{1}{60}$. For such parameters, $(\omega_s/a_\perp)^2 \simeq \frac{1}{2}$ and the Zeeman effect becomes of the same order as the orbital contribution in the $H_{\perp ab}$ case, and the orbital contribution becomes small in the $H_{\parallel ab}$ case since $(\omega_s/a_\parallel)^2 \simeq 30$. This order estimate agrees roughly with the calculation in Fig. 1.

We notice that the Aslamazov-Larkin orbital effect should give a large difference between $H_{\perp ab}$ and $H_{\parallel ab}$, of the order of $(\xi_{ab}/\xi_c)^2 \approx 60$. The two curves in Fig. 1 do not show such a large difference, at least for $\epsilon \approx 0.04$. This observation may also be explained by the dominance of the Zeeman effect on the Maki-Thompson and Aslamazov-Larkin terms for $H_{\parallel ab}$.

Some Zeeman effects on the magnetoresistance in disordered metals have already been discussed. Lee and Ramakrishnan[14] studied the magnetoresistance due to the Zeeman effect on the effective electron-hole interaction in the diffusion channel. This effect has no singular behavior near the superconducting transition temperature. The Zeeman effect changes the interaction in the Cooper channel also. It leads to a change of the electron density of states and therefore to a change of the resistance with magnetic field.[15] Neglecting the orbital effect, we will evaluate the Zeeman contribution to the density of states in two dimensions. By the relation between the density of states $\nu^{(c)}$ and the conductivity $\sigma^{(c)}$, the magnetoconductance $\Delta\sigma^{(c)}$ becomes

$$\Delta\sigma^{(c)} = \frac{e^2 D}{4kTd}\int_{-\infty}^{\infty} d\omega \frac{\Delta\nu^{(c)}(\omega)}{\cosh^2(\omega/2kT)} = \frac{e^2}{2\pi^2\hbar d\epsilon}\frac{3}{2}\zeta(3)\left(\frac{\omega_s}{\pi kT}\right)^2 , \quad (15)$$

where

$$\Delta\nu^{(c)}(\omega) = -\frac{1}{\pi^2\hbar D\epsilon}\int_0^\infty \frac{dt}{\sinh t}\cos\left(\frac{\omega t}{\pi kT}\right) \times \left[\sin\left(\frac{\omega_s t}{2\pi kT}\right)\right]^2 . \quad (16)$$

This quantity $\Delta\sigma^{(c)}$ is shown to be smaller than $\Delta\sigma'_{\mathrm{MTZ}}$ in two dimensions, of the following order:

$$\frac{\Delta\sigma^{(c)}}{\Delta\sigma'_{\mathrm{MTZ}}} \simeq \frac{12\zeta(3)}{\pi^4 (kT\tau_\varphi/\hbar)^2} \simeq \frac{0.15}{(kT\tau_\varphi/\hbar)^2} \ll 1 . \quad (17)$$

We have discussed the Zeeman effect on the Maki-Thompson and Aslamazov-Larkin terms for the magnetoresistance and found that this effect is dominant in parallel magnetic field $H_{\parallel ab}$ and that the magnetoresistance of $H_{\perp ab}$ is explained by a summation of the orbital AL term, MT term, and Zeeman terms on the MT and AL terms for a wide temperature region. The contribution of the Zeeman term in a Coulomb interaction is small. It has been shown that the phase relaxation time τ_φ can be obtained by a comparison with the experimental data of the magnetoresistance in a parallel magnetic field. The value of τ_φ obtained from the data of Matsuda *et al.* is 10^{-3} s at $T = 100$ K.

This value of τ_φ is consistent with the estimate for the magnetoresistance in a perpendicular field. We have also obtained the temperature dependence of the phase relaxation time, $\tau_\varphi \simeq 1.3\hbar/kT$. It is quite interesting to make analyses for other high-temperature superconductors such as Bi-Sr-Ca-Cu-O and Tl-Ba-Ca-Cu-O since these materials may have different parameters. The phase relaxation time τ_φ is small and it has a similar temperature dependence as the conductivity. The origin of the phase relaxation time is not known although the temperature dependence of the conductivity has been discussed by many different theories.[16] Our finding of the importance of the Zeeman effect is also of interest in the discussion of Anderson localization in normal metals, such as quasi-two-dimensional systems.

We thank Y. Matsuda, T. Hirai, and S. Komiyama for permitting us to use their experimental results prior to publication and also for useful discussions. S.H. is grateful to the hospitality of the Landau Institute for Theoretical Physics where this work has been done. He also acknowledges the Mitsubishi Foundation and Grants-in-Aid for Scientific Research from the Ministry of Education, Science, and Culture for the support of this research.

[1]S. Hikami and A. I. Larkin, Mod. Phys. Lett. B **2**, 693 (1988).

[2]L. G. Aslamazov and A. I. Larkin, Phys. Lett. **26A**, 238 (1968).

[3]K. Maki, Prog. Theor. Phys. **39**, 897 (1968).

[4]R. S. Thompson, Phys. Rev. B **1**, 327 (1970).

[5]Y. Matsuda, T. Hirai, and S. Komiyama, Solid State Commun. **68**, 103 (1988).

[6]B. Oh, K. Char, A. D. Kent, M. Naito, M. R. Beasley, T. H. Geballe, R. H. Hammond, A. Kapitulnik, and J. M. Graybeal, Phys. Rev. B **37**, 7861 (1988).

[7]T. Terashima, K. Iijima, K. Yamamoto, Y. Bando, and H. Mazaki, Jpn. J. Appl. Phys. **27**, L91 (1988).

[8]Y. Matsuda, T. Hirai, S. Komiyama, T. Terashima, K. Iijima, K. Yamamoto, K. Hirata, and Y. Bando (private communication).

[9]B. L. Altshuler, D. E. Khmelnitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[10]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).

[11]A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 239 (1980) [JETP Lett. **31**, 219 (1980)].

[12]B. L. Altshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmelnitskii, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].

[13]D. Saint-James, G. Sarma, and E. J. Thomas, *Type II Superconductivity* (Peragmon, New York, 1969); M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[14]P. A. Lee and T. V. Ramakrishnan, Phys. Rev. B **26**, 4009 (1982).

[15]B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 1.

[16]P. A. Lee and N. Read, Phys. Rev. Lett. **58**, 2691 (1987); P. W. Anderson and Z. Zou, Phys. Rev. Lett. **60**, 132 (1988).