Relation between Low-Temperature Quantum and High-Temperature Classical Magnetotransport in a Two-Dimensional Electron Gas

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Magnetotransport measurements in several GaAs/GaAlAs heterojunctions show a linear magnetoresistance at 30 K $< T <$ 120 K. At lower temperatures, where the quantum Hall effect is observed, the magnetoresistance is found to be proportional to the product of the magnetic field and the derivative of the Hall voltage with respect to the field. This can be seen as the extension of the classical linear magnetoresistance to the quantum regime. A relation between Hall voltage and magnetoresistance valid for all samples and temperatures is given.

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At low temperatures and at high magnetic fields, in a two-dimensional electron gas (2DEG), the Hall resistance shows quantized plateaus at values h/ie^2 (h is Planck's constant, e is the electron charge, and i is an integer). This quantization, the quantum Hall effect (QHE),[1,2] is generally believed to be due to localization as a consequence of inhomogeneities in the sample.[3] On the other hand, it is well known that in metals inhomogeneities can give rise to a term in the magnetoresistance which is linear in the magnetic field (linear magnetoresistance, LMR).[4] It is therefore interesting to investigate whether a similar LMR is also observed in semiconductor heterojunctions which are metallic in their conduction. A further stimulus for a possible connection between LMR and QHE comes from the fact that it has been shown that the mechanism of LMR in metals[4] can be due to inhomogeneities, in which case a part of the Hall voltage appears along the direction of the current. Therefore, the Hall voltage and the LMR are related. In particular, if the magnetoresistance shows only a linear field dependence, then the LMR is proportional to the product of the derivative of the Hall voltage with respect to the field and the magnetic field. In this paper we will show experimentally that in GaAs/GaAlAs heterojunctions (i) a clear linear magnetoresistance is observed at higher temperatures, where the Shubnikov–de Haas oscillations have disappeared; (ii) the proportionality between the derivative of the Hall voltage and LMR exists even at low temperatures, where the QHE is dominant; and finally (iii) that the proportionality factor, when scaled properly with the sample parameters, is the same for all investigated samples and at all investigated temperatures (0.35 K $< T <$ 120 K). Therefore, we find an apparently universal relation between Hall voltage and resistivity. Our results imply that scattering processes, usually only observable at low temperatures, manifest themselves in the LMR at high temperatures as well. Although we do not yet have full explanation of this behavior, we believe that these results are important for a better understanding of magnetotransport.

In the experiment, we measured ρ_{xx} and ρ_{xy} of several Hall bar-shaped standard modulation-doped GaAs/GaAlAs heterostructure samples of different carrier densities n and mobilities μ (see Table I) at temperatures from $T=1.4$ to $\simeq 120$ K. Hall measurements showed that up to $T \simeq 130$ K, no thermal creation of carriers took place (no parallel conduction in the highly doped GaAlAs layer). At low temperatures all samples show the usual oscillatory magnetoresistance (Shubnikov–de Haas oscillations) and quantum Hall plateaus, which disappear gradually as the temperature is increased. At temperatures above $\simeq 40$ K, only a smooth monotonic increase in the magnetoresistance is observed [Fig. 1(a)]. Focusing our attention on this high-temperature behav-

TABLE I. Carrier densities n and low-field mobilities μ_0 at 4.2 K. σ_∞: conductivities extrapolated to $B=\infty$ [see Fig. 1(b)]. The proportionality constant α_0 is defined by Eq. (2).

Sample	n (10^{11} cm^{-2})	μ_0 (10^4 cm^2/V s)	σ_∞ (e^2/h)	α_0
1	1.78	22.7	0.008	0.069
2	6.46	13.8	0.05	0.059
3	5.07	39.7	0.04	0.065
4	2.95	7.0	0.02	0.070

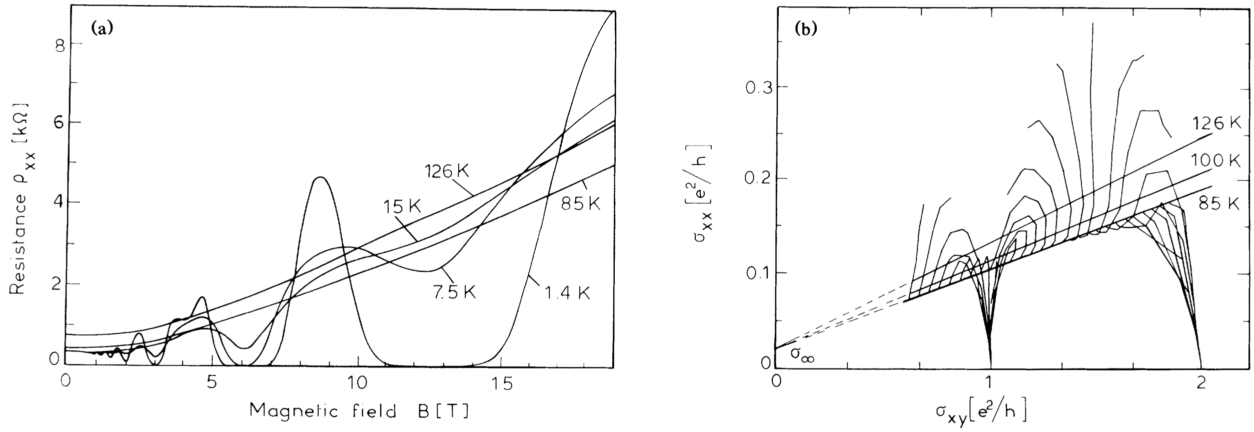

FIG. 1. (a) Magnetoresistance ρ_{xx} of sample No. 4 at temperatures from 1.4 to 126 K. (b) Conductivities σ_{xx} as a function of σ_{xy} (calculated from ρ_{xx} and ρ_{xy}). For temperatures between 1.4 and 85 K, magnetoconductivities evolve from quantum oscillations to a linear behavior. At these lower temperatures, the σ_{xx}/σ_{xy} points for the same magnetic field, but different temperatures, are connected. At higher temperatures, the magnetoconductivities follow the linear relation $\sigma_{xx}=\alpha(T)\sigma_{xy}+\sigma_\infty$. To demonstrate this behavior, we connected σ_{xx}/σ_{xy} points at the same temperature (85, 100, and 126 K) as a function of the magnetic field.

ior, it should be noted that on its own this magnetoresistance is already surprising, because from the classical theory of magnetotransport (if inhomogeneities are ignored), no magnetoresistance would be expected if one assumes parabolic bands and one-type carrier transport, conditions that are applicable almost ideally to the 2DEG in these heterojunctions.

Writing the field dependence of the resistance ρ_{xx} in the form of a power series in B, one gets

$$\rho_{xx}/\rho_0=1+\alpha(\mu B)+\beta(\mu B)^2 \text{ or } \rho_{xx}=\rho_0+\alpha(B/ne)+\beta\sigma_0(B/ne)^2, \quad (1)$$

where $\rho_0=1/ne\mu=1/\sigma_0$ is the zero-field resistance, n and μ are the carrier density and the mobility, respectively, and α and β are dimensionless coefficients. At high temperatures, where the quantum oscillations have disappeared, the Hall resistance ρ_{xy} is given by B/ne, and the expansion (1) expresses the magnetoresistance directly in this quantity. Concentrating on this regime first, in principle, we could fit Eq. (1) to the data of Fig. 1(a) using α and β as fit parameters. However, a more direct and illustrative way to express the data is in terms of the conductivities σ_{xx} and σ_{xy}, because by use of the tensor relations between the conductivity and the resistivity in the high-field approximation [$\sigma_{xx}=\rho_{xx}/(\rho_{xx}^2+\rho_{xy}^2)\simeq\rho_{xx}/\rho_{xy}^2$ and $\sigma_{xy}=\rho_{xy}/(\rho_{xx}^2+\rho_{xy}^2)\simeq 1/\rho_{xy}$], Eq. (1) can be rewritten as $\sigma_{xx}=\alpha\sigma_{xy}+\beta\sigma_0$. Representing the same data as σ_{xx} vs σ_{xy} [shown in Fig. 1(b)], α and $\beta\sigma_0$ can be determined directly from the slope and the intercept with the σ_{xx} axis for $\sigma_{xy}=0$. The figure shows how the oscillatory behavior evolves for $T\gtrsim 50$ K into a strictly linear relation between σ_{xx} and σ_{xy}. This linear dependence is observed in all samples studied and for filling factors $\nu\leq 2$; at lower magnetic fields, the slope α starts to decrease. With further increase of the temperatures, the behavior of the conductivity remains linear, only the slope α changes as a function of T. Experimentally, $\beta\sigma_0$ is found to be temperature independent; therefore, the straight σ_{xx} lines for different temperatures cross each other in one point $\sigma_{xy}=0$, $\sigma_{xx}=\beta\sigma_0=\sigma_\infty$. The ratio $\beta\sigma_0/n$ is approximately the same for all samples (see Table I), implying that β must be proportional to μ^{-1}. Note that in all cases β is so small that the quadratic term in (1) is only a small contribution to the magnetoresistance, and in the range of magnetic fields and temperatures studied, the linear term in B is always dominant.

At low temperatures, where quantum oscillations become observable, there are corrections to the classical Hall resistance due to localization effects, and B/ne in Eq. (1) cannot simply be replaced by ρ_{xy}. However, it has been found by Chang and Tsui[5] empirically that in this regime ρ_{xx} is proportional to $(d\rho_{xx}/dB)B$. Since we know from the high-temperature results that $\rho_{xx}\simeq\alpha(/ne)$, it is tempting to see the results of Chang and Tsui[5] as a generalization of our high-temperature classical behavior to the low-temperature classical regime. We therefore use Eq. (1), replace B/ne by $(d\rho_{xy}/dB)B$, and see whether the same proportionality constant connects these two regimes.

The coefficient α, as a function of temperature found in this way, is shown in Fig. 2. The values at high temperatures are obtained from the slope of the σ_{xx} vs σ_{xy} plot, and at low temperatures from the proportionality between ρ_{xx} and $(d\rho_{xy}/dB)B$. Several aspects from

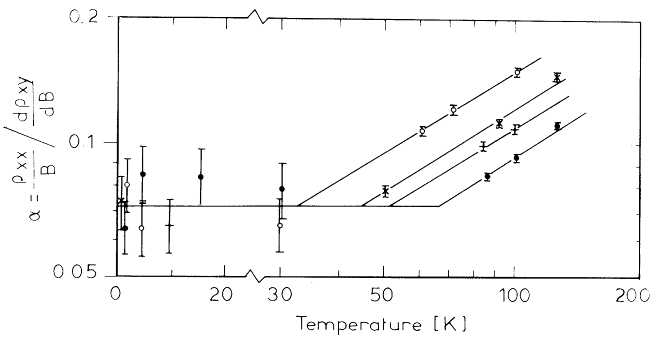

FIG. 2. The proportionality constant α as a function of temperature for the following samples: 1, ×; 2, +; 3, ○; 4, ●. At lower temperatures (linear scale), where μ =const, one finds $\alpha \simeq 0.07$; at higher temperatures (logarithmic scale), $\alpha \propto T^{3/4} \propto \mu^{-1/2}$.

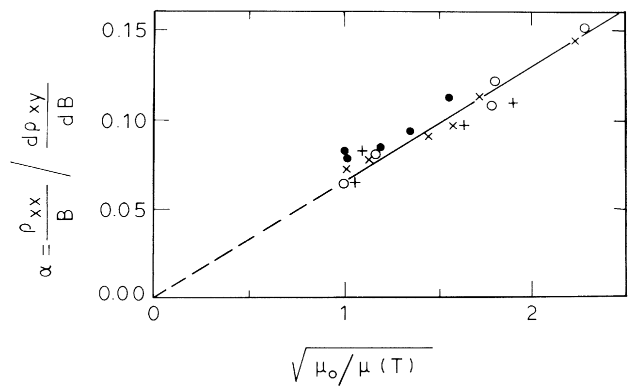

FIG. 3. The proportionality constant α vs $[\mu_0/\mu(T)]^{1/2}$. Symbols for different samples are the same as in Fig. 2.

these curves are worth noticing. The first is that at low temperatures α is [within the precision with which the proportionality between ρ_{xx} and $(d\rho_{xy}/dB)B$ is obeyed as indicated by the error bars in the figure] temperature independent and has the same value $\alpha \simeq 0.07$ for all our samples. The second aspect is that at the crossover between the two regimes at around 30 K, indeed the same value of α is found. The last aspect is that at high temperatures α rises proportionally to $T^{3/4}$ for all samples, but that the proportionality factor differs from sample to sample.

As the dc mobility is quite different for the different samples investigated, we try in the following to relate the observed sample behavior to this quantity. This mobility is known[6] to have a saturation value μ_0 at low T which depends on sample quality. At higher T, μ is approximately $\propto T^{-3/2}$ due to phonon scattering. For higher-mobility samples, the phonon scattering becomes the mobility-limiting process at lower temperatures. Finally, in the high-temperature range, all samples have roughly the same mobility. This standard behavior is also observed in our samples. We have noticed that α deviates from the constant value for each sample at approximately the same temperature where the mobility μ deviates from its value at low T. From this correlation, we assume that α and μ are related; the temperature dependence of α ($\propto T^{3/4}$) suggests that this relation is $\alpha(T) \propto [\mu(T)]^{-1/2}$. Furthermore, since the onset of the deviation of α from the low-T values occurs at lower T for better quality samples,[6] and since for a given (high) temperature one finds $\alpha \propto (\mu_0)^{1/2}$, one obtains

$$\alpha(T) = \alpha_0[\mu_0/\mu(T)]^{1/2}. \tag{2}$$

Note that these experimental results imply that the sample-dependent high-T behavior of α seems to be due to the same mechanism that is responsible for the sample-dependent saturation value μ_0 at low T. On the other hand, just in this high-T range all samples have the same μ, but they can be distinguished by their LMR. Furthermore, the temperature dependence of the LMR suggests that phonon scattering also contributes to the LMR.

Figure 3 shows the dependence predicted by Eq. (2) for all samples and temperatures. Despite the very different mobilities and densities the same factor $\alpha_0 \simeq 0.07$ is found. From the measurement of Chang and Tsui[5] a similar value of $\alpha = \alpha_0 = 0.068$ can be determined. The value for α_0 in Fig. 2 is determined from the low-T data with the proportionality between ρ_{xx} and $(d\rho_{xy}/dB)B$, but using Eq. (2) we can determine equally well α_0 from the high-T data. In this case we find values between 0.059 and 0.070 (Table I) in accordance with those shown in Fig. 2.

Therefore, we find that the relation between ρ_{xx} and ρ_{xy} is described by

$$\rho_{xx} \simeq 0.07 \left(\frac{\mu_o}{\mu(T)} \right)^{1/2} \frac{d\rho_{xy}}{dB} B, \tag{3}$$

which, at high fields, is valid for all investigated densities, mobilities, and temperatures in GaAs/GaAlAs heterostructures. The universality of this relation is striking in two respects, namely, the fact that all samples have the same proportionality constant and the same dependence on the mobility. We cannot offer a satisfactory explanation for these rather puzzling facts but only give some commentary on these two aspects.

In the theory of linear magnetoresistance for metals, it is customary to define a Kohler slope $S = d[(\rho_{xx} - \rho_0)/\rho_0]/d(\omega_c\tau)$, ω_c being the cyclotron frequency and τ the scattering time.[4] As can be seen from Eq. (1), noting that $\omega_c\tau = \mu B$, this is the same quantity as α. In the case of metals, S is determined by the relative difference in local carrier densities and the area (or volume) fraction covered by the regions of different carrier density. As LMR in metals is often related to inclusions, surface roughness, etc., S is found to be temperature independent since these defects can be seen as infinitely high potential barriers around which the current has to flow at all temperatures. Therefore, the current flow lines and thus the LMR do not change with temperature. In our case the sample inhomogeneities are believed to be rather shallow[3] potential fluctuations; therefore, the current lines may change with temperature either through the different thermal energies of the carriers or through the temperature dependence of the potential fluctuations (phonons). The width of the potential fluctuations is characterized by the Landau-level width. Theoretical calculations have shown[7-9] that in two-dimensional systems the Landau-level width Γ for different scattering mechanisms is proportional to $\mu^{-1/2}$. Let us assume that at low T, where $\mu=\mu_0$ (μ_0 being the low-temperature, sample-dependent saturation value of the mobility), Γ has some intrinsic value $\Gamma_0 \propto \mu_0^{-1/2}$ which depends on sample quality. At higher temperatures, an additional broadening mechanism due to phonons sets in. If, at these higher temperatures, $\Gamma \propto \mu^{1/2}$ (with μ now the phonon-scattering-limited mobility), as is theoretically suggested,[8,9] we could write $\alpha \propto \Gamma/\Gamma_0$, i.e., α is proportional to the relative broadening due to phonons, implying that α would become thermally activated when the phonon-broadened Landau-level width exceeds the intrinsic width Γ_0.

The second puzzling observation is the apparent universality of the constant $\alpha_0 \simeq 0.07$ in Eq. (4). In Si metal-oxide-semiconductor field-effect transistors, Tausendfreund and von Klitzing[10] have measured the proportionality between the minimal values of $d\rho_{xy}/dn$ and ρ_{xx} in the QHE plateaus, and from their results, the corresponding proportionality factor can be estimated as $\alpha_0 \simeq 0.17$, as compared to 0.07 in GaAs/GaAlAs heterostructures. These numbers seem to suggest a relation to the effective masses in these materials (which are 0.19 and 0.068, respectively) which would be a very striking observation.

In conclusion, we have shown that the 2DEG in GaAs/GaAlAs heterostructures shows a linear magnetoresistance where the slope is dependent on the temperature and sample mobility as $[\mu_0/\mu(T)]^{1/2}$. We have shown that this linear magnetoresistance can be seen as an extension of the relation between ρ_{xx} and ρ_{xy} first found by Chang and Tsui,[5] and that this behavior is universal for all samples with substantially different mobilities and carrier densities at all temperatures between 0.35 and 120 K. The validity of Eq. (3) with the same proportionality factor α_0 between ρ_{xx} and $(d\rho_{xy}/dB)B$ both in the QHE and in the LMR regime shows that the same inhomogeneities are responsible for both behaviors, which are thus clearly connected. $\alpha_0 \simeq 0.07$ for all our and Chang and Tsui's[5] samples, and comparison with data on Si metal-oxide-semiconductor field-effect transistors, seem to suggest a relation to the effective mass.

Although many questions remain unanswered, we believe that our data of LMR at high temperatures may contribute to a better understanding of the transport properties of the 2DEG in high magnetic fields.

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