Corrections to the Fermi Matrix Element for Superallowed β Decay

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Corrections to the Fermi matrix element for superallowed transitions due to isospin nonconservation are reexamined. The sources of theoretical uncertainty and the possibility of a previously neglected correction based on the mismatch between the spectator nucleons are investigated. The consequences to the conserved-vector-current hypothesis and the Kobayashi-Maskawa mixing angles are discussed.

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Superallowed Fermi β decay has been the subject of intense study for several decades (cf. Refs. 1–5 and references therein). According to the conserved-vector-current (CVC) hypothesis, their ft values should be constant for all nuclei, and given by

$$ft=\frac{K}{G_V^2|M_F|^2}, \qquad (1)$$

where $K=8.1201\times10^{-7}$ is a product of fundamental constants, G_V is the vector coupling constant for nucleon β decay [measured in units of $(\hbar c)^3$], and M_F is the Fermi matrix element, $M_F=\langle\psi_f|T_\pm|\psi_i\rangle$. By comparing the vector coupling constant for nucleon β decay to that of muon β decay, the Kobayashi-Maskawa mixing angle between u and d quarks (v_{ud}) can be determined,[5,6] and a test of the three-generation standard model of the electroweak interaction is possible.[7]

Two classes of nucleus-dependent corrections, however, must be applied to Eq. (1). The first is radiative corrections to the statistical rate function f, denoted by δ_R, giving $f_R=f(1+\delta_R)$. The second is corrections to the nuclear matrix element due to the presence of isospin-nonconserving (INC) forces in nuclei, and is denoted by δ_C; that is $|M_F|^2=|M_{F_0}|^2(1-\delta_C)$, where $M_{F_0}=[T(T+1)-T_{Z_i}T_{Z_f}]^{1/2}\delta_{if}$. With δ_R and δ_C, the "nucleus-independent" ft value for $(0^+,T=1)\rightarrow(0^+,T=1)$ transitions is

$$\mathcal{F}t=ft(1+\delta_R)(1-\delta_C)=\frac{K}{2G_V^2}. \qquad (2)$$

From these $\mathcal{F}t$ values, it is then possible to determine empirical values of G_V.

Until recently, the experimental ft values, together with the calculated corrections δ_R and δ_C for the eight most accurately measured ft values (^{14}O, ^{26m}Al, ^{34}Cl, ^{38m}K, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co), failed to yield constant $\mathcal{F}t$ values. In fact, the data gave two $\mathcal{F}t$ values,[4] one consistent for $Z<21$, and another for $Z\geq21$. This failure to give constant $\mathcal{F}t$ values persisted even after an intense effort to measure the ft values as accurately as possible.[8]

It appears that the primary reason for the lack of agreement with the CVC hypothesis was due to an incorrect evaluation of the $Z\alpha^2$ contribution to δ_R.[9] The study of superallowed Fermi transitions, however, cannot be satisfactorily concluded because the values of δ_C calculated previously by Towner, Hardy, and Harvey (THH)[3] and Wilkinson (W)[10] yield considerably different $\mathcal{F}t$ values than do those of the more recent reevaluation of Ormand and Brown (OB).[11,12] In this Letter, we reexamine the nuclear corrections δ_C, also investigating the possibility of a correction based on the mismatch between the spectator nucleons.

The formalism needed to perform microscopic calculations of δ_C is given in Refs. 3 and 11. Conventionally, δ_C has been factored into two components, i.e., $\delta_C=\delta_{IM}+\delta_{RO}$. The first, δ_{IM}, is due to isospin mixing between the different shell-model configuration states, while δ_{RO} is due to the deviation from unity of the radial overlap between the converted proton and corresponding neutron (i.e., mixing between states that lie outside of the shell-model configuration space). The OB study found that $\delta_{RO}\gg\delta_{IM}$. The essential ingredient of a calculation of δ_{IM} is an INC interaction that is added onto the standard shell-model Hamiltonian, while the calculation of δ_{RO} is based on radial wave functions that are obtained from a suitable parametrization of the mean field.

The present problem with superallowed Fermi β decay is that in the recent reevaluation both δ_{IM} and δ_{RO} were found to be considerably smaller than those obtained previously.[13] Although both sets of δ_C agree with the CVC hypothesis almost equally well, they yield inconsistent averaged $\mathcal{F}t$ values. The principal differences in these two calculations are as follows: The THH INC interaction was obtained by (i) adding Coulomb matrix elements onto the proton-proton Hamiltonian; (ii) increasing the $T=1$ part of the proton-neutron interaction by 2%; and (iii) determining the single-particle energies from closed-core plus proton and neutron nuclei, whereas the OB INC interaction was determined empirically by requiring that the parameters of a Coulomb plus phenomenological isovector and isotensor potential reproduce experimental isotopic mass splittings.[11,14] It was found that this procedure better determined the single-particle energies for nuclei away from a closed major shell, and was responsible for most of the decrease in δ_{IM}.[11] The THH radial wave functions were obtained with a Woods-Saxon plus Coulomb potential, while those of the OB study were determined with a self-consistent Hartree-Fock (HF) calculation using a Skyrme-type interaction that also included the Coulomb exchange term. The advantage of the HF procedure is that since the mean field is proportional to the nucleon densities, the Coulomb force induces a one-body isovector potential that tends to counter Coulomb repulsion, thereby reducing δ_{RO}.

In addition to these corrections, we should also account for the fact that all the parent and daughter nucleons feel the effects of different mean fields before and after the transition. With this in mind, we expect a correction to Eq. (1) of the form

$$\delta_S = 2\left[\sum_{i,\mu} n_{\mu_i}\Omega_{\mu_i} + (n_{\pi_v}-1)\Omega_{\pi_v} + n_{v_v}\Omega_{v_v}\right], \qquad (3)$$

where μ and i represent a sum over proton (π) and neutron (ν) core orbits, n_{μ_i} represents the number of protons or neutrons occupying the ith orbit, and v denotes the valence orbit in the parent nucleus. The overlaps Ω denote the deviation from unity of the parent (P) and daughter (D) radial wave functions, i.e., $\Omega_{\mu_j} = 1 - \int dr\, r^2 R^P_{\mu_j} R^D_{\mu_j}$.

Inserting radial wave functions obtained from a Hartree-Fock calculation utilizing a Skyrme-type force into Eq. (3), one finds δ_S to be appreciable. Using the SG II force,[15] we find $\delta_S = 0.18\%$ for ^{54}Co, as compared to $\delta_{RO} = 0.38\%$. It is known, however, that Hartree-Fock calculations do not conserve isospin under the interchange of the last nucleon in mirror nuclei even though the two-body force is isoscalar.[16] To determine the sensitivity of Eq. (3) to this type of spurious mixing, we have evaluated δ_S while excluding the Coulomb potential. For ^{54}Co, we find $\delta_S = 0.16\%$, indicating that the spurious mixing is important. In addition, a nonzero δ_S in the absence of the Coulomb potential violates the Behrends-Sirlon-Ademollo-Gatto theorem,[17] which states that corrections to Eq. (1) must be proportional to the mass difference between the initial and final states.

Given these considerations, an estimate of δ_S might be obtained in two ways. First, by taking the difference between the results obtained by Eq. (3) with and without the Coulomb potential, leading to corrections of the order 0.02%–0.05%. The second procedure, which avoids the problem encountered in the absence of the Coulomb potential, is to use wave functions obtained from the $T_Z = 0$ mean field. In this case, the effect of the Coulomb potential in the $T_Z \neq 0$ nucleus can be accounted for by using an effective charge $e' = e(1 + 2T_Z/A)$, where A is the number of nucleons, and T_Z for the proton is taken to be positive. Using these wave functions, we find $\delta_S < 0.01$. Given these considerations, δ_S is probably negligible: however, we account for it by including a conservative uncertainty of 0.05% in the total correction δ_C.

Other sources of uncertainty in δ_C lie in the procedures used to evaluate δ_{RO} and δ_{IM}. In order to determine the sensitivity in δ_{RO} to the Skyrme parameters, we have evaluated δ_{RO} using five different Skyrme forces: A,[18], SG I,[15] SG II,[15], Skyrme M,[19] and Skyrme M*.[20] The SG II values were found to be very nearly equal to the average obtained with all forces, and since this is the same force used previously[11,12,21] these values are reported here in Table I (under the heading HF). The variations in δ_{RO} were typically $\pm 0.03\%$, and are due to the fact that the explicit isovector properties of the Skyrme force cannot be determined unambiguously.[22] An additional uncertainty of $\approx 0.03\%$ in δ_{RO} arises from the selection of the shell-model Hamiltonian and truncations on the configuration space (see Ref. 21).

Just as in the case for δ_S mentioned above, we note that in the limit that the Coulomb potential is switched off, δ_{RO} is also nonzero. This effect is due to the fact that the HF binding energies of the $T_Z = 0$ and ± 1 nuclei are not equal, and is largest for the lightest nuclei, $\delta_{RO} \approx 0.04\%$ for ^{14}O, and decreases for the heavier nu-

TABLE I. Comparison of the values of δ_{RO} and δ_{IM}.

	δ_{RO} (%)				δ_{IM} (%)	
Nucleus	HF	ECHF	AVG	THH	OB	THH
^{14}O	0.15	0.21	0.18(5)	0.23(3)	0.01	0.05
^{26m}Al	0.27	0.18	0.23(6)	0.27(4)	0.01	0.07
^{34}Cl	0.47	0.36	0.42(7)	0.62(7)	0.06	0.23
^{38m}K	0.49	0.27	0.38(12)	0.54(7)	0.11	0.16
^{42}Sc	0.29	0.25	0.28(5)	0.35(6)	0.11	0.13
^{46}V	0.24	0.17	0.20(6)	0.36(6)	0.01	0.04
^{50}Mn	0.31	0.24	0.28(6)	0.40(9)	0.004	0.03
^{54}Co	0.38	0.30	0.34(6)	0.56(6)	0.005	0.04

clei, $\delta_{RO} \approx 0.01\%$ for ^{34}Cl. Further, in this case, if the binding energies of the parent and daughter states are constrained to be equal, δ_{RO} reduces to nearly zero. We have also evaluated δ_{RO} using the $T_Z=0$ effective-charge Hartree-Fock (ECHF) mean fields described above, and are also given in Table I. The ECHF values of δ_{RO} tend to be somewhat smaller than their HF counterparts. In particular, for ^{38}K where the ECHF value is $\approx 0.2\%$ smaller. This reflects an uncertainty in our ability to calculate δ_{RO}. Here, we use the average of the two values when evaluating the total correction δ_C, which is also given in Table I along with the THH values of δ_{RO}.

The configuration-mixing corrections δ_{IM} are sensitive to the choice of the shell-model configuration space, the isoscalar Hamiltonian, and the isovector single-particle energies. The uncertainties due to these quantities are somewhat difficult to estimate. However, in this work, the most recent isoscalar shell-model Hamiltonians were used within largest shell-model configuration space possible, and, further, the INC interactions were determined empirically for each shell-model Hamiltonian.[14] In all cases, δ_{IM} was found to be much smaller than δ_{RO}, with an upper limit being approximately 0.1%. In this regard, we have chosen to assign an uncertainty of 0.05% in δ_{IM}. For the purpose of comparison, both the THH values of δ_{IM} and those of the present work are also given in Table I.

The total uncertainties discussed up to this point add up to approximately 0.09% in most cases. This arises from the addition in quadrature of 0.05% in δ_S, 0.05% in δ_{IM}, and 0.06% in δ_{RO}. The uncertainties in δ_{RO} are 0.03% due to different Skyrme forces, 0.03% due to model-space truncations, and typically 0.04% (0.11% in ^{38m}K) from the difference in the ECHF and HF calculations.

Given in Table II are the experimental ft values,[8] the outer radiative correction δ_R,[9,23] and a comparison between the OB and THH[3,4,24] values of δ_C, and the corresponding "nucleus-independent" $\mathcal{F}t$ values. Both sets of corrections yield essentially constant, but inconsistent averaged $\mathcal{F}t$ values. The THH corrections give $(\mathcal{F}t)_{avg}=3071.5\pm1.6$ sec with $\chi^2/\nu=0.64$, while the corrections reported here yield $(\mathcal{F}t)_{avg}=3077.3\pm1.9$ sec with $\chi^2/\nu=0.63$.

From the averaged $\mathcal{F}t$ values the vector coupling constant for single nucleon β decay can be determined, and the Kobayashi-Maskawa mixing matrix element v_{ud} is then

$$v_{ud}=\frac{G_V}{G_\mu}(1+\Delta_\beta-\Delta_\mu)^{-1/2},$$

where $G_\mu/(\hbar c)^3=1.16637(13)\times10^{-5}$ GeV^{-2} (Ref. 25) is the vector coupling constant for muon β decay, and Δ_β and Δ_μ are the "inner" radiative corrections to both nucleon and muon β decay, with $\Delta_\beta-\Delta_\mu=0.023(2)$.[26] The THH and OB corrections give $v_{ud}=0.9746\pm0.0010$ and 0.9737 ± 0.0010, respectively. With v_{ud} determined, a test of the three-generation standard model is possible at the level of quantum corrections,[7] i.e., the Kobayshi-Maskawa matrix should be unitary ($v^2=v_{ud}^2+v_{us}^2+v_{ub}^2=1$). Taking $v_{us}=0.220\pm0.002$ (Ref. 27) and $v_{ub}<0.0075$ (90% confidence level),[28] the unitarity condition for the THH and OB values are $v^2=0.9982\pm0.0021$ and 0.9965 ± 0.0021, respectively. The THH and OB sets of nuclear corrections appear to be in agreement at this level. However, the error is dominated by the uncertainty in the "inner' radiative corrections. If this uncertainty were reduced to the level of the uncertainty in the nuclear corrections, the conclusions might be different. For instance, if the inner radiative correction does not change in value, the v^2 obtained with the THH nuclear corrections would agree better with the unitarity condition.

At this point, we wish to emphasize that both the OB and THH values of δ_C yield $\mathcal{F}t$ values that are essentially in equally good agreement with the CVC hypothesis, i.e., at the level of 0.06%. However, as is pointed out, they yield inconsistent averaged values, leading to somewhat different conclusions regarding a test of the three-generation standard model. The principal difference be-

TABLE II. List of ft values, corrections, and "nucleus-independent" $\mathcal{F}t$ values.

			δ_C (%)		$\mathcal{F}t$	
Nucleus	ft	δ_R (%)	OB	THH	OB	THH
^{14}O	3038.1(23)	1.53(1)	0.19(13)	0.33(10)	3078.7(47)	3074.4(37)
^{26m}Al	3034.5(14)	1.47(2)	0.24(13)	0.34(11)	3071.7(44)	3068.6(37)
^{34}Cl	3052.0(29)	1.45(3)	0.48(14)	0.85(12)	3081.4(53)	3070.0(47)
^{38m}K	3045.1(26)	1.44(3)	0.49(17)	0.70(12)	3073.7(59)	3067.3(48)
^{42}Sc	3048.7(63)	1.46(4)	0.39(13)	0.48(12)	3081.1(76)	3078.4(76)
^{46}V	3043.7(22)	1.46(4)	0.21(13)	0.40(12)	3081.6(49)	3075.7(46)
^{50}Mn	3039.9(40)	1.46(5)	0.28(13)	0.43(13)	3075.6(59)	3071.0(59)
^{54}Co	3044.7(23)	1.45(5)	0.35(13)	0.60(12)	3077.1(49)	3070.3(48)
Avg					3077.3(19)	3071.5(16)
χ^2/ν					0.63	0.64

tween these two estimates lies in the radial overlap correction. Here δ_{RO} is reduced considerably relative to the THH values. This reduction is primarily due to the different treatments of the Coulomb potential and the nuclear mean fields. The THH Coulomb potential was that of a uniformly charged sphere, while in this work it was obtained from the proton densities via a self-consistent Hartree-Fock calculation. Further, the influence of the Coulomb exchange term and the "induced" isovector potential described previously were included. The net effect of these differences is to produce a HF potential for the protons that is both deeper at the origin, and has a higher barrier at the surface than the Woods-Saxon potential, therefore, increasing the overlap between the converted proton and corresponding neutron.[11]

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