Unitarity Effects in W^+W^- Elastic Scattering

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Unitarity effects associated with the two-channel system consisting of the states $W_L^+W_L^-$ and $Z_L^0Z_L^0/\sqrt{2}$ are investigated using the K-matrix technique. In $W_L^+W_L^-$ elastic scattering, we find that for a Higgs-boson mass (m_H) of 0.5 TeV the unitarized s-wave amplitude is very similar to the Born amplitude modified by the addition of a Higgs-boson width. When m_H reaches 1.4 TeV, which is above the unitarity bound obtained from the Born term, the unitarized amplitude saturates at an absolute value of about $\frac{2}{3}$ of the unitarity limit. The implications of unitarity on W-pair production at Superconducting Super Collider energies are also discussed.

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Even though virtually all data available from high-energy experiments are consistent with the predictions of the standard model, the precise nature of its Higgs sector remains untested. It is certainly possible to attribute the masses of the $W^{\pm}$ and Z^0 gauge bosons to spontaneous symmetry breaking associated with a complex Higgs-boson doublet. Clearly, the validity of this interpretation would benefit from the discovery of a neutral spinless boson with the standard-model couplings to the leptons, quarks, and gauge bosons. The ability to detect such a particle is an important design criterion for proposed super colliders. However, at present there are no significant experimental constraints on the mass m_H of the neutral Higgs boson (H^0) within the minimal-doublet scheme. Since possible characteristic decay modes of the H^0 are sensitive to m_H, a variety of mass regimes must be explored in any systematic survey of Higgs-boson signatures. One important regime is defined by $m_H > 2m_{Z^0}$. In this case, the Higgs-boson decays

$$H^0 \to W^+W^- , \tag{1a}$$

$$H^0 \to Z^0Z^0 , \tag{1b}$$

are allowed and dominant. As a consequence, Higgs bosons can be produced by gauge-boson fusion[1] and their existence can be inferred by analysis of gauge-boson pair events.[2] A search of this type amounts to a study of, for example, W^+W^- elastic scattering.[3]

It so happens that the strength of gauge-boson scattering is sensitive to the value of m_H. Indeed, Dicus and Mathur[4] and Lee, Quigg, and Thacker[5] investigated various gauge-boson scattering amplitudes in the leading order of perturbation theory and showed that these amplitudes violate the unitarity bound for $m_H \gtrsim 1.0$–1.4 TeV in the limit that the center-of-mass energy $\sqrt{s} \to \infty$. Since the perturbative expansion of the scattering matrix is not unitary order by order, it is not possible to rigorously conclude that m_H is bounded from above. It is, however, evident that higher-order corrections are no longer negligible when m_H exceeds 1 TeV. Gauge-boson scattering may even become nonperturbative in this range of Higgs-boson masses. This possibility has been investigated using the Goldstone-boson equivalence theorem[6] and its extension to SU(N)×U(1) combined with a $1/N$ expansion.[7] There have also been recent calculations of one-loop corrections to the Higgs-boson width[8] and to the W^+W^- scattering amplitude,[9] as well as a suggestion to use like-charge W-pair events as a probe of electroweak symmetry breaking.[10]

In this Letter, we examine the effects of higher-order corrections by unitarizing the amplitudes associated with the two-channel system[5] consisting of $W_L^+W_L^-$ and $Z_L^0Z_L^0/\sqrt{2}$. The subscript L denotes a longitudinally polarized $W^{\pm}$ or Z^0. Purely longitudinal states are known to describe gauge-boson scattering amplitudes to the leading order in the center-of-mass energy.[11] We chose to unitarize the S matrix by introducing the K matrix as

$$S = \frac{1 - \frac{1}{2}iK}{1 + \frac{1}{2}iK} . \tag{2}$$

The matrix K is Hermitian and it can be expressed order by order in perturbation theory in terms of the Feynman-Dyson expansion of the S matrix.[12] In second order, the relation is

$$K^{(2)} = iS^{(2)} . \tag{3}$$

If S is expressed as $S = 1 + i\mathcal{M}$, and the resulting integral equation for $\mathcal{M}$ in terms of K is projected into partial waves, the unitarized Jth partial-wave amplitude t_J is related to Born amplitude a_J as

$$t_J = (1 - ia_J)^{-1} a_J . \tag{4}$$

When $J = 0$, a_0 for the two-channel system described

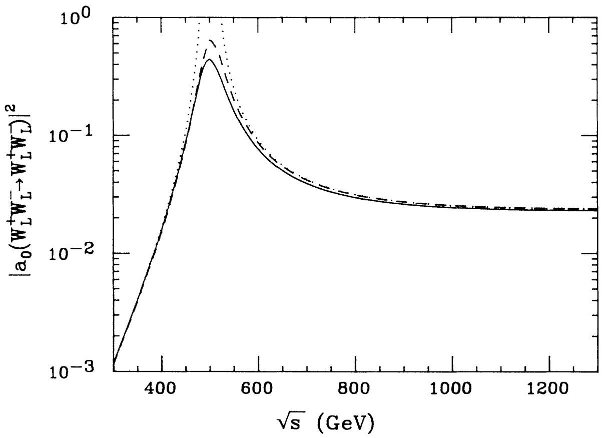

FIG. 1. The square of the $J=0$ partial-wave amplitude a_0 for $W_L^+ W_L^-$ elastic scattering plotted as a function of the center-of-mass energy $\sqrt{s}$ for $m_H = 0.5$ TeV. The solid line is the unitarized amplitude, the dashed line is the Born amplitude with the addition of a Higgs-boson width $\Gamma_H = 51.5$ GeV, and the dotted line is the Born amplitude with no modifications.

above takes the form

$$a_0 = \begin{pmatrix} a(s)+b(s) & \dfrac{a(s)}{\sqrt{2}} \\ \dfrac{a(s)}{\sqrt{2}} & \frac{1}{2}a(s)+b(s) \end{pmatrix},$$

where

$$a(s) = -\frac{g^2 m_H^2}{64\pi m_W^2}\left[\frac{s}{s-m_H^2}\right], \tag{5a}$$

$$b(s) = -\frac{g^2 m_H^2}{64\pi m_W^2}\left[1-\frac{m_H^2}{s}\ln\left[1+\frac{s}{m_H^2}\right]\right], \tag{5b}$$

and g is the weak coupling constant. Notice that we have not included a Higgs-boson width in the pole term of Eq. (5a). We rely on the unitarization scheme to generate the width. The matrix t_0 is easily obtained in terms of a_0, whose eigenamplitudes are $\frac{3}{2}a+b$ and b, as

$$t_0 = \frac{a_0 - i\det(a_0)}{\det(1-ia_0)}. \tag{6}$$

Its eigenamplitudes are

$$\lambda_1 = \frac{\frac{3}{2}a+b}{1-i(\frac{3}{2}a+b)}, \tag{7a}$$

$$\lambda_2 = \frac{b}{1-ib}, \tag{7b}$$

and the unitarized s-wave $W_L^+ W_L^-$ amplitude is

$$(t_0)_{W_L^+ W_L^-} = \frac{(a+b)-i(\frac{3}{2}a+b)b}{[1-i(\frac{3}{2}a+b)](1-ib)}. \tag{8}$$

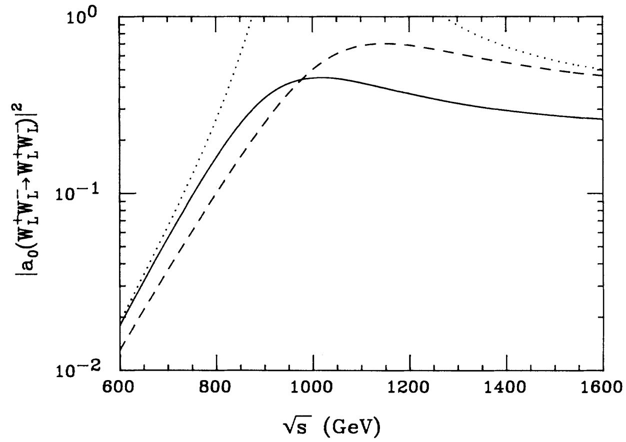

FIG. 2. Same as in Fig. 1 with $m_H = 1.0$ TeV and $\Gamma_H = 471$ GeV.

In order to assess the effect of unitarity, we show the square of the unitarized $W_L^+ W_L^-$ elastic amplitude compared to the corresponding Born amplitude in Figs. 1–3. In addition, we plot the Born amplitude including the value of the Higgs-boson width obtained from the process listed in Eqs. (1) in the pole term of Eq. (5a). Since a finite Higgs-boson width is generated by the unitarization procedure, a similar modification is unnecessary in the unitarized amplitude. From Fig. 1, it is clear that even for a moderate value of the Higgs-boson mass ($m_H = 0.5$ TeV) the effect of imposing unitarity is not negligible, particularly at the peak. However, the simple prescription of adding the Higgs-boson width to the pole term is still reasonably accurate. As m_H increases, the adequacy of this prescription decreases. For $m_H = 1.4$ TeV, which is in the mass range where the Born amplitude violates unitarity, the simple addition of a width is not sufficient to preserve unitarity. At this mass, the uni-

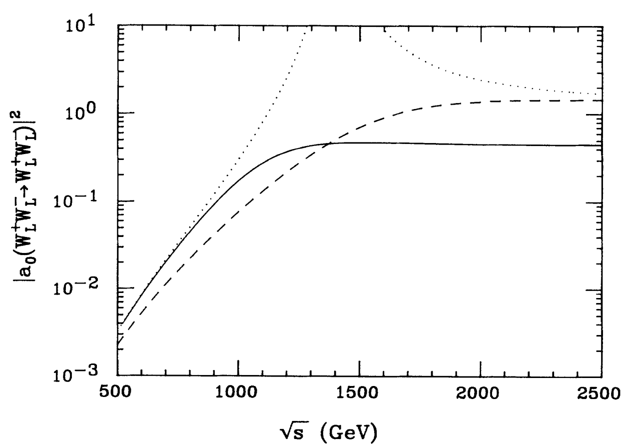

FIG. 3. Same as in Fig. 1 with $m_H = 1.4$ TeV and $\Gamma_H = 1320$ GeV.

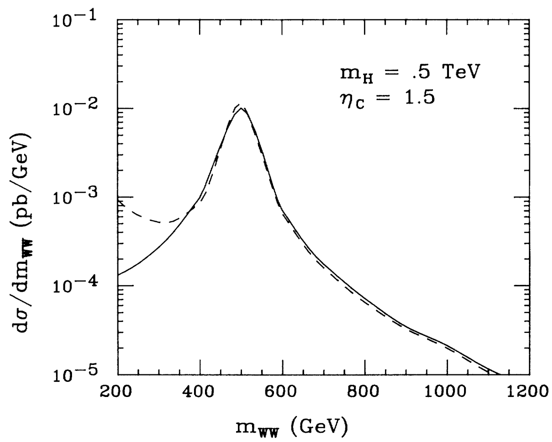

FIG. 4. The invariant-mass distribution for the process $pp \to W_L^+ W_L^- \to W_L^+ W_L^- X$ shown for a collider energy of $\sqrt{s} = 40$ TeV and a Higgs-boson mass $m_H = 0.5$ TeV. The solid line is obtained using the unitarized $J=0$ partial-wave amplitude and the dashed line is obtained using the full Born amplitude for $W_L^+ W_L^-$ elastic scattering. A rapidity cut $\eta_C = 1.5$ is imposed on both final W's.

tarized amplitude saturates at an absolute value of about 0.67 in the energy range shown, while the Born amplitude continues to increase.

In Figs. 4–6, the yield of $W_L^+ W_L^-$ pairs computed with the unitarized s-wave amplitude is compared to the yield obtained with the full Born amplitude for the process

$$pp \to W_L^+ W_L^- \to W_L^+ W_L^- X. \tag{9}$$

Both calculations are performed with the effective-gauge-boson approximation[13] at the proposed energy of the Superconducting Super Collider. For $m_H = 0.5$ TeV the two calculations are in excellent agreement. For $m_H \gtrsim 1.0$ TeV, the Born amplitude (with the addition of a Higgs-boson width) yields a pair cross section that exceeds the cross section given by the unitarized amplitude for large values of the invariant mass m_{WW}. The saturation effect evident in Fig. 3 for $m_H = 1.4$ TeV and large m_{WW} is suppressed in Fig. 6 due to phase-space constraints. It is important to note that the total number of pair events predicted using the unitarized amplitude is less than that predicted by the Born term for larger values of m_H.

We have shown that the imposition of unitarity has a discernible effect on the predicted yield of W pairs for moderate values of m_H. For these values of m_H, the advantage of imposing unitarity lies in the fact that the finite-width effect is generated from the Born amplitudes. This ensures that the equivalence of longitudinal-gauge-boson scattering and Goldstone-boson scattering[6,11,14] is preserved. It also affords a means of assessing the effect of one-loop corrections to the scattering amplitude in a systematic way. As m_H increases, the unitarized amplitude continues to predict a peak near $m_{WW} = m_H$ until m_H approaches the "unitarity bound" of $m_H \sim 1.4$ TeV. At this value of m_H, the pair cross section is rather featureless.

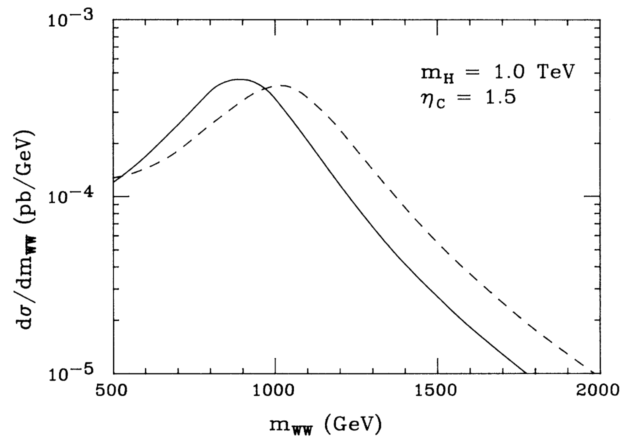

FIG. 5. Same as Fig. 4 with $m_H = 1.0$ TeV.

The discussion presented here is based on the K-matrix unitarization scheme which is, like *any* unitarization method employing S-matrix elements of a finite order, *ad hoc.* The technique amounts to summing bubble diagrams and is most appropriate in situations where two-body channels can give rise to saturation effects. This is certainly the case in gauge-boson scattering. Although the calculations outlined above could be refined by including additional channels and incorporating the real parts of the one-loop corrections to the amplitudes appearing in the matrices a_J, the general features associated with the unitarization of standard-model amplitudes illustrated here should persist.

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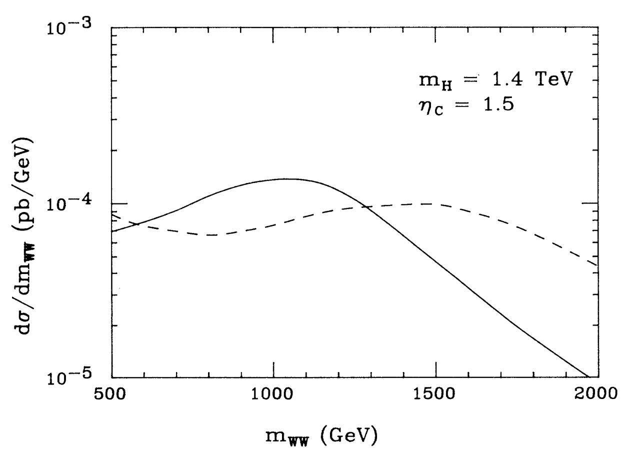

FIG. 6. Same as Fig. 4 with $m_H = 1.4$ TeV.

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[14]For a discussion of the ambiguities associated with *ad hoc* prescriptions for including the Higgs-boson width in perturbative amplitudes, see G. L. Kane and C. P. Yuan (to be published).