Invisible-Axion Emissions from SN 1987A

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Based on the equivalence theorem of the on-shell S-matrix elements independently of nonlinearly realized Goldstone fields, we clarify the discrepancies raised by the use of the pseudovector derivative and pseudoscalar pion-nucleon couplings in the nucleon-nucleon axion bremsstrahlung process which is often taken as the dominant axion emission mechanism from SN 1987A. In addition, we propose a new method to calculate the axion emission rate from $NN \to NN\pi$ data, and compare the result to the theoretical calculation based on the one-pion-exchange approximation.

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Recent studies[1] on the axion decay constant from SN 1987A seem to give quite a strong lower bound, $F_a \gtrsim 10^{10}$ GeV. In general, the axion decay constant is defined[2] by the axion-gluon-gluon interaction, $(a/32\pi^2/F_a)F\tilde{F}$. If one uses the vacuum expectation value of the singlet Higgs field, $F_a \cong v/N$, where N is the vacuum degeneracy in the axion literature.[2] These studies are important because the primary emission process from SN 1987A is nucleon-nucleon axion bremsstrahlung (NNAB) while direct evidence of the invisible axion from laboratory experiments is still lacking. Theoretically one invokes the one-pion-exchange approximation (OPEA) to the matrix element for the NNAB process. In addition, most of these calculations used the pseudovector pion-nucleon couplings and pseudoscalar axion-nucleon couplings following Ref. 3, while the pseudoscalar coupling was used for both pions and axions in Ref. 4. Both Refs. 3 and 4 precede the observation of the neutrino burst from SN 1987A.

Here we will comment on two aspects related to the axion-emission calculations from SN 1987A. First we introduce the equivalence theorem[5] of the on-shell S-matrix elements from the particular set of Goldstone boson fields that are realized nonlinearly in a local Lagrangian theory. From this we will see that both the pseudovector and pseudoscalar coupling schemes are possible for both pions and axions to the nucleons, and in particular they give the identical matrix element for the NNAB process independently of the coupling schemes employed as long as one makes approximations systematically consistent. This will clear the discrepancies raised before by the pseudovector[3] and pseudoscalar[4] pion-nucleon couplings. In addition, we will see that there are potentially important other effects coming from the nontrivial Peccei-Quinn (PQ) charge matrix for axions, the pion-axion mixing effect due to the gluon anomaly, and isospin-symmetry breaking, which are ignored in all axion-emission calculations hitherto. Secondly we suggest a new way of computing the NNAB amplitude from the experimental one-pion-production cross sections. The result will be compared to the theoretical NNAB calculation based on OPEA.

It is well known[6] how to treat pions as Goldstone bosons of the chiral $SU(2)_L \times SU(2)_R$ symmetry. Similarly, we can treat the axion as the Goldstone boson of the PQ symmetry. Let N be the nucleon doublet, and $\Sigma = \exp(2i\pi/f_\pi)$, where $\pi = \boldsymbol{\pi}\cdot\boldsymbol{\tau}/2$. Consider the following CP-conserving effective Lagrangian:

$$L = \bar{N} i \partial\!\!\!/ N - m(\bar{N}_L \Sigma N_R + \text{H.c.}) + \lambda[\bar{N}_L \Sigma i(\partial\!\!\!/ \Sigma^\dagger) N_L + \bar{N}_R \Sigma^\dagger i(\partial\!\!\!/ \Sigma) N_R] - (F_a)^{-1} \bar{N}(\partial\!\!\!/ a)\gamma_5 X N + (f_\pi^2/4)\mathrm{tr}\,\partial_\mu \Sigma\, \partial^\mu \Sigma^\dagger + \tfrac{1}{2}\partial_\mu a\, \partial^\mu a + \cdots , \quad (1)$$

where X denotes the real constant 2×2 diagonal matrix that depends on the PQ charges, λ is a real constant, m is the nucleon mass, and $f_\pi = 93$ MeV is the pion decay constant. Here the isospin-symmetry-breaking terms which are of order m_q (the light-quark mass) as well as the π-a mixing due to the gluon anomaly are neglected.

The choice of the nucleon basis is up to us depending on the convenience. Consider the a- and π-dependent phase transformation of the nucleon,

$$N_L' = U^\dagger N_L, \quad N_R' = U N_R , \quad (2)$$

with

$$U = \exp i(2h\pi/f_\pi + aY/F_a) , \quad (3)$$

where h is a constant and Y is a constant 2×2 diagonal matrix. Then it is well known[5] that both the exact and tree-level on-mass-shell amplitudes calculated from L of (1) and from the new Lagrangian defined by

$$L' = L(N_L \to U N_L, N_R \to U^\dagger N_R) \quad (4)$$

are identical independently of any specific choices of h

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and Y in (3); i.e., the S matrix is independent of U. It is then clear that there are many equivalent Lagrangians possible but with different coupling schemes from (4). This will clarify the discrepancies between the pseudovector and pseudoscalar pion-nucleon couplings used previously[3,4] for NNAB. In particular let us consider the interaction Lagrangian $L'_{\rm int}$ relevant to NNAB in OPEA[1,3,4] that is systematically consistent to the order $1/F_a f_\pi$,

$$\begin{aligned}L'_{\rm int} = &-(2m/f_\pi)(1-2h)\bar{N}i\gamma_5\pi N + (2/f_\pi)(h-\lambda)\bar{N}\,\partial\pi\,\gamma_5 N \\ &+(2m/F_a)\bar{N}i\gamma_5 aYN + (1/F_a)\bar{N}\,\partial\!\!\!/ a\,\gamma_5(Y-X)N - (2m/F_a f_\pi)(1-2h)\bar{N}a\{\pi,Y\}N \\ &+(1/F_a f_\pi)(h-2\lambda)\bar{N}ia[Y,\partial\!\!\!/\pi]N + (h/F_a f_\pi)\bar{N}i\,\partial\!\!\!/ a[\pi,Y-2X]N\,. \end{aligned} \tag{5}$$

Note for the axions with the PQ charge matrix $X=g1$, which is the case considered effectively in Refs. 1, 3, and 4, that the interaction Lagrangian (5) reduces to the derivative-coupling terms only for the convenient choice $h=\frac{1}{2}$ and $Y=0$,

$$L'_{\rm int} = (1/f_\pi)(1-2\lambda)\bar{N}\,\partial\!\!\!/\pi\,\gamma_5 N - (g/F_a)\bar{N}\,\partial\!\!\!/ a\,\gamma_5 N\,, \tag{6}$$

while we get the nonderivative pseudoscalar couplings only for another convenient choice $h=\lambda$ and $Y=X$,

$$L'_{\rm int} = -(2m/f_\pi)(1-2\lambda)\bar{N}i\gamma_5\pi N + (2mg/F_a)\bar{N}i\gamma_5 aN - (4mg/F_a f_\pi)(1-2\lambda)\bar{N}a\pi N\,. \tag{7}$$

Also the case[3] of the derivative pion coupling and pseudoscalar axion coupling can be obtained when $h=\frac{1}{2}$ and $Y=X$,

$$L'_{\rm int} = (1/f_\pi)(1-2\lambda)\bar{N}\,\partial\!\!\!/\pi\,\gamma_5 N + (2mg/F_a)\bar{N}i\gamma_5 aN\,. \tag{8}$$

Similarly, the case of the derivative axion coupling and pseudoscalar pion coupling follows for the choice $h=\lambda$ and $Y=0$. Since the S matrix is independent of U, any one of the possible coupling schemes from (5) is *correct* and in particular the three cases (6), (7), and (8) are all *equivalent* and give the identical NNAB amplitude with the usual coupling constants $(1-2\lambda)/f_\pi = f/m_\pi = g_A/2f_\pi$, g_A being the π-N axial-vector coupling constant 1.25, and $g_{an}=2mg/F_a$. We then see that in OPEA we must keep the dimension-five interaction term[7] $\bar{N}a\pi N$ which is of the order $1/F_a f_\pi$ in the case[4] of the pseudoscalar couplings for both pions and axions to be consistent with the pseudovector coupling scheme. In (6) and (8), there is no such term so that the use of at least one derivative-coupling scheme for pion or axion results the equivalent S matrix as (6) but even this is true only for the axions with the PQ charge matrix $X=g1$. Strictly speaking, axions must have the PQ charge matrix of the form $X=g1+g_3\tau_3$ with $g_3\neq 0$ because of the π-a mixing due to the gluon anomaly,[8] so that an additional dimension-six term, $-(1/F_a f_\pi)\bar{N}i\,\partial\!\!\!/ a[\pi,X]N$, contributes to the derivative-coupling scheme (6). This vertex would give rise to an additional n-p axion bremsstrahlung in OPEA and the π-a mixing effects can cause an important correction[9] to the axion-emission rate. In this paper, however, we will ignore such additional complications following the spirit of the previous works.[1,3,4]

Next we discuss a new method of estimating the NNAB amplitude as the one-pion exchange can only be a guideline for the complicated strong-interaction effects. Indeed the doubt on the validity of OPEA in the context of NNAB has already been raised.[10] The use of the experimental data measured in a similar and related process will be more reliable in principle and can provide a test for the validity of OPEA. Since both pions and axions are Goldstone bosons, we suggest the use of the production cross sections for $NN\to NN\pi$, which is most naturally related to NNAB. For $pp\to pp\pi^0$, there exist data[11] above $T_L=350$ MeV. The corresponding matrix element squared $|M|^2$ for NNAB can be obtained from $\sigma(pp\to pp\pi^0)$ by dividing the latter by the phase space times the flux factor and by replacing the pion-nucleon coupling constant $(1-2\lambda)/f_\pi = g_A/2f_\pi$ by the axion-nucleon coupling constant g/F_a. Near the threshold, which is the case of our interest, the Lorentz-invariant phase space of the final $pp\pi^0$ state can well be approximated by[12]

$$P_3 \cong \frac{1}{128\sqrt{2}\pi^2}\left(\frac{m_\pi}{m}\right)^{1/2}\frac{(\sqrt{s}-2m-m_\pi)^2}{(1+m_\pi/2m)^{3/2}}\,, \tag{9}$$

where $\sqrt{s}$ is the total c.m. energy and the flux factor is $(2s)^{-1}(1-4m^2/s)^{-1/2}$. Then the phenomenological matrix element is

$$\frac{1}{8}\sum_{\rm spin}|M|^2_{\rm phen} \cong 256\sqrt{2}\pi^2\left(\frac{2f_\pi g}{g_A F_a}\right)^2\sigma(pp\to pp\pi^0)\left(\frac{m}{m_\pi}\right)^{1/2}\frac{(1+m_\pi/2m)^{3/2}(1-4m^2/s)^{1/2}}{(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2}\,, \tag{10}$$

where the factor $\frac{1}{8}$ comes from the statistical factor for the average over the initial proton spin states times the symmetry factor for the identical protons in the final state and we used the approximation $|M^2|\cong$const, which is believed to be good in the limit of our interest $3mT \gg m_\pi^2$ for the supernova.

The matrix element for NNAB in OPEA is given by[13]

$$\frac{1}{8}\sum_{\text{spin}}|M|^2_{\text{1pion}} \cong 32\left(\frac{f}{m_\pi}\right)^4 g_{an}^2 m^2 = 128\left(\frac{g_A}{2f_\pi}\right)^4\left(\frac{g}{F_a}\right)^2 m^4 \qquad (11)$$

for the fully degenerate nuclear matter. To see the validity of OPEA, let us define the overestimation ratio r by dividing Eq. (11) by Eq. (10),

$$r=\frac{3.5\times 10^3}{\tilde{\sigma}}\frac{(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2}{(1-4m^2/\sqrt{s})^{1/2}}, \qquad (12)$$

where we used $\sigma(pp\to pp\pi^0)=\tilde{\sigma}\times 10^{-28}\ \text{cm}^2$. Numerically, we find from Ref. 11 that $\tilde{\sigma}\cong 1.35$ for the incident-proton kinetic energy $T_L=400$ MeV ($\sqrt{s}=2068$ MeV) and $\tilde{\sigma}\cong 5$ for $T_L=500$ MeV ($\sqrt{s}=2113$ MeV). For these, we find $r=4.4$ and 3.4, respectively, which are in agreement with the results of Ref. 12. Despite various approximations gone into (10) and (11), these r values indicate that OPEA to NNAB adopted in the previous works[1,3,4] is perhaps not unreasonably overestimating the $nn\to nna$ production rate near the supernova core temperature. A more accurate test should involve the exact phase space, the pion-momentum dependence of the cross sections, and more relativistic treatment of pions in Eq. (10) as well as more careful pion and perhaps axion-mass dependence in Eq. (11).

We have shown in this paper a systematic way of deriving the interaction terms of pions and axions as the Goldstone boson with the nucleons based on the equivalence of the on-shell S-matrix elements independently of the Goldstone fields realized nonlinearly in a local Lagrangian theory. In particular, both the pseudovector and pseudoscalar pion-nucleon couplings are not only possible but also give the identical on-shell S-matrix elements so long as one keeps all interaction terms consistently in the given order of approximation. This result has a broad implication in nuclear physics involving the Goldstone-boson interactions beyond just the application to NNAB in the nascent neutron star associated with SN 1987A. Also, we have suggested a new way of estimating NNAB amplitude from experimental pion-production cross sections. When the phenomenologically determined amplitude is compared to that of theoretical NNAB amplitude based on OPEA, we find that OPEA is not dangerously overestimating the axion-emission rate near the supernova core temperature despite the various crude approximations. A more accurate test should treat pions fully relativistically and require the pion momentum dependence of the production cross sections. In addition, we have noted other potentially important corrections to the interaction Lagrangian such as the π-a mixing effects that induce a nontrivial PQ charge matrix to cause an additional interaction term even in the derivative-coupling scheme. Assuming that these corrections are minor and the approximations gone into Eqs. (10) and (11) are reasonable, we conclude that OPEA overestimates the axion-emission rate by about a factor of 3 so that the modification to the lower bound of F_a seems to be insignificant.

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