## Evidence for Intermittent Patterns of Fluctuations in Particle Production in High-Energy Interactions in Nuclear Emulsion

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The method of scaled factorial moments is used to study short-range fluctuations in the pseudorapidity distributions of particles produced in high-energy interactions in nuclear emulsion. An intermittent behavior of the fluctuations is clearly observed in both proton (200 and 800 GeV) and oxygen (60 and 200 GeV/nucleon) beam interactions in emulsion.

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Unusually large fluctuations in rapidity density have been observed in high-energy interactions of cosmic rays,<sup>1</sup> as well as in accelerator experiments.<sup>2,3</sup> Such fluctuations may be a reflection of a phase transition from ordinary matter to a quark-gluon plasma, predicted by QCD to occur in high-energy nucleus-nucleus collisions, or may be due to minijets at very high energies, or to other collective phenomena. Various methods for identification of nonstatistical fluctuations have been proposed,<sup>4</sup> ranging from the total deviation of the measured rapidity distribution from an idealized smooth distribution, to the height of high-ranked peaks in the distribution. The analysis consists of estimating the corresponding statistical probability through Monte Carlo simulations.

Recently, a new method of analysis based on the use of scaled factorial moments has been proposed.<sup>5</sup> This method allows not only the detection of large nonstatistical fluctuations but also investigation of the pattern of the fluctuations, which could lead to a physical interpretation of their origin. The behavior of these factorial moments was suggested in analogy with the phenomenon known as "intermittency" in the hydrodynamics of turbulent fluid flow. This phenomenon is characterized by the presence of fluctuations at different scales, limited, however, to a small part of the available phase space. This intermittency analysis was originally applied to the pseudorapidity distribution of a single very-highmultiplicity cosmic-ray event recorded in the Japanese-American Cooperative Emulsion Experiment.<sup>1</sup> A positive indication of intermittency was found.<sup>5</sup> Here, we use the method to search for intermittency in the interactions of accelerator protons and oxygen nuclei in nuclear emulsion.

The scaled factorial moment  $F_i$  of the *i*th order is defined for a fixed multiplicity event and for a particular partition of an overall rapidity interval  $\Delta Y$  into M bins of width  $\delta Y = \Delta Y/M$  as

$$F_{i} = \frac{1}{M} \sum_{m=1}^{M} M^{i} \frac{k_{m}(k_{m}-1)\cdots(k_{m}-i+1)}{N(N-1)\cdots(N-i+1)}, \quad (1)$$

where  $k_m$  is the number of particles in the *m*th bin and N is the multiplicity (within the  $\Delta Y$  interval) of the event.<sup>5</sup> For a sample of events with various multiplicities, the definition changes to

$$F_{i} = \frac{1}{M} \sum_{m=1}^{M} \frac{M^{i}}{\langle N \rangle^{i}} k_{m} (k_{m} - 1) \cdots (k_{m} - i + 1), \quad (2)$$

where  $\langle N \rangle$  is the mean multiplicity in the interval  $\Delta Y$ .<sup>6</sup> It has been shown that the factorial moments  $F_i$  averaged over many events are equal to the moments of a

	No. of	No. of events	Investigated	$\langle N_s \rangle$ in the
Data set	events	with $N_s > 10$	$\eta$ range	η range
p 200 GeV	2595	1542	0.5-5.5	$17.3 \pm 0.4$
p 800 GeV	1749	1336	0.5-6.5	$22.2 \pm 0.6$
<sup>16</sup> O 60 GeV/nucleon	226	226	0.5-4.5	$93.1 \pm 6.0$
<sup>16</sup> O 200 GeV/nucleon	146	146	0.5-5.5	$154 \pm 13$

TABLE I. Characteristics of the four data sets.



FIG. 1. The dependence of the averaged moments of orders 2 ( $\Box$ ), 3 ( $\blacksquare$ ), 4 ( $\diamond$ ), and 5 ( $\blacklozenge$ ) on the bin size, for protonemulsion interactions at (a) 200 GeV and (b) 800 GeV. Solid lines represent linear fits to the data.

true probability distribution of the particle density in rapidity space.<sup>5</sup> Thus the problem of statistical fluctuations, always present because of a finite number of particles per event, is reduced.

Differential patterns of nonstatistical fluctuations change not only the absolute value of moments, but even more important, they influence the variation of moments with the size of the rapidity window,  $\delta Y$ . Two cases of different fluctuation patterns have been analyzed.<sup>5</sup> Fixed-range fluctuations of a scale S make the moments depend on the resolution  $\delta Y$  only when  $\delta Y$  is close to or greater than S, while the moments are independent of  $\delta Y$  when  $\delta Y < S$ . On the other hand, an intermittent pattern leads to a power-law behavior of the moments:

$$\langle F_i \rangle = \left(\frac{\Delta Y}{\delta Y}\right)^{\phi_i},\tag{3}$$

and so to a characteristic linear rise of  $\ln \langle F_i \rangle$  with  $-\ln \delta Y$  for all windows  $\delta Y$ , down to the smallest.

In a model with an intermittent pattern of particle density fluctuations,  $\phi_i$  acquires a physical meaning. For a model of random cascades,<sup>5</sup> the probability distribu-



FIG. 2. The dependence of the averaged moments of orders 2 ( $\Box$ ), 3 ( $\bullet$ ), 4 ( $\diamond$ ), 5 ( $\diamond$ ), and 6 ( $\triangle$ ) on the bin size, for oxygen-(Ag,Br) interactions at (a) 60 GeV/nucleon and (b) 200 GeV/nucleon. Solid lines represent linear fits to the data.

tion of particles in rapidity space is formed as a result of consecutive partitions of the full rapidity interval  $\Delta Y$  into clusters of decreasing rapidity size, with rapidity densities drawn from a distribution of random variable W at each step of the cascade. In the framework of this model,

$$\phi_i = \frac{\ln\{W^i\}}{\ln\lambda} , \qquad (4)$$

where  $\{W^i\}$  are the moment averages of the probability distribution of W, and  $\lambda$  is the number of parts into which clusters are divided. Thus, one can study the parameters of the cascade of decaying clusters in highenergy particle production through the determination of slopes  $\phi_i$  from the analysis of the experimental rapidity distribution.<sup>5</sup>

The experimental data analyzed here come from interactions in nuclear emulsion of protons at 200 and 800 GeV from the Fermi National Accelerator Laboratory



FIG. 3. Comparison of (a)  $\ln\langle F_4 \rangle$  and (b)  $\ln\langle F_2 \rangle$  dependence on  $-\ln \delta \eta$  for measured (**a**) and Monte Carlo-simulated (**b**) proton-emulsion interactions at 200 GeV.

and of <sup>16</sup>O at 60 and 200 GeV/nucleon obtained by the EMU-07 experiment performed by the Krakow-Louisiana-Minnesota collaboration at CERN. The proton interactions were analyzed in an inclusive manner<sup>7,8</sup> while the oxygen interactions were restricted to semicentral collisions (small impact parameter) with heavy emulsion nuclei, Ag or Br.<sup>9</sup> The measurements in emulsion allow very accurate determination of the polar angle  $\theta$  of the secondary particles produced. Hence the pseudorapidity ( $\eta = -\ln \tan \theta/2$ ) is measured without serious bias over the whole range of values encountered. The estimated error in the angular measurement does not exceed 0.1 in pseudorapidity units.

The fluctuations are studied in a limited pseudorapidity region, cutting off fragmentation tails where the statistics are low. Further, the sample of proton interactions was restricted to those with the number of relativistic charged particles  $N_s$  greater than 10. For the central collisions of <sup>16</sup>O the event multiplicities were always much larger than 10. Table I gives the characteristics of the data sets.

The moments  $F_i$  with *i* between 2 and 5 for proton and between 2 and 6 for oxygen interactions were calculated



FIG. 4. Comparison of (a)  $\ln\langle F_4 \rangle$  and (b)  $\ln\langle F_2 \rangle$  dependence on  $-\ln \delta \eta$  for measured (**a**) and Monte Carlo-simulated (**b**) oxygen-(Ag,Br) interactions at 200 GeV/nucleon

according to Eq. (2), with the maximum order of the moments being limited by the available statistics in the samples. The dependence of the average moments on bin size  $\delta\eta$  is shown in Figs. 1 and 2. The errors were estimated, with standard statistical procedures, from the dispersion of the values of the moments used to calculate the average.<sup>10</sup> The linear rise of the average moments with decreasing bin width  $\delta\eta$  is evident in both figures.

To prove that this linear dependence is not a spurious effect produced by the method itself, comparable samples of Monte Carlo-generated events were analyzed. The simulation consisted of the random generation of event multiplicities, according to the observed multiplicity distribution, and pseudorapidities for individual events from the pseudorapidity distribution of the data set being considered. The results of this analysis are shown in Figs. 3 and 4. The Monte Carlo-generated events exhibit no dependence on  $\delta\eta$ .

Since there is no evidence of a tendency to level off at any  $\delta \eta$  in the experimental data (Figs. 1 and 2), one can conclude that indeed an intermittent pattern is being observed. To obtain the characteristic parameter of the in-

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Data set	<i>\$</i> 2	<i>\$</i> 3	φ4	<i>\$</i> 5	φ6		
p 200 GeV	$0.027 \pm 0.002$	$0.063 \pm 0.011$	$0.129 \pm 0.030$	$0.202 \pm 0.060$			
p 800 GeV	$0.023 \pm 0.002$	$0.062 \pm 0.006$	$0.094 \pm 0.017$	$0.100 \pm 0.031$	• • •		
<sup>16</sup> O 60 GeV/nucleon	$0.017 \pm 0.002$	$0.039 \pm 0.006$	$0.066 \pm 0.014$	$0.089 \pm 0.027$	$0.113 \pm 0.044$		
<sup>16</sup> O 200 GeV/nucleon	$0.020\pm0.001$	$0.052\pm0.004$	$0.099 \pm 0.008$	$0.158 \pm 0.014$	$0.234 \pm 0.022$		

TABLE II. Slopes  $\phi_i$  of fits for each data set.

termittency, namely the slope  $\phi_i$ , the data were fitted to

$$\ln\langle F_i \rangle = A - \phi_i \ln \delta \eta \,. \tag{5}$$

The results are summarized in Table II. The moments of the highest order exhibit some irregular behavior which may be due to the small statistics of these samples. With increasing order, the slopes increase in all cases. No systematic correlations between the value of the slopes and the energy of the projectile were found.

Concluding, we find evidence for an intermittent pattern of rapidity density fluctuations in both *p*-nucleus and nucleus-nucleus data over a wide range of projectile energy. The search for such effects in collisions of still heavier nuclei is in progress. The origin of the intermittent fluctuations is still unclear and becomes a new challenge for both theoretical and experimental studies. The standard models of multiparticle production do not predict intermittent behavior.<sup>6</sup> On the other hand, a jet model with a scale-invariant decay function seems to be consistent with intermittent behavior.<sup>11</sup> The investigation of intermittency may provide decisive information on the hadronization process and may be important in discriminating between different particle-production models.

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<sup>10</sup>There are two components contributing to the measured dispersion. One,  $D_{\rm fix}$ , is constant for a given sample of events and originates in the variation of the event multiplicity and the statistical fluctuations of the number of particles in a bin. The other component,  $D_{\rm var}$ , is a reflection of the dynamical fluctuations in which we are interested. Since the intermittency effects are expected to occur at small bin sizes, we assume that  $D_{\rm var}$  vanishes at  $\delta \eta \ge 1.0$ . For the total dispersion given by  $D^2 = D_{\rm fix}^2 + D_{\rm var}^2$ , and taking  $D_{\rm fix}$  equal to the dispersion at  $\delta \eta \ge 1.0$ , we calculate  $D_{\rm var}$  at each bin width. In contrast to  $D_{\rm fix}$  which is responsible for the error in the absolute value of  $\langle F_i \rangle$ , only  $D_{\rm var}$  is important when studying the dependence of moments on the resolution  $\delta \eta$ .

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