Double Scattering in Nuclei at High Energy

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We discuss double scattering in high-transverse-momentum cross sections on nuclei in the context of QCD. We give results for the A dependence of jet and one-particle inclusive cross sections in hadron and lepton reactions, in terms of the distribution of gluons in a bound nucleon.

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Enhanced A dependence, A^α with $\alpha > 1$, is a familiar feature of high-transverse-momentum cross sections in hadron-nucleus scattering.[1] At least a partial explanation of this behavior may be found in multiple scattering of a parton on separate nucleons within the target nucleus.[2] Detailed models of this type have been constructed,[3] based on sequential scatterings in the QCD Born approximation. As emphasized in Ref. 4, however, cross sections calculated in this way are infrared sensitive when one of the scatterings becomes soft, so that these cross sections describe only multiple hard scatterings accurately. This suggests that it might be useful to reexamine hadron-nucleus scattering in perturbative QCD, without assuming multiple Born scatterings at the outset.

Consider first the incoherent scattering model of Ref. 3, and let the incoming hadron h have momentum P_h^μ. A parton of type k and momentum $p^\mu = x_k P_h^\mu$ from h scatters incoherently from two nucleons of the target, after which a parton of type l is detected, with momentum p'^μ. The corresponding cross section is

$$\omega_{p'} \frac{d\sigma^{kl}_{\text{hard}}}{d^3p'} = Ah^{kl}(p,p') + \frac{9A^{4/3}}{16\pi R_0^2} \int_0^1 dx_k f_{k/h}(x_k) \int \frac{dq^+ d^2\mathbf{q}_\perp}{p^+ + q^+} \sum_j [h^{kj}(p,p'-q)h^{jl}(p'-q,p') - h^{kl}(p,p')h^{lj}(p',p+q) - h^{kj}(p,p+q)h^{kl}(p,p')] . \quad (1)$$

Here R_0 is the internucleon separation, while the function $h^{ij}(p,l)$ is the single-particle inclusive cross section for parton i of momentum p to be scattered by a nucleon into parton j of momentum l. $f_{k/h}(x_k)$ is the distribution for parton k in hadron h. The first term in brackets represents the contribution of two physical scatterings and is positive. The two remaining terms are negative, and correspond to a single physical scattering along with the effects of absorption in either the final or initial state. By itself, Eq. (1) describes jet production; for single-particle inclusive cross sections it should be convoluted with a fragmentation function. In principle, it gives quantitative predictions for high-p_T cross sections when parton-model cross sections are used for the h's. It is then infrared sensitive,[4] however, and requires a cutoff. We must therefore deal with soft-gluon corrections.

One might think that in QCD any number of soft-gluon corrections would contribute at the same power in A and Q. We argue, however, that this is not the case.[5] Instead, we suggest that the complete cross section at order $\alpha_s(Q^2)A^{1/3}/Q^2R_0^2$ relative to single scattering is determined by diagrams in which two soft gluons, with physical polarizations, attach to the active parton from the projectile. Essentially, this is an extension of the factorization program[6] to the first higher-twist contribution in hadron-nucleus scattering.

To proceed, we assume that the target acts as a collection of uncorrelated nucleons, and that its matrix elements break up into matrix elements for individual nucleons, neglecting, for instance, shadowing. The remaining information on nuclear structure is contained in the single-particle density matrix, which we approximate by

$$\langle \mathbf{x}' | d | \mathbf{x} \rangle = \frac{9}{16\pi^2 R_0^6} \rho \left(\frac{\mathbf{x}+\mathbf{x}'}{2} \right) r(|\mathbf{x}-\mathbf{x}'|) . \quad (2)$$

For constant nuclear density, $\rho(\mathbf{z})$ is $\theta(R - |\mathbf{z}|)$, with R the nuclear radius. r is a function which is normalized to $r(0)=1$, and which we assume to decay on a scale which is small compared to the nuclear radius.

With this model, we find[5] that the soft exchange contribution to the jet cross section is given by a sum of products of nonperturbative functions, I and J, times perturbative functions C and B, which play the role of ultraviolet coefficients,

$$\omega_{p'} \frac{d\sigma^{kl}_{\text{soft}}}{d^3p'} = \frac{A^{4/3}}{R_0^2} \int_0^1 dx_k f_{k/h}(x_k) [I(C_i^{kl} + C_f^{kl}) + (J - 2I)B^{kl}] . \quad (3)$$

Let us turn our attention first to the nonperturbative functions I and J. After relevant diagrams are combined, the cross section can be reexpressed in terms of matrix elements of the gluon field strength. These, in turn, are reinterpreted as distribution functions[7] for the gluon in a bound nucleon. I and J are given in terms of gluon distributions by

$$J(Q^2,x_k s)=v4\pi^2\alpha_s(Q^2)\int d^2P_\perp P^-\tilde{r}(P)\int_{q_\perp^2<Q^2}d^2q_\perp(\mathbf{Q}_\perp\cdot\mathbf{q}_\perp/2p^+P^-)\mathcal{D}_{g/N}(\mathbf{Q}_\perp\cdot\mathbf{q}_\perp/2p^+P^-,\mathbf{q}_\perp)\,,$$
$$I(Q^2,R)=8\pi^2\alpha_s(Q^2)\int d^2P_\perp dP^-P^-\tilde{r}(P)\int_0^1 dx\,xf_{g/N}(x,Q^2)\tilde{u}(-xP^-,R)\,, \tag{4}$$

where P^μ is the momentum of a single nucleon in the target nucleus. $\tilde{r}(P)$, the Fourier transform of the function $r(|\mathbf{x}-\mathbf{x}'|)$ in Eq. (2), is the single-particle momentum distribution. In (4), as above, $p_k=x_kP_h$, and we define $s=\sqrt{2}P_h^-m$, with m the nucleon mass. In our simple model for the target, the distributions $f_{g/N}$ are the same as for free nucleons. In a more sophisticated treatment, including nuclear effects on nucleon structure, it might be necessary to absorb $\tilde{r}(P)$ into the definition of the gluon distribution.

$\mathcal{D}$ is the joint distribution of gluons at fixed momentum fraction x and transverse momentum $q_\perp$,[8]

$$\mathcal{D}_{g/N}(x,\mathbf{q}_\perp)=\frac{1}{xP^-}\int\frac{d^4\xi}{(2\pi)^3}\delta(\xi^-)e^{-i(xP^-\xi^+-\mathbf{q}_\perp\cdot\xi_\perp)}\langle P|F^-{}_\mu{}^d(\xi/2)F^{\mu-d}(-\xi/2)|P\rangle\,. \tag{5}$$

Equation (5) defines $\mathcal{D}$ in the $A^-=0$ gauge.[8] From $\mathcal{D}$ we may define the x distribution of gluons,[8] which, to leading logarithm, is

$$f_{g/N}(x,Q^2)=\int_{q_\perp^2<Q^2}d^2q_\perp\mathcal{D}_{g/N}(x,\mathbf{q}_\perp)\,. \tag{6}$$

In Eqs. (4), nuclear structure also enters through the functions

$$\tilde{u}(xP^-,R)=\frac{9}{16\pi^2R^4}\int\frac{d\xi^+}{2\pi}e^{-ixP^-\xi^+}\int d^3y\,\rho(\mathbf{y})\int_{-2R}^{-|\xi^+|/2\sqrt{2}}d\alpha\,\rho(\mathbf{y}+\alpha\mathbf{n})\,,$$
$$v=\frac{9}{16\pi^2R^4}\int d^3y\,\rho(\mathbf{y})\int_{-2R}^{2R}d\alpha\,\rho(\mathbf{y}+\alpha\mathbf{n})\,, \tag{7}$$

with $\mathbf{n}$ the unit vector in the projectile direction. For a spherical, homogeneous nucleus, as in Eq. (1), $v=9/8\pi$.

C_i, C_f, and B in Eq. (3) are ultraviolet functions in the presence of initial-state, final-state, and mixed initial- and final-state soft-gluon corrections, respectively. To lowest order, C_i and C_f are given in terms of parton-model cross sections by

$$C_i^{kl}(p,p')=\frac{C_F}{32}\int d^2P_\perp dP^-\tilde{r}(P)\int dx_mf_{m/N}(x_m)\sum_{i=1}^2\frac{d^2}{dq_{i\perp}^2}\omega_{p'}\frac{d\sigma^{kl}}{d^3p'}(x_m,p+\hat{q}_i,p')\,|_{q=0}\,,$$
$$C_f^{kl}(p,p')=\frac{C_F}{32}\int d^2P_\perp dP^-\tilde{r}(P)\int dx_mf_{m/N}(x_m)\sum_{i=1}^2\frac{d^2}{dq_{i\perp}^2}\omega_{p'}\frac{d\sigma^{kl}}{d^3p'}(x_m,p,p'-\hat{q}_2)\,|_{q=0}\,, \tag{8}$$

where in QCD, $C_F=\frac{4}{3}$. B is given by

$$\begin{aligned}B^{kl}(p,p')=\int d^2P_\perp dP^-\tilde{r}(P)\frac{1}{64(2\pi)^2}\sum_{i=1}^2\frac{d^2}{dq_{i\perp}^2}\int_0^1dx_mf_{m/N}(x_m)(4p^+x_mP^-)^{-1}\\ \times[T_bM^{kl*}(p;p'+\hat{k}_3-\hat{q}_3)\delta((p+x_mP-p'+\hat{q}_3)^2)T_b'M^{kl}(p+\hat{q}_3,p')\\ -T_bM^{kl*}(p-\hat{q}_4;p'+\hat{k}_4-\hat{q}_4)\delta((p+x_mP-p')^2)T_b'M^{kl}(p,p')]\,|_{q=0}\,.\end{aligned} \tag{9}$$

Spin and color sums and averages are understood, and T_b and T_b' are the color generators appropriate to the projectile (k) and scattered (l) partons, respectively. $M(p;p')$ is the parton-model amplitude for the scattering of partons with momenta x_mP and p into a parton of momenta p'. In Eq. (9), $\hat{q}_i^\mu=(0^+,q_i^-,\mathbf{q}_\perp)$, where $q_i^-=q_3^-=-q_4^-=q_\perp^2/2l^+$, $q_2^-=-(-2\mathbf{Q}_\perp\cdot\mathbf{q}_\perp+q_\perp^2)/2l^+$, and $\hat{k}_j^\mu=(q_j^--q_2^-)\delta_{\mu-}$.

The expression for the complete double-scattering cross section, including hard as well as soft double scattering, is

$$\begin{aligned}\omega_{p'}\frac{d\sigma^{kl}}{d^3p'}=\omega_{p'}\frac{d\sigma^{kl}_{\text{soft}}}{d^3p'}+\omega_{p'}\frac{d\sigma^{kl}_{\text{hard}}}{d^3p'}-\frac{vA^{4/3}}{2R_0^2}\int_0^1dx_kf_{k/h}(x_k)\int_{\mathbf{q}_\perp^2<Q^2}\frac{dq^+d^2q_\perp}{p^++q^+}q_\perp^2\\ \times[h_t^{kk}(p,p+q)C_i^{kl}(p,p')+C_f^{kl}(p,p')h_t^{ll}(p',p+q)]\,,\end{aligned} \tag{10}$$

where h_t^{ii} is the t-channel part of h^{ii}. For small $\mathbf{q}_\perp$, where $d\sigma_{\rm soft}$ gives the correct result, the final term cancels $d\sigma_{\rm hard}$, while for large $\mathbf{q}_\perp$, where $d\sigma_{\rm hard}$ is appropriate, it cancels $d\sigma_{\rm soft}$. Note in particular that the infrared sensitivity of $d\sigma_{\rm hard}$ is canceled by the final term. Equation (10) gives a prediction for the quark double-scattering contribution to high-transverse-momentum hadron-nucleus scattering in terms of the gluon distribution as a function of x, and of $q_\perp$, as well as the nuclear density matrix, Eq. (2). In principle, these distributions can be measured in other reactions. In practice, however, the $q_\perp$ distribution of gluons is not well known, so that J should probably be thought of as a free parameter.

The situation is simpler for high-transverse-momentum reactions involving leptons. Equation (10) may be adapted by inspection to several such processes. These include high $p_\perp$ jet and single-hadron-inclusive cross sections in deeply inelastic lepton scattering, and the single-lepton inclusive cross section in the Drell-Yan process. In the former case the cross section is found by keeping only the final-state term, proportional to C_f, and in the latter by keeping only the C_i term and replacing $f_{k/h}(x_k)$ by $\delta(1-x_k)$. These cross sections are given as absolute predictions, once $f_{g/N}$ is specified. To the extent that the transverse momentum distribution of Drell-Yan pairs[9] is short-distance dominated,[10] the A dependence of this cross section is determined in the same way. It is worth noting that, as developed in Ref. 10, the pair transverse-momentum distribution is sensitive to the transverse momentum of the incoming partons through the dimensionless variable $b_{\rm SP}q_\perp$, where $b_{\rm SP}$ is the saddle-point value of an impact-parameter integral involving summed Sudakov logarithms. $1/b_{\rm SP}$ scales as $\Lambda_{\rm QCD}$ times a fractional power of Q^2, and is generally much smaller than Q. This is the explanation, in this formalism, of the fact that the pair transverse-momentum distribution is more sensitive to multiple scattering than is the pair mass distribution.[11]

Finally, we note that the A dependence in Eq. (10) is sensitive to the low-x behavior of the gluon distribution, through the nonperturbative function I in Eq. (7). In fact, if $xf_{g/N}(x)$ behaves as $x^{-\Delta}$ for small x,[12] then

$$\omega_{p'}\frac{d\sigma_{\rm soft}}{d^3p'} \propto A^{(4+\Delta)/3}. \tag{11}$$

We leave the observability of this effect, along with other phenomenological issues, for later work.

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