One-Jet Inclusive Cross Section at Order α_s^3: Gluons Only

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(Received 5 December 1988)

The next-to-leading-order jet cross section is calculated for the simplified case in which there are only gluons. The general structure of the differences from the lowest-order cross section are described. The important new effects at this order are the explicit dependence on the definition of the jet and the reduced dependence on the choice of the renormalization scale μ^2. While only the purely glue results are discussed here, we expect that the dependence on the jet definition and on μ^2 will be quite similar in the complete calculation, which we are now undertaking.

PACS numbers: 13.87.Ce, 12.38.Bx

One of the dramatic features of the recent data[1] from hadron colliders, both at CERN and Fermilab, is the obvious appearance of hadron jets[2] as a characteristic feature of a sizable fraction of the final states. These jets are an essential tool for organizing and analyzing the data and are potentially important as a test of our quantitative understanding of the underlying strong-interaction theory, QCD. This particularly true if one wants to look[3] for a breakdown of the standard model due, for example, to the possible composite structure of the particles now thought to be elementary. One would like to analyze the scattering of these elementary partons at the largest p_T scale possible. The signal for hard parton-parton scattering is jet production and the most straightforward jet cross section is that for the inclusive production of a jet.

Unfortunately, there remain important ambiguities which limit our ability to perform detailed quantitative studies with the observed jet cross sections. A central difficulty is that fact that a jet is *not* intrinsically well defined. In particular, for reasons of color, energy, and momentum conservation, a jet of hadrons *cannot* be the residue of a *single* parton. Ambiguities arise on the experimental side both from the question of how to define a jet and from the systematic uncertainties inherent in jet energy measurements. The differences in jet definitions are presumably responsible for at least some of the approximately 50% difference between the reported[1] jet cross sections from UA1 and UA2. On the theoretical side there is the underlying uncertainty in the parton structure functions,[4] which will be improved only by further deeply inelastic lepton scattering data and, for the gluons, by the sort of jet analysis discussed here. Further issues are the choice of the renormalization-factorization[5] scale μ^2, the value of the so-called "K factor" (characterizing the uncertainty in magnitude of the cross section due to higher-order contributions) and, of course, the question of matching the theoretical jet definition[3] with the experimental one. These latter three points are essentially all the question of higher-order corrections. Thus we can improve the situation by performing a complete calculation of one order beyond the Born approximation (i.e, at order α_s^3), leading to a theoretical uncertainty smaller than the current experimental error. In earlier theoretical studies,[6] only incomplete QCD matrix elements at order α_s^3 were available. Recently, the full order α_s^3 matrix elements in $4-2\epsilon$ dimensions have been calculated.[7]

The present Letter gives a brief summary of the results of an analysis[8] of jet cross sections using these full matrix elements applied to the simplified case of gluons only. The quantity that we calculate is the inclusive cross section $d\sigma/d\eta dE_T$ for production of a jet with pseudorapidity η $(=-\ln\tan\theta/2)$ and transverse energy E_T plus anything. Subsequent publications[9] will provide a more complete description of the pure gluon calculation and of the forthcoming results of a full analysis of $p\bar{p}$ collisions including quarks.

It is important to note that, strictly speaking, the cross section at the Born level is not a jet cross section but is rather just the parton cross section. At order α_s^3 there is an explicit dependence on the jet definition: One must decide when two partons count as two jets and when they count as one. Thus the calculation at this order allows us to account for the power of the "experimental microscope" to resolve one parton into two. It is exactly this careful treatment of the finite size of the jet which renders the jet cross section finite at all orders in perturbation theory, in analogy to what happens for similar quantities in e^+e^- physics.[10] Also, when two partons do count as one jet, one must define the resulting jet axis and jet E_T. In the experimental measurement, the differences between jet definitions algorithms are now expected to matter, in that they can change the measured cross section at the same level as the α_s^3 corrections in the theory.

Let us consider for a moment the criteria which characterize "good" jet definition. In general it has the following properties: (1) It is simple to implement in experimental analysis; (2) it is simple to implement in the theoretical calculation; (3) it is defined at any order of perturbation theory; (4) it yields infinite cross sections at any order of perturbation theory; and (5) it yields a cross section that is insensitive to hadronization. The definition we use is as follows.[11] Let the calorimeter consist of cells i, in which the transverse energies $E_{T,i}$ are measured. Define a jet cone of radius R in η-ϕ space, $R=(\Delta\eta^2+\Delta\phi^2)^{1/2}$. The E_T of the (trial) jet is then

$$E_{T,J}=\sum_{i \text{ in cone}} E_{T,i}\,. \tag{1}$$

The jet axis is defined by the weighted averages:

$$\eta_J=\frac{1}{E_{T,J}}\sum_{i \text{ in cone}} E_{T,i}\eta_i\,,$$
$$\phi_J=\frac{1}{E_{T,J}}\sum_{i \text{ in cone}} E_{T,i}\phi_i\,. \tag{2}$$

Finally, the cone axis must agree with the jet axis determined by Eq. (2). If it does not, one simply iterates the process until there is agreement. With this definition, a single isolated parton with parameters (E_T,η,ϕ) will be "reconstructed" as a jet with these same parameters. Two nearby partons with parameters $(E_{T,1},\eta_1,\phi_1)$ and $(E_{T,2},\eta_2,\phi_2)$ will be identified as a single jet with $E_{T,J}=E_{T,1}+E_{T,2}$ and with a direction between the directions of the two partons if the two partons fit inside a cone of radius R around the combined jet axis. Note that sometimes the two parts of such a jet can also qualify as acceptable jets separately. By definition, we count only the combined jet when this happens and ignore the subjects. (This somewhat arbitrary choice is taken because we want to "see" the highest-E_T jet, which reflects the most interesting physics.) This ambiguous situation does not generally arise for unequal jets but it can arise for jets of nearly equal E_T. There is a potential problem here, since some jet-finding algorithms used by the experimental groups may not locate the combined jet because there is no "seed" near the jet axis. A final concern is the allocation of energy in cases when the cones of two jets which cannot be merged into a single jet still overlap. One could allocate the E_T of the cells in the overlap region to the nearest jet or the largest jet or just allocate it to both jets. This problem does not occur at order α_s^3, and so we have not needed to specify how it is to be resolved in our calculation. The differences resulting from different experimental choices should be small, $\mathcal{O}(\alpha_s^4)$.

The calculation of the inclusive jet cross section at order α_s^3, given the relevant matrix elements, is largely a (massive) exercise in sophisticated bookkeeping and organization. The primary point is to ensure that the singularities are correctly canceled and that the finite remainder is correctly evaluated. Only the results will be presented here, while the methods will be published separately.[9]

There is some question as to just what to calculate for our gluon only (number of quark flavors, $N_f=0$) study. We have chosen to adjust $\Lambda_{\overline{\rm MS}}$ ($\overline{\rm MS}$ denotes the modified matrix minimal-subtraction scheme) in the second-order formula for $\alpha_s(\mu)$ so that $\alpha_s(50\text{ GeV})\sim 0.13$, consistent with the value of $\alpha_s(50\text{ GeV})$ observed in the real world with quarks. The appropriate value turns out to be $\Lambda_{\overline{\rm MS}}=1600$ MeV. This choice is intended to ensure that the relative size of the higher-order contributions is "physically" relevant. When we compare the α_s^3 cross section to the Born-level calculation, we compute the Born cross section using the first-order formula for $\alpha_s(\mu)$ with this same value of $\Lambda_{\overline{\rm MS}}$. [This makes the lowest-order version of $\alpha_s(\mu)$ about 30% larger than the second-order version, a not insignificant effect.] The schematic form of the cross section is

$$\frac{d\sigma}{dE_T\,d\eta}=\int\!\!\int dx_1\,dx_2\,G(x_1,\mu^2)G(x_2,\mu^2)\frac{d\hat\sigma(x_1,x_2,s,E_T,\eta;\mu^2)}{dE_T\,d\eta}\,, \tag{3}$$

where $d\hat\sigma$ is the order-α_s^3 parton cross section including the appropriate phase-space integrals. For the gluon structure functions (of the colliding "glueballs") we use a reference structure function of the form

$$G(x,\mu_0^2)=\frac{6.0}{x}(1-x)^5 \tag{4}$$

and then let G evolve from $\mu_0^2=5\text{ GeV}^2$ using the case of Collins and Wu-ki Tung.[12] Note that since the perturbative calculation is for $N_f=0$, it is important to perform the evolution of the structure function also for $N_f=0$ in order to obtain the correct overall μ^2 dependence. In the complete calculation with quarks included, it will be important to use structure functions for which the evolution (from the values of μ where the underlying data are obtained) is carried out to second order. However, since our functions $G(x,\mu^2)$ are illustrative only, we have been content to use first-order evolution.

As a first illustration of results we exhibit the dependence on the cone size R. This is illustrated in Fig. 1 where the inclusive cross section is plotted as a function

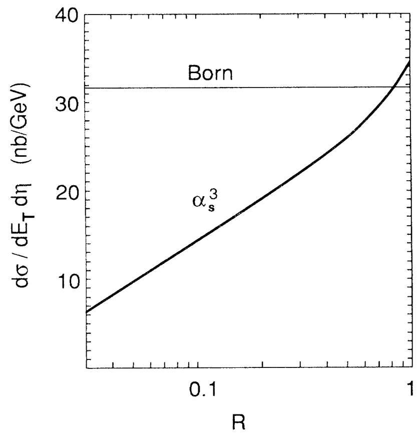

FIG. 1. Inclusive jet cross section, $d\sigma/dE_T d\eta$, for $E_T=50$ GeV, $\eta=0$, and $\sqrt{s}=1800$ GeV at both the Born and α_s^3 level vs the logarithm of the jet size R.

of R for $\eta=0$, $E_T=50$ GeV, and $\sqrt{s}=1800$ GeV. For comparison the *R-independent* Born cross section is also indicated. As expected the dependence is basically logarithmic. Since a cancellation is occurring between a (negative) infinity in the virtual correction to the $2\rightarrow 2$ process and a (positive) collinear singularity in the $2\rightarrow 3$ process, the cross section diverges to negative infinity as $R\rightarrow 0$ ($d\sigma/dE_T d\eta \rightarrow a+b\ln R$). We observe that the corrections to the Born cross section are becoming larger as R becomes small, indicating that fixed-order perturbation theory is inadequate for small R. Physically, this is because the higher-order perturbative structure of the jets ("showering") is playing an important role at these small R values.

We wish to emphasize that the dependence of the cross section on R is a physical effect predicted by QCD. This prediction should be reliable when R is not too small, and can be tested experimentally.

We turn now to the question of the overall size of the cross section and the magnitude of the μ^2 dependence for the Born result versus the order-α_s^3 result. The answers to these questions are illustrated in Figs. 2 and 3. In Fig. 2 we plot the inclusive jet cross section for $\eta=0$, $E_T=50$ GeV, and $\sqrt{s}=1800$ GeV vs μ both at the Born level and at order α_s^3. Similar results are obtained at other jet energies, $20<E_T<200$ GeV, when plotted versus the scaled variable μ/E_T. In particular, the Born and order-α_s^3 cross sections are equal at a μ value in the range $1.2<\mu/E_T<1.5$ (the larger ratios coming at the lower energies) and the order-α_s^3 cross section exhibits a maximum at $\mu/E_T\sim 0.5$.

One expects on general grounds that μ should be chosen of order E_T. Thus if one plots the range of calculated cross sections as μ varies in the range $0.5E_T<\mu<2E_T$ (the range indicated by the vertical dashed lines in Fig. 2), one obtains both the predicted cross section and some estimate of the theoretical uncertainty.

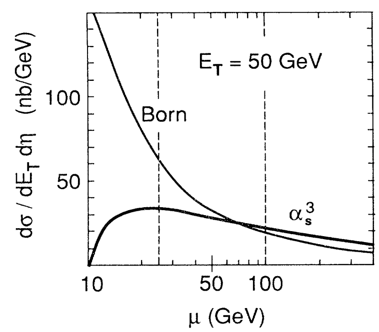

FIG. 2. Inclusive jet cross section, $d\sigma/dE_T d\eta$, for $E_T=50$ GeV, $\eta=0$, and $\sqrt{s}=1800$ GeV at both the Born and α_s^3 level vs the scale μ.

This range of μ values yields the invariant cross-section ($E_T^3 d\sigma/dE_T d\eta$) values versus E_T indicated by the two bands in Fig. 3. Over this range of μ the variation in the higher-order cross section is approximately $\frac{1}{3}$ to $\frac{1}{2}$ of the variation of the Born cross section at all E_T values of interest. In this sense the uncertainty in the theoretical cross section has been reduced by a factor of 2 to 3 by including the next-order contributions.

In conclusion, we have calculated the inclusive jet cross section at order α_s^3 for the case of gluons only. This step allows two dramatic improvements in our understanding of the theoretical jet cross section. First, the cross section at this order displays *explicit* dependence on the jet cone size, so that explicit account can be taken of the differences in jet definitions employed by different

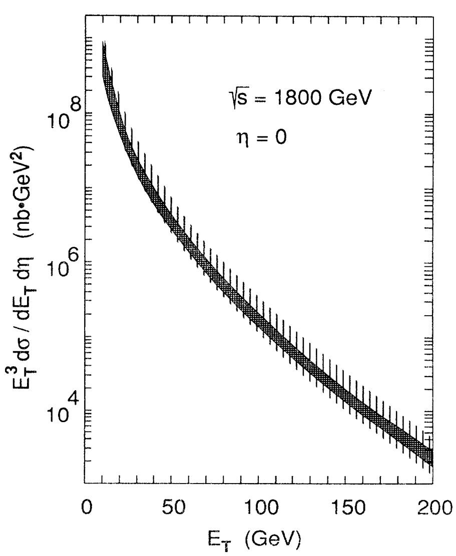

FIG. 3. Invariant jet cross section, $E_T^3 d\sigma/dE_T d\eta$, for $\eta=0$ and $\sqrt{s}=1800$ GeV vs E_T at both the Born (vertical lines) and α_s^3 level (hatched area) for $0.5<\mu/E_T\leq 2.0$ as indicated by the bands of values.

experiments. Second, the magnitude of the uncertainty of the theoretical cross section due to the (arbitrary) choice of the factorization scale μ^2 has been reduced by a factor of 2 to 3. We anticipate similar conclusions to hold for the forthcoming[9] complete analysis including quarks.

This work was supported in part by the U.S. Department of Energy, Contracts No. DE-AT06-88ER40423 and No. DE-FG06-85ER40224 and by the National Science Foundation, Contract No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration. We thank our many experimental and theoretical colleagues for useful discussions, especially J. C. Collins, R. K. Ellis, and F. I. Olness. We would like to acknowledge the assistance of participants and staff at the Institute for Theoretical Physics where much of this work was performed. S.D.E. and D.E.S. would also like to acknowledge the hospitality of the staff of the Eidgenössische Technische Hochschule where a part of this work was performed.

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