High-β, Sawtooth-Free Tokamak Operation Using Energetic Trapped Particles

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It is shown that a population of high-energy trapped particles, such as that produced by ion cyclotron heating in tokamaks, can result in a plasma completely stable to both sawtooth oscillations and the fishbone mode. The stable window of operation increases in size with plasma temperature and with trapped-particle energy, and provides a means of obtaining a stable plasma with high current and high β.

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The resistive internal kink mode has great influence on the evolution of a tokamak discharge. Repetitive sawtooth oscillations limit the current on axis and cause large-scale transport in the plasma center. In the presence of an energetic trapped-particle component such as that produced by neutral beam injection or ion cyclotron heating, this mode is described by the dispersion relation[1]

$$\delta W_c+\delta W_k-\frac{8i\Gamma((\Lambda^{3/2}+5)/4)[\omega(\omega-\omega_{*i})]^{1/2}}{\Lambda^{9/4}\Gamma((\Lambda^{3/2}-1)/4)\omega_A}=0, \tag{1}$$

where

$$\Lambda=-i[\omega(\omega-\hat{\omega}_{*e})(\omega-\omega_{*i})]^{1/3}\gamma_R,$$

$\gamma_R=S^{-1/3}\omega_A$ is the resistive growth rate, ω_{*e} and ω_{*i} are diamagnetic frequencies, S is the magnetic Reynolds number, and ω_A is the shear Alfvén frequency $\omega_A=v_A/(\sqrt{3}Rrq')$ with v_A the Alfvén velocity, R and r the major and minor radii, respectively, and $q'=dq/dr$ with q the safety factor. δW_c is the minimized ideal variational energy for the internal kink, defining the ideal-internal-kink-mode growth rate $\gamma_I=-\omega_A\delta W_c$, and δW_k is the kinetic contribution coming from the trapped-particle distribution F,

$$\delta W_k=\frac{2^{3/2}}{B^2}m\pi^2\left[\int d(\alpha B)\int\frac{dE\,E^{5/2}K_2^2\omega(\partial/\partial E+\hat{\omega}_*/\omega_d)F}{K_b(\omega_d-\omega)}\right], \tag{2}$$

with $[y]=(2\int yr\,dr)/r_s^2$, r_s the $q=1$ radius, $\alpha=v_\perp^2/v^2$, $\hat{\omega}_*$ a differential operator associated with the hot-particle diamagnetic drift frequency, and K_2 and K_b elliptic functions arising from bounce averaging. The dispersion relation thus depends on the form of the distribution function and on the six frequencies ω_{*i}, ω_{*e}, γ_I, γ_R, ω_A, and ω_d. Details of the derivation of this dispersion relation are found in Ref. 1 and in papers cited there. In this Letter we report on the existence of a domain in which the plasma is stable to both the low-frequency internal resistive kink mode and the high-frequency fishbone mode. This domain exists for a large class of distribution functions and includes values of parameters typical of high-power ion-cyclotron-heating experiments.

Solutions to Eq. (1) were examined with Monte Carlo–generated particle distribution functions. Qualitative results are, however, reproduced by using model distribution functions and approximating the radial dependence of $\omega_d(r)$ by $\omega_d(r^*)$ with r^* a mean value, allowing analytical evaluation of δW_k.

It is instructive to consider the ideal limit, $\Lambda\to\infty$, but to keep the effects of ω_{*i}. First, consider a model slowing-down distribution for the hot trapped particles:

$$F(E,\mu)=n(r)\delta(\mu/E-\alpha)E^{3/2},\quad E<E_m, \tag{3}$$

with μ the magnetic moment and E the energy. Equation (1) then becomes

$$\delta W_c-\frac{i[\omega(\omega-\omega_{*i})]^{1/2}}{\omega_A}+\frac{\beta_h}{\epsilon}\frac{\omega}{\omega_{dm}}\ln\left(1-\frac{\omega_{dm}}{\omega}\right)=0, \tag{4}$$

where ω_{dm} is the precession rate of a particle with $E=E_m$. The parameter $\epsilon=r_s/R$ is the inverse aspect ratio at the $q=1$ surface. The β_h term is the leading-order contribution to δW_k, by use of Eq. (2). The condition for threshold (ω real) gives for $\omega_{*i} < \omega < \omega_{dm}$ an equation for the threshold frequency,

$$\gamma_I = \frac{[\omega(\omega-\omega_{*i})]^{1/2}}{\pi} \ln\left(\frac{\omega_{dm}}{\omega} - 1\right), \tag{5}$$

with $\gamma_I = -\delta W_c \omega_A$ the ideal-kink-mode growth rate, and the associated value of β is $\beta_h = \epsilon\omega_{dm}(1-\omega_{*i}/\omega)^{1/2}\pi\omega_A$, a monotonically increasing function of ω. The ideal growth rate is related to the background plasma. For example, for a quadratic q profile and circular flux surfaces, γ_I was found by Bussac *et al.*[2] to have the form

$$\gamma_I = \frac{\omega_A r_s^2 3\pi}{R^2}[1-q(0)](\beta_p^2 - \tfrac{13}{144}), \tag{6}$$

where β_p is the poloidal β of the background plasma. In general, γ_I depends on the full q profile and plasma shape.

The right-hand side of Eq. (5) is zero for $\omega = \omega_{*i}$ and $\omega = \omega_{dm}/2$, and is positive and real for $\omega_{*i} < \omega < \omega_{dm}/2$. For γ_I small and positive, there are two solutions. For $\gamma_I > \omega_{*i}/2$, the lower-frequency one corresponds to the stabilization of the kink-mode branch and the higher-frequency one corresponds to the destabilization of the fishbone. The case $\gamma_I < \omega_{*i}/2$ corresponds instead to the stabilization of the ion branch[3] of the dispersion relation. This branch is marginally stable in the absence of trapped particles, and the resistive version is normally stable, and so we will not consider it here. A detailed treatment of all branches of the dispersion relation will be published elsewhere. We are interested in the case in which the two threshold frequencies, and associated values of β_h, are widely separated, and so we consider $\omega_{*i} \ll \omega_{dm}/2$. This condition puts a lower bound on the trapped-particle energy. If also $\omega_{*i}/2 < \gamma_I \ll \omega_{dm}/2\pi$, the threshold frequencies are given approximately by

$$\omega_1 = \omega_{*i}/2 + \{\omega_{*i}^2 + 4[\pi\gamma_I/\ln(\omega_{dm}/\gamma_I)]^2\}^{1/2}/2,$$

$$\omega_2 \approx \omega_{dm}/2,$$

and the corresponding values of β_h indicate that a stable gap can exist between the two modes. The stabilization of the ideal internal kink, which has also been pointed out by Pegoraro, Porcelli, and Hastie[4] occurs for

$$\beta_h = \epsilon\omega_{dm}[1 - \omega_{*i}\ln(\omega_{dm}/\gamma_I)/2\pi\gamma_I]/\pi\omega_A.$$

For $\omega_{*i} = 0$, the stabilization occurs at $\beta_h = \epsilon\omega_{dm}/\pi\omega_A$, as pointed out previously.[5]

Now consider resistive modification of these results. For the gap to continue to exist, it is necessary and sufficient to require that the arguments of the Γ functions to be large (ideal limit) at the threshold locations ω_1, ω_2. If the arguments of the Γ functions are small, it is easy to show that the growth rate tends asymptotically to zero for $\beta_h \to \infty$, so that complete stabilization does not occur. With use of ω_1 for $\gamma_I \gg \omega_{*i}$, the condition $\Lambda \gg 1$ gives

$$S \gg \frac{\ln^3(\omega_{dm}/\gamma_I)\omega_A^3}{\pi^2\gamma_I^3}. \tag{7}$$

Since $\omega_2 \gg \omega_1$, this condition automatically ensures that $\Lambda \gg 1$ at $\omega = \omega_2$. It is desirable to achieve a stable gap for S not too large. From Eq. (7) we see that this requires taking γ_I large, consistent with there being two solutions to Eq. (5), i.e., γ_I must remain small compared to $\omega_{dm}/4\pi$.

We can now write necessary and sufficient conditions for the existence of a range of hot-particle density between the sawtooth stabilization value and that for fishbone destabilization for $\gamma_I > 0$. The limits on the ideal growth rate γ_I are given by Eq. (7) and by the maximum value permitting a solution to Eq. (5). This gives

$$\frac{\omega_A \ln(\omega_{dm}/\gamma_I)}{S^{1/3}\pi} < \gamma_I \lesssim 0.09\omega_{dm}. \tag{8}$$

It seems perhaps strange that to achieve stabilization with trapped particles the kink mode must be above its ideal threshold ($\gamma_I > 0$), but this is understandable in that it is precisely this instability which preserves the ideal character of the mode. In the resistive limit the mode cannot be stabilized, and for small values of S the gap described by Eq. (8) vanishes. Note, however, that although for $\gamma_I \approx 0$ the mode is unstable, the growth rate is very small in the presence of a trapped-particle population.[1] The upper bound of Eq. (8) is the point at which the kink branch and the fishbone branch coalesce in the complex ω plane. For γ_I larger than this there are no thresholds for any value of β_h. The kink and fishbone branches exchange roles; the kink branch is destabilized by the trapped particles and takes on a large real frequency, and the fishbone is stable.

The range of trapped-particle β in which both modes are stable is given approximately by

$$\frac{\omega_{dm}}{\pi\omega_A}\left(1 - \frac{\omega_{*i}}{\omega_1}\right)^{1/2} < \frac{\beta_h}{\epsilon} < \frac{\omega_{dm}}{\pi\omega_A}. \tag{9}$$

Since ω_{dm} is proportional to the particle energy, Eq. (9) is seen to be a condition on the density of the hot-trapped species. The upper limit in Eq. (9) is the fishbone threshold, and the lower limit is the internal kink stabilization point. The stability domain in the (β_h, γ_I) plane is approximately triangular. Outside this triangle one or both of the branches are unstable. If γ_I is below the lower limit in Eq. (8) and β_h/ϵ is above the upper limit of Eq. (9), sawtoothing and fishbones can occur simultaneously.

Now consider a model Maxwellian distribution function

$$F(E,\mu,r) = \delta(E/\mu - \alpha)e^{-E/T}n(r). \tag{10}$$

Again neglecting the dependence of ω_d on r we find

$$\delta W_k = \frac{4\beta_h}{3\epsilon}\Omega[\tfrac{1}{2}+\Omega+\Omega^{3/2}Z(\Omega^{1/2})]\,, \tag{11}$$

where $\Omega = \omega/\langle\omega_d\rangle$, $\langle\omega_d\rangle$ is the precession frequency of a particle with energy T, and Z is the plasma dispersion function. Consider again the ideal limit $\gamma_R \to 0$. Substituting into Eq. (1) we find

$$0 = -\frac{\gamma_I}{\omega_d} - i[\Omega(\Omega-\Omega_*)]^{1/2} + \frac{4}{3}\frac{\beta_h}{\epsilon}[\tfrac{1}{2}+\Omega+\Omega^{3/2}Z(\Omega^{1/2})]\,, \tag{12}$$

with $\Omega_* = \omega_{*i}/\langle\omega_d\rangle$. The threshold condition is

$$\gamma_I = \frac{3[\Omega(\Omega-\Omega_*)]^{1/2}}{4\Omega^{3/2}}[\tfrac{1}{2}+\Omega+\Omega^{3/2}\mathrm{Re}Z(\Omega^{1/2})]\,, \tag{13}$$

and the value of β_h at threshold is given by

$$\beta_h = \frac{3\epsilon[\Omega(\Omega-\Omega_*)]^{1/2}e^{\Omega}}{4\pi^{1/2}\Omega^{5/2}}\,. \tag{14}$$

The behavior of the roots is similar to that found for the slowing-down distribution, provided $\Omega_* > \frac{1}{2}$. For $\Omega_* \ll 1$ the function β_h is not a monotonic function of Ω and the fishbone branch can destabilize at a smaller value of β_h than that at which the kink-mode branch becomes stable, depending on the value of γ_I.

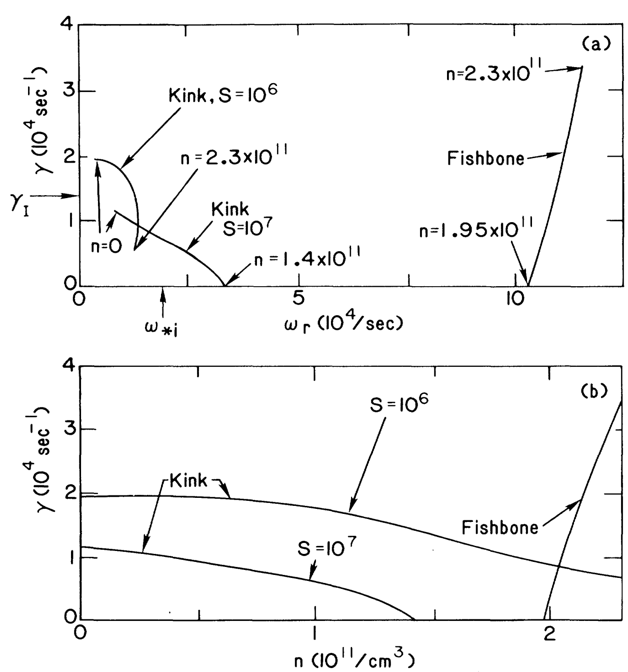

FIG. 1. (a), (b) The complex frequency plane, showing kink and fishbone branches for a range of hot-trapped-particle density and a plot of growth rate vs trapped-particle density. A slowing-down distribution was used.

A few points are worth noting. First, the stable window of operation is accessible with present-day tokamaks. The condition $\omega_{dm} \gg 2\omega_{*i}$ requires a fairly high-energy trapped-particle population, but the 1-MeV particles produced in JET easily satisfy this requirement. Beam-injection experiments on PDX and other machines involved particle energies not sufficiently high to observe this effect. The internal kink stability threshold is also readily accessible. Second, the lower bound of this stability domain decreases with increasing temperature, giving a stable window suitable for reactor parameters. For example, the energy of the trapped particles can be chosen so that the upper bound on γ_I approximates the Troyan limit. Further, if a sawtooth cycle were desirable (e.g., for impurity ejection), it could be produced by a temporary lowering of the trapped-particle density.

We have examined the solution of Eq. (1) numerically for analytic and Monte Carlo–generated distributions. Results are shown in Figs. 1 and 2 for a slowing-down and a Maxwellian distribution. Parameters were chosen to approximate a hydrogen minority species in JET. A root finding procedure with a Stokes plot[6] was used to follow unstable roots. Shown in Fig. 1 are the complex frequencies, and the growth rates, as a function of trapped-particle density for an approximate JET equilibrium with $R=296$ cm, for two different values of the magnetic Reynolds number, $S=10^6$ and 10^7, and $\gamma_I = 1.4\times10^4$ sec^{-1}. Both the resistive internal kink branch and the fishbone branch are shown. The trapped-particle density n ranges from zero to 2.3×10^{11}/cm^3. The particles were taken to be a slowing-down distribution with an average energy of 700 keV. The toroidal field was $B=24$ kG and the averaged trapped-particle precession rate was $\langle\omega_d\rangle \approx 1.3\times10^5$/sec. The shear Alfvén frequency was $\omega_A = 2\times10^6$/sec, and

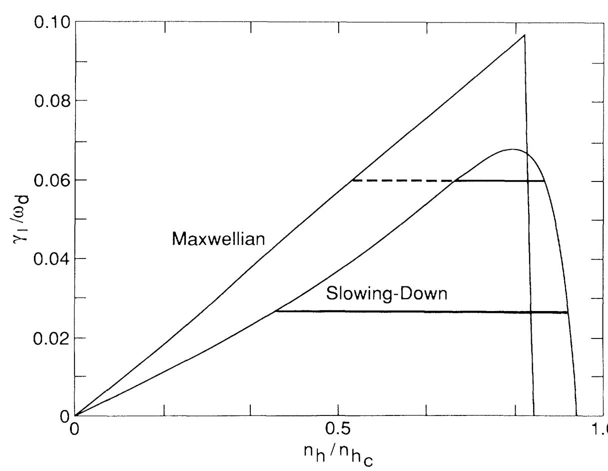

FIG. 2. The stable domain in the (γ_I, n) plane, for JET parameters, with both a slowing-down and a Maxwellian distribution function.

the diamagnetic frequencies were $\hat{\omega}_{*e} = -3\times 10^4$/sec and $\omega_{*i} = 2\times 10^4$/sec.

As seen, the $S=10^7$ case is almost ideal, with $\gamma \approx \gamma_I$ and $\omega_r \approx \omega_{*i}/2$ at $n=0$. There is a range of trapped-particle density in which both the kink-mode branch and the fishbone branch are stable. For smaller values of S the resistive modification of the kink branch makes it more unstable and eliminates this stable gap. For $S=10^6$, the decrease of the growth rate with increasing hot particle density, results in values of $\Lambda \approx 1$. The behavior for large β_h is then dominated by the small argument of the Γ functions, and the growth rate only asymptotically approaches zero for $\beta_h \rightarrow \infty$, as found previously,[1] and complete stabilization does not occur. The fishbone branch is not noticeably modified by resistive effects, because although the growth rates of the two branches are comparable, the real frequency of the fishbone branch is much larger, giving $\Lambda \gg 1$. In Fig. 2 are shown the stability domains, again for JET parameters, for slowing-down and Maxwellian distributions. Both the ideal internal kink and the fishbone are stable inside the triangular domains. The upper stability limit for the density is given approximately by

$$\frac{\beta_h \omega_A}{\epsilon \omega_d} = \begin{cases} 1, & \text{Maxwellian,} \\ \frac{1}{3}, & \text{slowing down,} \end{cases} \tag{15}$$

where ω_d is the toroidal precession rate of a particle of energy $E_{\max}$ (slowing down) or T (Maxwellian). The triangular stability domains in the (β_h, γ_I) plane are thus seen to be approximately equal for the two distributions if $E_{\max} = 3T$. For Maxwellian with $T=700$ keV, the upper critical density n_{hc} is 2×10^{11} cm^{-3}. The Maxwellian is the appropriate distribution to compare to the JET sawtooth-free discharges. Within the triangular domain, but below a horizontal line whose position depends on S, the resistive kink mode is weakly unstable.[1] The horizontal lines shown correspond to $S=4\times 10^8$, for the Maxwellian and $S=10^7$ and, at a lower value of γ_I, $S=10^8$, for the slowing-down distribution.

In conclusion, we find that for typical JET parameters there exists a domain of hot-trapped-particle density in which both the sawtooth mode and the fishbone mode are stable. This stable domain requires that the plasma be ideally unstable to the internal kink mode ($\gamma_I > 0$), i.e., operation at fairly high plasma β, and that the trapped-particle population have a sufficiently high energy ($\omega_d \gg \omega_{*i}$). This domain of stable operation appears to be suitable for use in a fusion reactor.

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