Planar-Transverse Amplitude-Phase Pattern in Nonelastic Reactions

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(Received 7 November 1988)

The first evidence is presented that the phase pattern of the planar-transverse optimal reaction amplitudes found previously for elastic-scattering strong-interaction reactions also holds for nonelastic reactions. The pattern is observed in the reaction $p+p \rightarrow d+\pi$ in the energy range between 300 and 800 MeV.

PACS numbers: 13.88.+e, 13.75.Cs, 25.10.+s

The amplitude analysis of strong-interaction reactions in terms of the optimal formalism[1] has indicated that the amplitudes in the planar-transverse system have a strong tendency to be pure real or pure imaginary with respect to each other.

The optimal formalism describes reactions in terms of a choice of basis for the spin space in which the relationship between the bilinear products of amplitudes and the observables is as close to diagonal as it is compatible with Hermiticity requirements. The formalism contains an infinite set of representations which differ from each other by the quantization directions of the particles in the reaction. The planar-transverse system is the one in which the quantization direction of each particle is in the reaction plane and perpendicular to the helicity direction.

So far this pattern was observed in all elastic-scattering reactions in which sufficient data were available to be able to make an amplitude analysis. Thus the evidence comes from elastic nucleon-nucleon scattering, elastic pion-nucleon scattering, and elastic pion-deuteron scattering.[2]

For each of these reactions, the pattern has been observed in the entire range of kinematic variables for which sufficient polarization data are available to make such an analysis. As new complete sets of data are generated on such reactions (such as the impending release of data from Saclay on elastic proton-proton scattering in the low-GeV region or data on *p-d* elastic scattering from the University of California, Los Angeles), efforts are continuing to explore the limits of validity of this pattern.

In this Letter we want to report an extension of the validity of this pattern along a different parameter, namely whether the pattern also holds for nonelastic reactions. In particular, we analyzed recent data[3] on the reaction $p+p \rightarrow d+\pi$ in the range of 300–800 MeV. The set of measurements allows a determination of the six reaction amplitudes. While the direct determination of amplitudes from the data may be most efficacious in some other optimal representation (e.g., the transversity frame), once the determination is completed a simple linear transformation yields the amplitudes in any other optimal frame and hence also in the planar-transverse frame. The planar-transverse amplitudes used here were constructed from the phase-shift analysis of the data.

Some sample amplitudes arising from the above procedure are given in Table I. In addition, data on all the amplitudes are shown in Figs. 1–4, in which the histogram is given of the phase differences among the six reaction amplitudes for $p+p \rightarrow d+\pi$ in the entire angular range and in four energy ranges in which these new data are available. The data can be thus aggregated because more detailed checks indicate that the pattern also tends to hold at each individual energy and angle in the measured range.

The figures indicate a strong tendency for the relative phases to be either 0° or 180°. Such a property is the same type as the one observed for elastic-scattering reactions. The tendency is very pronounced at 330 MeV and in the range between 600 and 800 MeV, while in the range of 400–578 MeV the pattern, although still discernable, appears to have a considerable background.

Though the peaks are very pronounced, they have considerable widths. One needs to remember, however, that the determination of the phases of reaction amplitudes (especially if there are a fairly large number of them) involves considerable uncertainties. The magnitude of these uncertainties depends, of course, on the magnitude of the experimental errors in the primary data. For a given size of these experimental errors, however, the determination, in a given formalism, of the magnitudes of the amplitudes from the experimental data can be done (compared to the determination of the phase

TABLE I. Some sample planar-transverse amplitudes for the reaction discussed in this paper, at selected energies and angles. The definition of the amplitudes are as follows: $A=(+1,+,+)$, $B=(0,+,+)$, $C=(-1,+,+)$, $D=(0,-,+)$, $E=(+1,-,+)$, and $F=(+1,+,-)$. The first argument in the parentheses is the spin projection of the deuteron, and the other two give the signs of the spin projection of the two protons, all in the planar-transverse direction. The normalization of the amplitudes (which, for the purposes of this paper, is irrelevant) is such that the totally unpolarized differential cross section times 1600π is the sum of the absolute value squares of the six amplitudes. The upper number is the magnitude, and the lower number the phase angle in degrees.

			330	MeV		
REACTION	A	B	C	D	E	F
ANGLE	1.63	2.73	2.09	7.84	3.65	7.49
30	217	343	170	141	183	183
	1.99	0.96	2.38	7.69	3.35	1.69
60	217	248	334	170	343	237
	1.98		1.98		6.12	2.96
90	221		320		357	42
	2.38	0.96	1.99	7.69	3.35	1.69
120	207	2 93	294	11	343	237
	2.09	2.73	1.63	7.84	3.65	7.49
150	198	294	333	11	141	183
			451	MeV		
REACTION	A	B	C	D	E	F
ANGLE	3.11	2.76	9.45	21.71	11.49	17.99
30	213	247	353	153	131	165
	2.62	5.4	5.99	20.05	6.66	2.77
60	72	14	341	151	0	259
	2.79		2.79		13.33	9.31
90	61		120		18	44
	5.99	5.42	2.57	20.05	6.66	2.77
120	200	167	109	29	0	259
	9.45	2.76	3.11	21.71	11.49	17.99
150	188	294	328	27	131	165
			650	MeV		
REACTION	A	B	C	D	E	F
ANGLE	7.64	8.76	19.6	25.96	14.95	19.13
30	179	72	39	113	284	121
	7.96	20.29	6.33	21.91	7.67	1.08
60	76	46	41	103	45	268
	11.45		11.45		14.26	8.55
90	52		129		76	78
	6.32	20.26	7.97	21.91	7.67	1.08
120	140	135	105	78	45	268
	19.56	8.76	7.64	25.96	14.95	19.13
150	142	109	2	68	283	121
			800	MEV		
REACTION	A	B	C	D	E	F
ANGLE	3.92	8.22	13.72	14.61	10.64	12.45
30	125	249	260	260	315	293
	6.61	15.86	4.51	11.83	2.59	2.29
60	239	253	239	307	88	21
	9.48		9.48		7.44	4.54
90	249		292		221	252
	4.51	15.86	6.62	11.83	2.6	2.29
120	302	288	302	235	89	20
	13.72	8.22	3.92	14.62	10.64	12.45
150	281	291	56	261	316	293

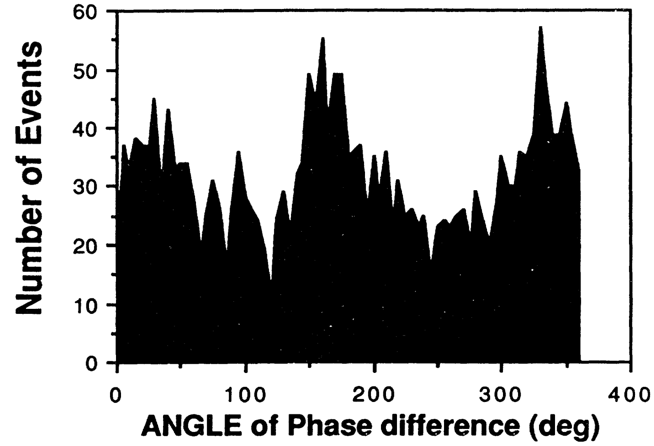

FIG. 1. Histogram of the phase differences among planar-transverse optimal amplitudes for the reaction $p+p \rightarrow d+\pi$ in the complete angular region and in the energy range between 300 and 800 MeV. Data at 330, 400, 451, 493, 578, 650, 700, 750, and 800 MeV have been used, and at each of these energies the data covered the angular range from 10° to 170°.

differences) with relatively small uncertainties, and without discrete ambiguities, since the magnitude squares are linear functions of the experimental observables. Furthermore, the determination of these magnitudes can be carried out on a subset of the experimental observables, without reference to the relative phases of these amplitudes.

In contrast, the determination of the relative phases can be carried out only in conjunction with the determination of the magnitudes and entails, compared to the accuracy of determination of the magnitudes, larger uncertainties as well as possible discrete ambiguities.

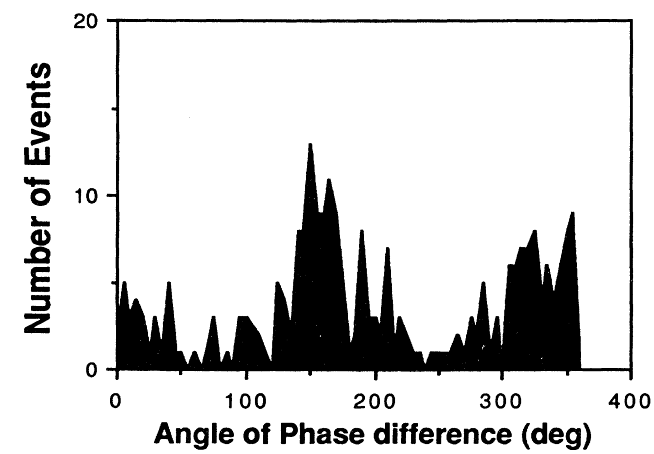

FIG. 2. The same histogram as in Fig. 1 but only at 330 MeV.

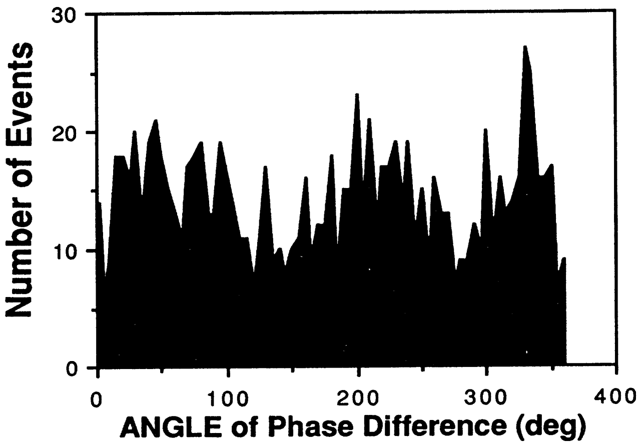

FIG. 3. The same histogram as in Fig. 1 but only for the energy range of 400–578 MeV.

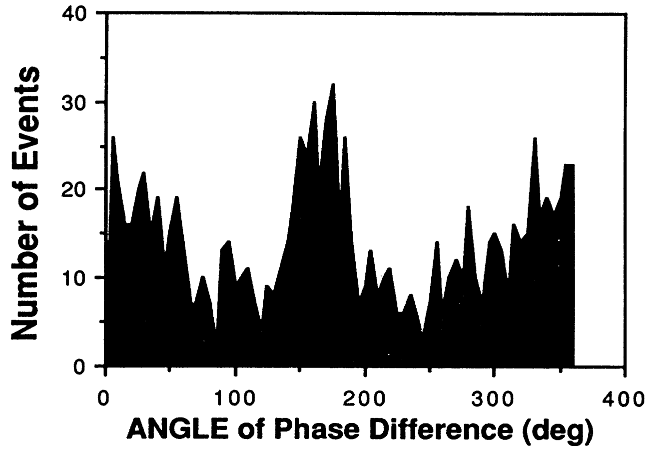

FIG. 4. The same histogram as in Fig. 1 but only for the energy range of 650–800 MeV.

As a consequence, it is possible that all or at least much of the width of the peaks in Fig. 1 comes from the uncertainty of determining the phase differences from the data. In particular, the diagonal elements of the error matrix on the phases of the planar-transverse amplitudes indicate a purely statistical uncertainty on the order of 5°. The actual uncertainty is larger in that correlated changes in the amplitude parameters can be effected by amounts larger than the magnitudes of the diagonal elements of the error matrix alone.

Furthermore, there are systematic errors, and these can be expected to increase the uncertainties by a factor on the order of 2. Finally, in computing the differences of phases, one increases the uncertainties further by something on the order of 1.4. In summary then, one can say that the uncertainties on the individual phase differences can be estimated to be on the order of 15°–20°.

It is of course important to ascertain whether such a pattern can be a consequence of some symmetry or dynamical mechanism already known to us. So far no such connection has been found. If the efforts along such lines remain fruitless, it would also be interesting to construct some other symmetry or dynamical mechanism that does yield such a pattern. This has not been accomplished so far either.

While there is no explanation at this time for the pattern, it is possible to correlate the pattern with other phenomenological descriptions. In particular, at 330 MeV one might expect some such pattern because there the single NN angular momentum state $^3P_1$ dominates the dynamics.

In the range of 600–800 MeV, on the other hand, one may expect the dominant contribution coming from a Δ intermediate state which may also produce such a pattern. Since in the range between 400 and 578 MeV the Δ intermediate-state mechanism is competing with an intermediate state consisting of a nucleon and a pion-nucleon system in an S wave, one might expect that the pattern is somewhat disturbed. These and other possibilities are now being studied in greater detail.

Finally, it would be important to have more reactions and in more extended kinematic domains so as to test the bounds of validity of this pattern. Such an experimental assignment is, however, an arduous one and hence it is unlikely that the present data set indicating this pattern will be greatly enlarged within a short period of time.

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[2]Various indications regarding the pattern appear in G. R. Goldstein and M. J. Moravcsik, Phys. Lett. **102B**, 189 (1981); M. J. Moravcsik, F. Arash, and G. R. Goldstein, Phys. Rev. D **31**, 2360 (1985); F. Arash, M. J. Moravcsik, and G. R. Goldstein, Int. J. Mod. Phys. A **2**, 739 (1987); G. R. Goldstein and M. J. Moravcsik, Phys. Lett. B **199**, 563 (1987); F. Arash, M. J. Moravcsik, and Gary R. Goldstein, Mod. Phys. Lett. A (to be published).

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